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Quark models of hadrons and issues in quark dynamics

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1.1 Chromostatics

The discovery of quarks in inelastic electron scattering experiments, following their hypothesized existence to explain the spectroscopy of hadrons, led rapidly to the quantum chromodynamic (QCD) theory and the Standard Model, which has underpinned particle physics for three decades. Today, all known hadrons contain quarks and/or antiquarks.

The QCD Lagrangian implies that gluons also exist, and the data for inelastic scattering at high energy and large momentum transfer confirm this. What is not yet established is the role that gluons play at low energies in the strong interaction regime characteristic of hadron spectroscopy. QCD implies that there exist 'glueballs', containing no quarks or antiquarks, and also quark–gluon hybrids. The electromagnetic production of hybrids is one of the aims of JLab. Glueballs, on the other hand, are not expected to have direct affinity for electromagnetic interactions; hence hadroproduction of a meson that has suppressed electromagnetic coupling is one of the ways that such states might be identified.

Quarks are fermions with spin $\frac{1}{2}$ and baryon number $\frac{1}{3}$. A baryon, with half-integer spin, thus consists of an odd number of quarks (q) and/or antiquarks ($\bar{q}$), with a net excess of three quarks. Mesons are bosons with baryon number zero, and so must contain the same number of quarks and antiquarks.

The simplest configuration to make a baryon is thus three quarks, qqq; a meson most simply is $q\bar{q}$. Within this hypothesis well over two hundred hadrons listed by the Particle Data Group (PDG) [1] can be described. The question of whether there are hadrons whose basic constitution is more complicated than these, such as mesons made of $qq\bar{q}\bar{q}$ or baryons made of $qqqq\bar{q}$, is an active area of research,

Electromagnetic Interactions and Hadronic Structure, eds Frank Close, Sandy Donnachie and Graham Shaw. Published by Cambridge University Press.

which we shall summarize later. First we examine what property of the attractive forces causes such combinations to occur and then discuss how the multitude of hadrons are described.

The fundamental theory of the strong forces between quarks is QCD. The details of this theory and rules of calculation may be found in dedicated texts such as [2]. A quark carries any of three colours – which we label *RBG*. They are the charges that are the source of the force between quarks. The rules of attraction and repulsion are akin to those of electrostatics where like repel and unlike attract. Associate positive charges with quarks and negative with antiquarks. The attraction of plus and minus then naturally leads to the $q\bar{q}$ configurations, the mesons, for which the colour charges have counterbalanced.

In quantum electrodynamics, QED, the electromagnetic force is transmitted by photons; analogously, in QCD the forces between quarks are transmitted by gluons. This far is analogous to the formation of electrically neutral atoms. The novel feature arises from the three different colour charges. Two identical colours repel one another but two different, namely *RG*, *RB* or *BG*, can mutually attract. A third quark can be mutually attracted to the initial pair so long as its colour differs from that pair. This leads to attraction between three different colours: *RBG*. A fourth quark must carry the same colour charge as one that is already there and will be repelled by that, meanwhile being attracted to the dissimilar pair.

The above pedagogic illustration needs better specification. The rules of attraction and repulsion depend on the symmetry of the pair under interchange. Thus symmetric combinations repel, antisymmetric attract. Two identical colours, being indistinguishable, are trivially symmetric. Two differing colours can be either symmetric or antisymmetric:

$$[RB]_S \equiv \sqrt{\tfrac{1}{2}}(RB + BR), \tag{1.1}$$

$$[RB]_A \equiv \sqrt{\tfrac{1}{2}}(RB - BR). \tag{1.2}$$

Thus any pair of quarks in a baryon is in an antisymmetric symmetry state for the saturation of the attractive forces. The full wave function for the colour of a three-quark baryon is thus

$$\sqrt{\tfrac{1}{6}}\,((RB - BR)G + (BG - GB)R + (GR - RG)B)\,. \tag{1.3}$$

The three colours form the basic **3** representation of SU(3); the three 'negative' colours of antiquarks are then a $\bar{\mathbf{3}}$. The rules of combining representations give

