#### BAROCLINIC TIDES Theoretical Modeling and Observational Evidence

Internal waves are water motions originating from density variations within the water body in which they propagate. When an oceanic tidal wave that is primarily active on the water surface impinges on the ocean shelf or passes a region with a seamount, this wave is modified by the topographic disruption. It is split into a less energetic surface wave and other internal modes with different wavelengths and propagation speeds. This cascading process, from the barotropic tides to the baroclinic components, eventually leads to the transformation of tidal energy into turbulence and heat. This is an important process for the dynamics of the lower ocean.

BAROCLINIC TIDES demonstrates how analytical and numerical methods can be used to study the generation and evolution of baroclinic tides and, by comparison with experiments and observational data, shows how internal waves can be distinguished and interpreted. Particular attention is paid to the investigation of strongly nonlinear solitary internal waves, which are generated by internal tidal waves at the final stage of their evolution. This book is intended for researchers and graduate students of physical oceanography, geophysical fluid dynamics, and hydroacoustics.

VASILIY VLASENKO received a Ph.D. from the the Marine Hydrophysical Institute of the National Academy of Sciences of Ukraine (Sevastopol) in 1987 and a Doctor of Sciences degree from the same Institute in 2000. Dr. Vlasenko was at the Institute of Mechanics of the Darmstadt University of Technology, Germany, from 1999 to 2003. He currently lectures in environmental modeling in the School of Earth, Ocean and Environmental Sciences at the University of Plymouth. Dr. Vlasenko is the co-author of three books and more than 70 papers on various aspects of baroclinic wave dynamics in the ocean. His research interests are in physical oceanography: internal gravity waves, coastal ocean dynamics, baroclinic tides, flow–topography interaction, and large-amplitude solitary internal waves.

NATALIYA STASHCHUK received a Ph.D. from the Marine Hydrophysical Institute of the National Academy of Sciences of Ukraine (Sevastopol) in 1990. From 1999 to 2004 she worked at the Institute of Mechanics of the Darmstadt University of Technology, Germany, and she came to the University of Plymouth in 2005.

Dr. Stashchuk has taken part in six cruises to the Mediterranean and the Atlantic Ocean and is the co-author of three books and more than 50 papers on oceanic internal waves. Her research interests are in mathematical modeling of oceanic processes: strait dynamics, internal waves, coastal ocean dynamics, baroclinic tides, and flow-topography interaction.

KOLUMBAN HUTTER received a Ph.D. from Cornell University, Ithaca, New York, in 1973 and has held the position of Professor of Mechanics at Darmstadt University of Technology, Germany, since 1987. His research interests are in geophysical fluid mechanics with applications in the dynamics of glaciers and ice sheets, the mechanics of granular materials, avalanching flows of snow, debris, and mud, and physical limnology and the foundations of continuum mechanics and thermodynamics. Professor Hutter is author or co-author of more than 340 papers and has written or edited 16 books; he also serves on the editorial boards of two journals. He was awarded the Max-Planck Prize of the Max-Planck Society and the Alexander von Humboldt Foundation (Germany) in 1994, the Alexander von Humboldt Prize of the Foundation of Polish Science in 1998, and the Seligman Crystal of the International Glaciological Society in 2003.

# **BAROCLINIC TIDES**

Theoretical Modeling and Observational Evidence

VASILIY VLASENKO SEOES, University of Plymouth

NATALIYA STASHCHUK SEOES, University of Plymouth

KOLUMBAN HUTTER

Technische Universität Darmstadt, Institut für Mechanik



> CAMBRIDGE UNIVERSITY PRESS Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

> > Cambridge University Press The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org Information on this title: www.cambridge.org/9780521843959

© V. Vlasenko, N. Stashchuk, and K. Hutter 2005

This book is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

Printed in the United Kingdom at the University Press, Cambridge

A record for this book is available from the British Library

Library of Congress in Publication data

ISBN-13 978-0-521-84395-9 ISBN-10 0-521-84395-2

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this book, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

To our parents

### Contents

	List	of tables	page x
	Prefe	xi	
	Ackn	Acknowledgements	
	List	XV	
	List	of abbreviations	xix
	Prea	mble	1
1	General background		7
	1.1	Introduction	7
	1.2	Governing equations: basic assumptions and	hypotheses 12
	1.3	Problem formulation: boundary and initial c	onditions 23
	1.4	Linear wave equation	28
	1.5	Linear boundary value problem and dispersi	on relation 32
		1.5.1 Formulation of the boundary value p	roblem 32
		1.5.2 Linear vertical mode analysis	34
	1.6	Nonlinear wave problem	40
2	Line	ar baroclinic tides over variable bottom topogr	aphy 44
	2.1	Analytical solution for "small" bottom featu	res 47
		2.1.1 Generation of internal waves by an o	scillating
		tidal flux	52
		2.1.2 Scattering of internal waves by a bot	tom obstacle 59
	2.2	Numerical model for large bottom obstacles	63
	2.3 Wave dynamics over oceanic ridges: applicability of the		bility of the
	perturbation method		
		2.3.1 Generation of internal waves	75
		2.3.2 Internal wave scattering	79
	2.4	Wave dynamics in slope-shelf regions	85
		2.4.1 Generation of baroclinic tides	85
		2.4.2 Transformation of baroclinic tides	88

Cambridge University Press
0521843952 - Baroclinic Tides: Theoretical Modeling and Observational Evidence
Vasiliy Vlasenko, Nataliya Stashchuk and Kolumban Hutter
Frontmatter
More information

vii	i		Contents	
	2.5	Internal	waves near steep bottom topography	94
	2.6	Internal	waves near the critical latitude	100
3	Com	oined effe	ct of horizontal density gradient and bottom	
	top	ography o	on the dynamics of linear baroclinic tides	104
	3.1	Semiana	alytical two-layer model	110
	3.2	Wave ch	naracteristics derived from the two-layer model	117
		3.2.1	Generation of internal waves	118
		3.2.2	Internal wave scattering	122
	3.3	Applica	bility of layer models	123
	3.4	Rieman	n method for a continuously stratified fluid	127
	3.5	Propaga	tion of internal waves through a frontal zone	133
	3.6	Generat	ion of baroclinic tides in the presence of a	
		fronta	al zone	142
4	Topo	graphic ge	eneration of nonlinear baroclinic tides	146
	4.1	Experim	nental evidence for nonlinear baroclinic tides	148
	4.2	Numerio	cal model for the description of nonlinear waves	156
	4.3	-	ive analysis of the excitation mechanism	163
	4.4	Generat	ion mechanism at low Froude numbers: baroclinic tides	165
	4.5	Influenc	e of the intensity of the tidal forcing and dissipation	168
	4.6	Critical	Froude numbers: excitement of unsteady lee waves	177
5		•	tages of baroclinic tides	182
	5.1	•	cal models for the evolution of baroclinic tides	182
	5.2	•	internal waves as manifestations of the	
			ent structure of baroclinic tides	188
			Long's equation	191
			First-order weakly nonlinear theory	192
			Second-order weakly nonlinear theory	197
	5.3		e of large-amplitude solitary internal waves	201
			Numerical model for stationary wave solutions	203
			Characteristics of large waves	204
			Observational evidence of large waves	211
	5.4		ion of large-amplitude SIWs with bottom topography	216
			Scenarios of wave-topography interaction	217
			Strong wave–topography interaction: breaking criterion	228
		5.4.3	Generation of high baroclinic modes by	
-	~		wave-topography interaction	244
6			chanism for different background conditions	260
	6.1		related to the rotation of the Earth	260
			Barents Sea Polar Front experiment	261
		6.1.2 l	Baroclinic tides	264

			Contents	ix
		6.1.3	Short internal waves	266
		6.1.4	Dependence on the rotation of the Earth	271
	6.2	Influe	nce of the fluid stratification	274
		6.2.1	Variation of the vertical position of the pycnocline	279
		6.2.2	Effect of horizontal density gradients	284
6.3 Bai			linic tides over steep bottom features: "mode" and	
		"be	am" approaches	289
6.4 Strong high-mode baroclinic response over steep bottom				
		top	ography	294
	6.5	Gener	ation mechanism at large Froude numbers	301
	6.6	Summ	nary of generation mechanism	304
7	Three	e-dimen	sional effects of baroclinic tides	308
	7.1	Influe	nce of wave refraction	310
		7.1.1	Observations of SIWs on the Portuguese Shelf	310
		7.1.2	Generation of waves at the Oporto Seamount	313
		7.1.3	Far-field generation from a shelf edge	315
	7.2	Baroc	linic tides in narrow channels and straits	318
		7.2.1	Modification of the model for straits	321
		7.2.2	Dynamics of internal waves in the Skarnsund Strait	323
		7.2.3	Residual currents produced by nonlinear waves	327
		7.2.4	Experiments on the dynamics of a passive admixture	329
	Refer	ences		335
	Index			348