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8};\ \ \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}. \tag{1.4}$$

It is the colour-singlet representations that are the formally correct SU(3) expressions of the above heuristic combinations. Hadrons are thus colour singlets of colour SU(3). Building a relativistic quantum gauge field theory of colour SU(3) leads to QCD. The baryons are thus in the antisymmetric representation of colour SU(3). The above argument shows how this is a natural consequence of the attractive colour forces in chromostatics. The antisymmetry under interchange of colour labels, combined with the Pauli principle that requires fermions, such as quarks, to be antisymmetric under the exchange of all their quantum numbers, leads to essential constraints on the pattern of hadrons and their properties.

1.2 Mesons as bound states from $b\bar{b}$ to light flavours

For heavy-flavour mesons, such as $c\bar{c}$ and $b\bar{b}$, a non-relativistic potential model description of meson spectroscopy is realized phenomenologically and may be justified theoretically. The ground state $1S$, the first excited state with orbital excitation $L = 1$, denoted by $1P$, and the radial excitation of the S-state, $2S$ with the $1D$ level being slightly higher than this, are qualitatively in accord with the pattern of a linear potential $V(r) = Kr$. Here r is the radial separation of the q and $\bar{q}$, and K (known as the string tension) has dimensions of (Energy)^2. Empirically $K \sim 1$ GeV/fm ~ 0.18 GeV^2.

The qualitative features of the spectrum of states survive for all flavours (with some exceptions, such as 0^{++}, which we discuss later). This has enabled a successful phenomenology to be built in applying the non-relativistic constituent-quark model to light flavours even though the a priori theoretical justification for this remains unproven. A widely-used approach has been to approximate the dynamics to that of the harmonic oscillator, with Gaussian wave functions of form $\exp(-r^2\beta_M^2/2)$ multiplied by the appropriate polynomials and β treated as a variational parameter in the Hamiltonian H for each of the $1S$, $1P$, $2S$, $1D$, ... states.

There are also spin-dependent energy shifts among states with the same overall L, the forms of which phenomenologically share features with those generated by the Fermi–Breit Hamiltonian in QED. This is widely interpreted as evidence for analogous chromomagnetic effects in QCD.

The $q\bar{q}$ picture is only literally true for states that are stable. If the number of colours $N_c \to \infty$, the amplitudes for $q\bar{q} \to q\bar{q} + q\bar{q} \sim 1/N_c \to 0$. In the real world $N_c = 3$ and the coupling to meson channels must distort the simple $q\bar{q}$ picture. A particular example occurs for $c\bar{c}$ where the $D\bar{D}$ thresholds cause non-trivial admixtures of $c\bar{c}u\bar{u}$ and $c\bar{c}d\bar{d}$ in the 'primitive' $c\bar{c}$ wave functions of the $J^{PC} = (0, 1, 2)^{++}$ χ states [3–5] and the 1^{--} $\psi(3685)$ which are all below the $D\bar{D}$ threshold. In the simple $c\bar{c}$ non-relativistic-potential picture, electromagnetic transitions among these states, such as

$$\psi(3685) \to \gamma\chi_J; \chi_J \to \gamma\psi, \tag{1.5}$$

are electric dipole, E1, transitions. In this case if $\psi(3685) \equiv {}^3S_1$, then apart from phase-space effects the relative widths $\psi(3685) \to \gamma\chi_J$ would be in the ratio $2^+ : 1^+ : 0^+ = 5 : 3 : 1$, whereas for a ${}^3D_1$ initial state these ratios become $2^+ : 1^+ : 0^+ = 20 : 15 : 1$ [6,7]. While data are qualitatively in accord with the predictions of such a model for the ${}^3S_1$ case, future precision data can reveal the presence of relativistic effects, ${}^3D_1$ components and of $D\bar{D}$ mixing in the $c\bar{c}$ states. This is a particular example where electromagnetic transitions can give precision information on hadron wave functions and dynamics. More discussion of this can be found in chapter 4.