### Tables

1.1	Important tides and their characteristics	page 9
2.1	Errors with which the perturbation method defines the maxima and positions of zeros of the function $a_{\xi}^{(1)}(l)$	77
2.2	Amplitudes $a_{\xi}^{(j)}$ (in meters) of the first ten internal modes in zone III	
	with $H_1 = 100$ m and various values of angle arctan $\gamma_0$	96
2.3	Amplitudes $a_{\xi}^{(j)}$ (in meters) of the first ten internal modes in zone I at	
	arctan $\gamma_0 = 44.1^\circ$ for different values of the shelf depth $H_1$	98
3.1	Parameters used in the numerical calculations of the wave and frontal	
	zone interaction	118
3.2	Maximum relative discrepancy $\delta_{max}$ for different values of width 2 <i>l</i>	
	and height $H_{\rm max}$ of the ridge depending on the pycnocline width $\Delta H_{\rm p}$	127
3.3	Normalized amplitudes $\bar{a}_i$ of the baroclinic modes generated in zone	
	III $(2l = 100 \text{ km})$ as a result of wave–front interaction in a basin of	
	constant depth ( $H_0 = 4 \text{ km}$ )	141
4.1	Regimes of internal wave generation	165
5.1	Values of the parameters characterizing three types of water	
	stratifications given by formula (5.17)	188
5.2	Values of several characteristic quantities of large-amplitude solitary	
	internal waves north and south of the Strait of Messina as measured	
	by the CTD chain and the ADCP, and as simulated by the numerical	
	model	214
5.3	Model parameters used in the numerical experiments on wave breaking	230

#### Preface

The scientific literature on baroclinic tides in the ocean is as abundant as is the general mathematical theory of internal gravity waves. The majority of research papers on this topic written since the late 1970s deal mainly with the attempts to provide a theoretical description of the vast number of experimental data obtained on the dynamics of baroclinic tides. Various analytical and semianalytical models were developed for the case of infinitesimal waves when the external tidal forcing is negligible. However, observations show that strong nonlinear baroclinic tides are the more common phenomena; they demand not only theoretical but also numerical methods to describe the wave processes adequately.

The idea of the book is to present the theoretical basis of baroclinic tides at all stages of their genesis and evolution (generation, transformation, dissipation), and to give, by comparison with experiments, advice and help regarding the theoretical interpretation and prognosis of baroclinic tides. Such an exhaustive and complete theoretical review does not exist at present, nor is there a scientific book which describes this phenomenon multilaterally. So we are intending to fill this gap.

A second reason for writing a book on this topic is to gather in one place our experience, which extends over more than 20 years, in investigating internal waves in general and baroclinic tides in particular. This includes the development of different linear and nonlinear mathematical models of internal tidal motions and the application of these to concrete oceanic situations. In so doing, we will report not only on our own research, but also on many other publications on the studied topic and subject them to a careful analysis. It is impossible to mention thousands of papers published on baroclinic tides, so our list of references is not exhaustive. We apologize to those colleagues whose references are not quoted.

This book is the outgrowth of our common research activity on the internal dynamics of the ocean and of lakes, in particular as exhibited by waves; this subject has been studied separately by Kolumban Hutter since the mid 1970s in Lakes Zurich, Lugano, and Constance, and by Vasiliy Vlasenko and Nataliya Stashchuk

xii

#### Preface

in the World Ocean since 1980. A lucky break brought us together in 1999 to work on research projects funded by the Deutsche Forschungsgemeinschaft on the propagation of waves in stratified rotating basins. Some topics in the present book are the direct result of our joint investigations. The research cooperation was so constructive and our achievements were so fruitful that after three years of collaboration we decided to summarize our work in a book. Vasiliy Vlasenko and Nataliya Stashchuk, who were the active researchers and less occupied by administrative duties, took on the burden of designing the first skeleton of the book's outline; they also were the sole writers of the first drafts of all the chapters. The role of Kolumban Hutter was to act as *advocatus diaboli* and to read, criticize and revise the various versions of the individual chapters. This process went back and forth at least three, more likely five, times. During approximately two years of writing, we learned to appreciate one another, and we now present a book that is a joint effort in which the first two authors carried the heavier burden than the third, but which all three of us agree with. We hope the product is fairly free from misconceptions and professional errors. However, we must ask the reader to bear with our English wording, a mixture of what two Ukrainians and a Swiss with their school English have tried to do. If the English is good, it is due to the copy-editorial staff of Cambridge University Press.

### Acknowledgements

Work on this book was started at the Darmstadt University of Technology (Germany) with financial support from the Deutsche Forschungsgemeinschaft, and was continued at a later stage at Plymouth University (UK), although many of the scientific results were obtained in collaboration with colleagues from the Marine Hydrophysical Institute of the National Academy of Science of the Ukraine (Sevastopol). The authors express their gratitude to the co-authors of joint publications, whose valuable contributions made this book possible. Vlasenko and Stashchuk feel deeply indebted to Professor Leonid Cherkesov, who was their first teacher in wave theory. They profited not only from the generous sharing of his ideas and wisdom, but also from his friendship, support, and encouragement.

In the course of the preparation for the book we visited each other, and the hospitality of Plymouth University and Darmstadt University of Technology is gratefully acknowleged. Special thanks are due to the Head of the School of Earth, Ocean and Environmental Sciences, Professor David Huntley, who generously helped and encouraged our efforts.

Most figures included in this book have previously been published by the authors in various scientific journals. We are grateful to copyright holders for granting their permission to reproduce their figures in the text: the American Meteorological Society (Figures 1.1, 1.7, 5.9, 5.10, 5.12–5.19, and 5.27–5.34); Brill Academic Publishers (Figures 3.10, 4.10, 4.11, and 6.18–6.22); Elsevier (Figures 2.22, 4.9, 4.14, 4.15, 4.21, 6.1–6.10, and 7.1–7.14); the European Geophysical Union (Figures 5.20–5.26 and 5.35–5.46); MAIK "Nauka/Interperiodica" (Figures 4.1–4.5, 4.7, 4.8, 4.16–4.18, 5.2, and 6.23–6.26); Nauka (Figures 2.6, 2.7, 3.1–3.3, 4.12, 4.15, 5.5, 5.6, and 6.14–6.17); and Transworld Research Network (Figures 2.3, 2.4, 2.6, 2.7, 2.21, 4.19, and 4.20).

xiv

#### Acknowledgements

We also acknowledge the work of three anonymous reviewers who took the time to read the manuscript and who have made many fruitful comments on the improvement of the book's contents. Finally, we thank the staff of Cambridge University Press for their fruitful cooperation and attention to detail through the various stages of the publishing process.

## Symbols

$A^{\mathrm{H}}, A^{\mathrm{V}}$	coefficients of horizontal and vertical turbulent viscosity
A(x, t)	profile of internal wave used in a weakly nonlinear theory
$a_j$	amplitude of stream function of <i>j</i> th mode generated in zone III
$a_{\xi}$	amplitude of vertical isopycnal displacement of solitary internal wave
$a_{\xi}^{(j)} = \frac{a_j k_j}{\sigma}$	amplitude of isopycnal displacement
$a_m^{ m inc}$	of <i>j</i> th mode generated in zone III
$a_m$	amplitude of stream function of incoming <i>m</i> th mode in zone I
$b_j$	amplitude of stream function of <i>j</i> th mode generated in zone I
$b_{\xi}^{(j)} = \frac{b_j k_j}{\sigma}$	amplitude of isopycnal displacement
ς σ	of $j$ th mode generated in zone I
$c_1, c_2, c_3$	
	phase speed
Cg	group speed
$f = 2\Omega_{\rm E} \sin \varphi$	Coriolis parameter
g	acceleration due to gravity

xvi

#### List of symbols

Н	depth of ocean
$H_0$	average depth of ocean
$H_{\rm max}$	height of bottom obstacle
$H_1, H_2(x), H_3$	water depth in zones I, II, and III, respectively
$H_{\rm p}, \Delta H_{\rm p}$	depth and width of the model pycnocline
$h_+, h$	thickness of upper and lower layers in
	two-layer model
$h = z_1(-H)$	water depth in $(x, z_1)$ variables
$h_0 = z_1(-H_0)$	average depth of a basin in $(x, z_1)$ variables
$n_1 = z_1(-H_1)$ $h_1(x) = z_1(-H_1(x))$	water depth in zones I II III in (r. z.) veriebles
$ \left. \begin{array}{l} h_1 = z_1(-H_1) \\ h_2(x) = z_1(-H_2(x)) \\ h_3 = z_1(-H_3) \end{array} \right\} $	water depth in zones I, II, III in $(x, z_1)$ variables
$h_3 = z_1(-H_3)$ $h_{\text{max}}$	height of bottom obstacle in $(x, z_1)$ variables
$\mathscr{X} = x + z_1$	characteristic line
$\omega = \kappa + z_1$	
$J(a,b) = a_x b_z - a_z b_x$	Jacobian operator
$K^{\mathrm{H}}, K^{\mathrm{V}}$	coefficients of horizontal and vertical
	turbulent diffusivity
$k_{1j}, k_{3j}$	wave number of <i>j</i> th baroclinic mode in
	areas I and III, respectively
1	half-width of bottom obstacle
$\mathscr{Z} = x - z_1$	characteristic line
$\sim$ $^{\prime\prime}$ $^{\prime\prime}$	
$M = \frac{g}{\rho_0} \frac{\partial \rho}{\partial r}$	horizontal analog of buoyancy frequency
$m = \frac{1}{\rho_0} \partial x$	
т	number of incident baroclinic mode
$(a \partial a)^{1/2}$	
$N = \left(-\frac{g}{\rho_0}\frac{\partial\rho_0}{\partial z}\right)^{1/2}$	buoyancy frequency
$\langle \rho_0 \sigma_{\lambda} \rangle = N_0$	value of buoyancy frequency for monotonic
1,0	stratification
$N_p$	maximum value of buoyancy frequency
P	
Р	pressure
$P_{a}$	atmospheric pressure
$ ilde{P}$	wave disturbance of pressure
$q_j(z_1)$	eigenfunction of <i>j</i> th baroclinic mode