An example of this is the conundrum of the state $X(3872)$ [8]. This charmonium state is degenerate with the neutral $D\bar{D}^*$ threshold and as such is suspected of having $u\bar{u}$ admixture in its $c\bar{c}$ wave function. The $u\bar{u}$ and absence of $d\bar{d}$ will lead to significant isospin violation in its decays. If this state is 1^{++}, then one may anticipate such a light flavour asymmetry at a small level in the wave function of the $\chi_1(3500)$. High-statistics data on the hadronic decays of the $\chi_1(3500)$ could reveal if this is the case.

Such subtle effects could occur more widely in the charmonium states. The basic idea is as follows. The mass difference between $d\bar{d}$ and $u\bar{u}$, although small and widely neglected in analysis, can have measurable effects when the dynamics involves the differences among various energies. For example, the mixing of $d\bar{d}$ and $u\bar{u}$ in the χ states may be driven by $M(D\bar{D}) - M(\chi)$. For $u\bar{u}$ mixing into the χ_0 for example, it will be the neutral threshold $D^0\bar{D}^0$ that is relevant, while for the $d\bar{d}$ mixing it is D^+D^-. The difference in the energy gaps in the two cases is $\sim$5%; for states that are nearer the threshold such flavour-dependent effects can become highly significant. In the case of the $c\bar{c}$ state $X(3872)$ [8] one has almost perfect degeneracy with the $D^0\bar{D}^{*0}$ threshold such that admixture of $u\bar{u}$ is expected to dominate $d\bar{d}$ utterly [9,10].

High-statistics data on χ hadronic decays should be studied to see if there is an asymmetry between the neutral and charged particles in the final states, which would show a failure of simple isoscalar decay. These data can be taken in the e^+e^- facilities CLEO-c at Cornell University and BES in Beijing.

These mixing effects may be studied in precision data for heavy flavours and the resulting insights applied to light flavours. In the latter we already have qualitative understanding of where the limits of the $q\bar{q}$ model occur. The strategy is to quantify these en route to a more mature dynamical picture of the light flavoured hadrons. This has interest in its own right but also is needed when building Monte Carlo models for the decays of B and D heavy flavours into channels involving light hadrons.

1.2.1 Linear plus Coulomb potential

The phenomenological picture of the linear potential deduced from the pattern of the energy levels gives insight into the nature of the force fields acting on the constituents. As the force field $\sim dV(r)/dr$, then $V(r) \sim Kr$ implies the field is constant as a function of the distance r between the colour sources. This immediately contrasts with the behaviour in QED where $V(r) \sim 1/r$ implies that $E(r) \sim 1/r^2$, whereby the fields spread freely into all directions of three-dimensional space. The message for QCD is that the field lines concentrate along the line connecting the colour source q and sink $\bar{q}$. Thus the 'linear' potential is aptly named!

This is in accord with the picture that emerges from lattice QCD [11,12]. The potential is predicted to be linear and by implication the field lines collectively form a tube-like configuration. This has led to 'flux-tube models' [13,14] of $q\bar{q}$. These models underpin the potential, which is all that is needed for many calculations. However, they and the lattice computations also imply that the flux-tube provides an independent degree of freedom, which can be excited. The resulting states that form when the flux-tube is excited in the presence of $q\bar{q}$ are known as 'hybrids' [13–18]. We consider their dynamics later.

For the conventional $q\bar{q}$ states one views the flux-tube as the source of the linear potential, at least at distances comparable to the confinement scale ($r \sim 1$ fm). At short range, say $r \sim 0.1$ fm, QCD theory implies that the colour force is transmitted by gluons, which act independently of one another analogously to the way that photons behave in QED. This gives a Coulombic contribution to the potential as $r \to 0$. The exchange of a single gluon gives perturbative corrections to the simple potential, generating analogues of the spin-dependent hyperfine shifts that are familiar in QED.