	List of symbols	xvii
$\begin{aligned} \mathrm{Ri} &= N^2/u_z^2 \\ \mathcal{R} \end{aligned}$	Richardson number Riemann function	
S	salinity	
$T$ $T$ $T_0 = \lambda/c_p$ $t_0$ $t_b$	temperature tidal period time scale for solitary internal waves reference time instant of wave breaking	
U <sub>0</sub> <i>u</i> <i>u</i> +, <i>u</i> -	amplitude of barotropic tidal velocity component of velocity vector $\mathbf{v}$ in <i>x</i> -direction velocities of upper and lower layer, respectively, in <i>x</i> -direction in two-layer model	
v $v_+, v$	component of velocity vector $\mathbf{v}$ in y-direction velocities of upper and lower layer, respectively, in y-direction in two-layer model	
w	component of velocity vector $\mathbf{v}$ in z-direction	
$J_0 \alpha(s)$	$z_1$ -transformation $z_2$ -transformation	
$lpha, lpha_+, lpha$	slopes of characteristic lines	
eta	coefficient of dispersion	
$\begin{aligned} \gamma &= dH/dx\\ \gamma_0 &= dh/dx \end{aligned}$	bottom inclination bottom inclination in $(x_1z_1)$ variables	
$\varepsilon = a_{\xi}/H$ $\varepsilon_0 = h_{\max}/h_0$ $\varepsilon_1$	nondimensional wave amplitude nondimensional height of bottom obstacle parameter of nonlinearity	
$\theta = x - c_{\rm p} t$	variable in coordinate system moving with speed	C <sub>p</sub>

 $z_2 =$ 

xviii

List of symbols			
λ	wavelength		
$\mu = (H/\lambda)^2$	parameter of dispersion		
$\xi \ \xi_+, \xi$	vertical isopycnal displacement vertical displacements of free surface and interface, respectively (two-layer model)		
$\rho$ $\rho_0(z)$ $\tilde{\rho}(x, z, t)$ $\bar{\rho}_0 = \text{const.}$ $\rho_+, \rho$	wave disturbance of density		
$\sigma_t = \rho - 1000$	wave frequency conventional density		
$arphi \ arphi \ $	latitude critical latitude		
$\psi, \Psi$	stream function		
$\omega \Omega_{ m E}$	vorticity angular rotation rate of Earth		

## Abbreviations

ADCP	acoustic Doppler current profiler
ADIM	alternative direction implicit method
BBL	bottom boundary layer
BSPF	Barents Sea Polar Front
BVP	boundary value problem
CFL	Courant Friedrich Levi
CTD	conductivity, temperature, depth (profiler)
ERS-1, ERS-2	European Remote-Sensing Satellite
GW	gigawatt
JASIN experiment	Joint Air–Sea Interaction Experiment
JUSREX-92	Joint United States-Russian Experiment
K–dV	Korteweg–de Vries
eK–dV	extended Korteweg-de Vries
ODE	ordinary differential equation
POM	Princeton Oceanographic Model
R/V	research vessel
SAR	synthetic aperture radar
SIW	solitary internal wave
TOPEX	Topography Experiment
TOPEX/Poseidon	Joint US-French orbital mission, launched in 1992 to
	track changes in sea-level height with radar altimeters
TW	terawatt
WKB	Wentzel, Kramers, Brillouin

www.cambridge.org