The effective potential arising from QCD is thus taken as [3,7]

$$V(r) = Kr - \frac{4}{3}\frac{\alpha_s}{r} + C \tag{1.6}$$

(the factor $\frac{4}{3}$ is a normalization factor arising from the SU(3) colour matrices at the quark vertices). In calculations it is often useful to make a Gaussian approximation to the wave functions, which may be found variationally from the Hamiltonian

$$H = \frac{p^2}{\mu} + Kr - \frac{4}{3}\frac{\alpha_s}{r} + C, \tag{1.7}$$

where $\mu = m_q m_{\bar{q}}/(m_q + m_{\bar{q}})$ is the reduced mass, with standard quark-model parameters $m_q = 0.33$ GeV for u and d quarks and 0.45 GeV for s quarks, $K = 0.18$ GeV^2 and $\alpha_s \sim 0.5$.

Not only are the patterns of the energy levels preserved as one goes from heavy to lighter flavours, but many of the energy gaps are quantitatively approximately

Cambridge University Press
978-0-521-84420-8 - Electromagnetic Interactions and Hadronic Structure
Edited by Frank Close, Sandy Donnachie and Graham Shaw
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independent of flavour mass. Thus the gap between the 1^{--} $q\bar{q}$ in the $1S$ to $2S$ levels is 563 MeV for $b\bar{b}$ ($\Upsilon(10023) - \Upsilon(9460)$) and 589 MeV for $c\bar{c}$ ($\psi(3686) - \psi(3097)$). For the light flavours the analogous gap between $\rho(1460)$ and $\rho(770)$ is only some 10% larger while the absolute mass scales have changed by over an order of magnitude.

For a constituent of mass m in a potential that behaves as $V(r) \sim r^N$, this gap would vary as $m^{-N/(N+2)}$, hence $\sim m$ for the Coulomb potential and $\sim m^{-1/3}$ for linear. Mass independence would ensue for a potential $V(r) \sim \ln r$, which is approximately how the linear + Coulomb appears in the region of r most sensitive to the bound states.

However, there are also clear mass-dependent effects, notably in the splittings between the ${}^3S_1 - {}^1S_0$ levels ($1^{--} - 0^{-+}$). These vary from over 600 MeV for $\rho(770) - \pi(140)$, through 400 MeV for $K^*(890) - K(490)$ to significantly less for $\psi(3095) - \eta_c(2980)$ (we adopt the naming conventions for particles of the PDG [1]).

1.2.2 Hyperfine shifts

Although the mass gaps between successive orbital excitation levels of the effective potential are empirically approximately flavour-independent, there is a marked flavour dependence of the splitting between the S-wave states of differing total spin. Specifically this concerns the 1^- and 0^- states of $q\bar{q}$ and the $J^P = \frac{3}{2}^+$ and $\frac{1}{2}^+$ baryons.

Early evidence that mesons and baryons are made of the same quarks was provided by the remarkable successes of the Sakharov–Zeldovich constituent quark model [19], in which static properties and low-lying excitations of both mesons and baryons are described as simple composites of asymptotically free quasiparticles with a flavour-dependent linear mass term and hyperfine interaction,

$$M = \sum_i m_i + \sum_{i>j} \frac{\sigma_i \cdot \sigma_j}{m_i \cdot m_j} \cdot v^{hyp}, \tag{1.8}$$

where m_i is the effective mass of quark i, σ_i is a quark spin operator and v_{ij}^{hyp} is a hyperfine interaction. This form has analogy with the source of hyperfine splitting in atomic hydrogen and suggests for hadrons that there is a QCD source in single-gluon exchange. As in the QED case, this (chromo)magnetic interaction is inversely proportional to the constituent masses.

In QCD the colour couplings are proportional to $\lambda_i \cdot \lambda_j$, with λ the SU(3) matrices [20]. The spin-dependent $\sigma_i \cdot \sigma_j$ term then causes the lowest-spin combinations to be further attracted, their high-spin analogues suffering a relative repulsion. This leads to a strong chromomagnetic attraction between a u and a d flavour when the ud diquark is in the $\bar{\mathbf{3}}$ of the colour SU(3) and in the $\bar{\mathbf{3}}$ of the flavour SU(3) and has $I = 0, S = 0$.

The relative magnitudes of the spin-dependent shifts for a common set of flavours depend on the net colour of the interacting constituent pair. Since the colour expectation values are

$$\langle\lambda_1 \cdot \lambda_2\rangle_{qq(\bar{\mathbf{3}})} = \tfrac{1}{2}\langle\lambda_1 \cdot \lambda_2\rangle_{q\bar{q}(\mathbf{1})}, \tag{1.9}$$

the relative shifts for colour-singlet mesons and baryons are

$$J = 0 \to -3; J = 1 \to +1; J = \tfrac{3}{2} \to +\tfrac{3}{2}; J = \tfrac{1}{2} \to -\tfrac{3}{2}, \tag{1.10}$$

whence $m(\Delta) > m(N)$ and $m(\rho) > m(\pi)$. These need to be rescaled by the appropriate masses following (1.8) when comparing the flavour dependence of the energy shifts [20,21], such as for $m(\Sigma^*) > m(\Sigma)$ and $m(K^*) > m(K)$.

These spin- and flavour-dependent energy shifts are manifested not only in the different masses of hadrons with different spins, but also cause the splitting of $\Sigma - \Lambda$ baryons. This is because the ud in $\Sigma_Q(Qud)$ have, by the Pauli correlation, $S = 1$ and are hence pushed up in energy relative to their counterparts in the $\Lambda_Q(Qud)$, which are in $S = 0$. Details are in [21].

These attractive forces can generate correlations among pairs of quarks and/or antiquarks, which are manifested as spin-dependent effects in inelastic scattering and in the appearance of colour-singlet hadrons with content $qq\bar{q}\bar{q}$ or $qqqq\bar{q}$. These attractive correlations arise when a qq or $\bar{q}\bar{q}$ are antisymmetric in each quantum number, thus qq in colour $\bar{\mathbf{3}}$ ($\bar{q}\bar{q}$ in $\mathbf{3}$), spin-zero and flavour singlet. This has significant implications for the structure of mesons with $J^{PC} = 0^{++}$ below 1 GeV [22,23]. It also can lead to the possibility of 'pentaquark' states $(ud)(ud)\bar{Q}$, where the (ud) denotes a correlated pair and the antiquark has a distinct flavour $Q = s, c, b$. A particular example of the latter would be $udud\bar{s}$, which would form a baryon with positive strangeness, which is thus manifestly exotic in that it cannot be formed from any combination of qqq.

1.3 Flavour mixings

Any flavour of quark Q and its antiquark $\bar{Q}$ when attracted together form a state with no net flavour, in particular having zero electric charge and strangeness. How then are we to tell what combination of flavours occurs in any given physical state? We begin with some theoretical expectations.

Consider two heavy flavours, say $b\bar{b}$ and $c\bar{c}$. The mass matrix will have on its diagonal $2m_b$ and $2m_c$, and if this was the whole story these would be the physical eigenstates. But there is also the possibility that these neutral states can annihilate through some common channel, for example gluons. Let the strength of this annihilation be A. It may connect $b\bar{b}$ to $c\bar{c}$ and it can also connect either of these to itself. The matrix thus becomes

$$\begin{pmatrix} 2m_b + A & A \\ A & 2m_c + A \end{pmatrix}. \tag{1.11}$$

The eigenstates depend on the relative size of $(m_b - m_c)/A$. If this is large the eigenstates are $b\bar{b}$ and $c\bar{c}$; this is indeed the case if we identify the c, b with the physical charm and bottom quarks where the vector mesons, for example, are the $\psi(c\bar{c})$ and $\Upsilon(b\bar{b})$. If it is small, which would be the case if we chose u,d instead of c,b, they tend to the equally mixed states $(u\bar{u} \pm d\bar{d})/\sqrt{2}$, which are the familiar isospin eigenstates. The $I = 1$ state $(u\bar{u} - d\bar{d})/\sqrt{2}$ decouples from the annihilation A channel, while the $I = 0$ $(u\bar{u} + d\bar{d})/\sqrt{2}$ couples with an enhanced amplitude $\sqrt{2}$ times that of an individual flavour and the mass gap is proportional to A.

Thus we have a qualitative understanding of why the $c\bar{c}$ and $b\bar{b}$ spectroscopies are distinct (or 'ideal') while their u,d counterparts are mixed into the 'isospin' basis. Now consider the latter systems but in the presence of the strange quark, which can form $s\bar{s}$ states.

Consider the limit where $m_u \sim m_d$ but $m_s - m_d >> A$. The eigenstates will then be the same $u\bar{u} \pm d\bar{d}$ as above with the third state being $s\bar{s}$. This is realised in the vector mesons where the isoscalar mesons are $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\phi = s\bar{s}$. The evidence for this will be described shortly; the implication of it is that A is small for the vector meson channel [20].

Now consider the limit where $m_s > m_d$ and $A > m_s - m_d$. The eigenstates are now orthogonal mixtures of $u\bar{u} + d\bar{d}$ and $s\bar{s}$. This is what is observed for the 0^{-+} mesons where $\eta(550)$ and $\eta'(960)$ are mixtures of these flavours. Specifically, it was found in [20] that $A(0^{-+})$ is in the range 80−600 MeV while $A(1^{--})$ is 5–7 MeV, in both cases there being a slight hint that the magnitude falls with increasing energy. The annihilation contribution thus seems to be much smaller for the vector mesons than for the pseudoscalar and a question for dynamics is why?

Determining which J^{PC} states are 'ideal' (that is like the 1^{--}) and which are strongly mixed is one of the unresolved issues in the spectroscopy of light flavours. Answering it may help to identify the dynamics that controls this mixing. Electromagnetic interactions can play a significant role in addressing these issues as we now illustrate by showing how they have already been seminal in the case of the 1^{--} and 2^{++} multiplets, at least.

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Cambridge University Press
978-0-521-84420-8 - Electromagnetic Interactions and Hadronic Structure
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1.3.1 The 1^{--} *and* 2^{++} *nonets*

That the vector and tensor multiplets are near ideal can be seen from the pattern of their masses. One $I = 0$ state has mass similar to that of the isovector, while the other $I = 0(s\bar{s})$ is heavier with the strange $K(u\bar{s})$ midway between them. As the $I = 1$ state contains only u and d flavours, this pattern suggests that the lighter isoscalar is $n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$, while the heavier is $s\bar{s}$. Examples of such nonets are

$$1^{--} : \rho(770) \sim \omega(780); K^*(890); \phi(1020), \tag{1.12}$$

$$2^{++} : a_2(1320) \sim f_2(1270); K^*(1430); f_2(1525). \tag{1.13}$$

This pattern of flavours is also confirmed by the strong decays, in the approximation that the dominant hadronic decay is driven by the creation of $q_k\bar{q}_k$ in the field lines between the original $q_i\bar{q}_j$ (where i,j,k denote the flavour labels) such that

$$q_i\bar{q}_j \to q_i\bar{q}_k q_k\bar{q}_j \to (q_i\bar{q}_k) + (q_k\bar{q}_j). \tag{1.14}$$

Thus $s\bar{s}$ can decay to $s\bar{u} + u\bar{s}$, which is $\equiv K\bar{K}$, but it does not decay to $\pi\pi$. This rule underpins the suppressed decays of the ϕ and $f_2(1525)$ to $\pi\pi$. The relative strengths of the electromagnetic couplings of these states also fit with this ideal picture.

For 1^{--} one has the direct coupling $q\bar{q}(1^{--}) \to \gamma \to e^+e^-$. Thus the leptonic widths, after phase-space corrections, give a measure of the flavour contents. This is discussed further in chapter 5. The amplitude is proportional to the sums of electric charges of the $q\bar{q}$ contents weighed by their amplitude and phases. Thus for the relative squared amplitudes

$$\rho(d\bar{d} - u\bar{u})/\sqrt{2} : \omega(u\bar{u} + d\bar{d})/\sqrt{2} : \phi(s\bar{s}) = 9 : 1 : 2, \tag{1.15}$$

which may be compared with their e^+e^- widths in keV

$$\Gamma^{e^+e^-}[\rho : \omega : \phi] = 6.8 \pm 0.11 : 0.60 \pm 0.02 : 1.28 \pm 0.02. \tag{1.16}$$

The differences in phase space are only small and so do not affect the arguments here. However, it is noticeable that the ratios seem to apply to the widths in that they hold also for the $\Gamma^{e^+e^-}(\psi(c\bar{c}); \Upsilon(b\bar{b}))$

$$\Gamma^{e^+e^-}[\psi(c\bar{c}) : \Upsilon(b\bar{b})] = 5.26 \pm 0.37 : 1.32 \pm 0.05, \tag{1.17}$$

which are experimentally in accord with $4\Gamma^{e^+e^-}(\phi(s\bar{s}))$ and $\Gamma^{e^+e^-}(\phi(s\bar{s}))$ respectively.

For future reference, it is useful to show these states in the **1–8** basis of $SU(3)_F$:

$$\omega_1 \equiv (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}; \quad \omega_8 \equiv (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6} \tag{1.18}$$

whereby (denoting the light quark component by $n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$)

$$\begin{aligned} \phi(s\bar{s}) &= \sqrt{\tfrac{1}{3}}\omega_1 - \sqrt{\tfrac{2}{3}}\omega_8, \\ \omega(n\bar{n}) &= \sqrt{\tfrac{2}{3}}\omega_1 + \sqrt{\tfrac{1}{3}}\omega_8. \end{aligned} \tag{1.19}$$

The electromagnetic couplings of the 2^{++} states also confirm their tendency towards ideal flavour states. Here the decays to $\gamma\gamma$ have amplitudes proportional to the sum of the squares of the electric charges of the quarks weighted by their relative phases. Thus for the relative squared amplitudes

$$a_2(d\bar{d} - u\bar{u})/\sqrt{2} : f_2(u\bar{u} + d\bar{d})/\sqrt{2} : f_2(s\bar{s}) = 9 : 25 : 2, \tag{1.20}$$

which may be compared with their $\gamma\gamma$ widths in keV

$$\begin{aligned} &\Gamma(a_2(1320) \to \gamma\gamma) : \Gamma(f_2(1270) \to \gamma\gamma) : \Gamma(f_2(1525) \to \gamma\gamma) \\ &\qquad = 100 \pm 8 : 261 \pm 30 : 9.3 \pm 1.5. \end{aligned} \tag{1.21}$$

The $a_2(1320) : f_2(1270)$ ratio is in excellent agreement with this; the $f_2(1525)$ is about a factor 2 smaller. The agreement between the mass-degenerate $a_2 : f_2$ states is in accord with ideal flavour states and then, if the heavier f_2 is $s\bar{s}$, its strange quark masses will suppress its magnetic contribution to the $\gamma\gamma$ amplitude and thus be consistent with the reduced strength.

To the extent that the vector mesons are ideal states, the radiative transitions of $C = +$ states $(C = +) \to \gamma V (= \rho : \omega : \phi)$ may be used to determine the flavour contents of the initial $C = +$ states (see chapter 4). As flavour is conserved in electromagnetic transitions to leading order, decays to $\gamma\rho$ weigh the $n\bar{n}$ component of the initial $C = +$ state, and those to $\gamma\phi$ weigh its $s\bar{s}$ component. This can be used as a further measure of the flavours of tensor mesons, where the reduced widths (phase space removed) would be expected to satisfy

$$\frac{\Gamma_R(f_2 \to \rho\gamma)}{\Gamma_R(f_2' \to \phi\gamma)} = \frac{9}{4}. \tag{1.22}$$

Empirically there is only an upper limit on these transitions. Obtaining their magnitudes is thus important, both as a check of this flavour filter and also for comparison with the analogous transition magnitudes for $f_{0,1} \to \gamma V$ as these can test the single-quark transition hypothesis for radiative transitions [24].

Complementary to this is the question of to what extent these ideal flavour states are realized for excited vector mesons such as $\rho(1460)$, $\rho(1700)$, $\omega(1420)$, $\omega(1650)$