INTRODUCTION: THE GEOMETRY OF SOUND

It is the very same taste which relishes a demonstration in geometry, that is pleased with the resemblance of a picture to an original and touched with the harmony of music. All these have unalterable and fixed foundations in nature, and are therefore equally investigated by reason and known by study; some with more, some with less clearness, but all exactly in the same way.[1]

Musical sounds are elusive things. They are among the most ephemeral of the objects of human perception and the productions of human art. Not even a single musical phrase, let alone an entire melody, is accessible to the ear all at once, as a painting is to the eye. But this is only one of the difficulties. Even when musical sounds are considered in isolation, they remain extraordinarily resistant to analysis. Each of us, as much today as in antiquity, recognises some sounds, and some combinations or sequences of sounds, as more musical, more beautiful, more concordant than others, but we may also find it difficult to say precisely what makes them so, or to define these categories in a way that will account for differences of individual taste, culture, or the age in which we live. Determining to what extent categories such as the 'musical', the 'beautiful' and the 'concordant' overlap presents even greater challenges.

One approach to the problem of defining and explaining musical beauty is to assume that the realm of music is not unique or self-contained, and that when we judge sounds as beautiful, we do so on the basis of a broader definition of beauty which applies to other perceptible things as well. Claudius Ptolemy (*fl.* AD 146–c. 170), one of the best representatives of this view among those who wrote on harmonics in Greek antiquity, argued that of all the senses, hearing and sight are the most closely connected to the faculty of reason, and that this accounts for the fact that while

[1] Joshua Reynolds, *Discourses on Art*, discourse VII, delivered to the Students of the Royal Academy, on the Distribution of the Prizes, 10 December 1776.

other senses take pleasure from their objects, sight and hearing alone find beauty in them: a smell may be fragrant, a taste may be delicious, a thing may be soft to the touch, but only a sight or a sound can be beautiful.[2] Thus when the fifth-century BC sculptor Polyclitus locates beauty in proportionality (*symmetria*), by which he means the proportional relationship of parts to one another and to the whole,[3] the student of music who accepts this thesis will say that beauty in music must arise from some sort of proportionality between sounds. If our student of music also accepts Ptolemy's view, he will say that we can appreciate beauty in music because hearing, like sight, communicates more directly with the faculty of reason than the other senses. Beauty, then, will be a kind of rational judgement made through our two most rational senses. And one way to extend the argument into the realm of music is to suggest that musical sounds are more beautiful than non-musical ones.

Without this set of preliminary assumptions, the thesis that musical sounds are musical in virtue of being proportional seems almost laughable. One reason for this is that proportion, in Polyclitus' sense at least, is something we identify and assess primarily with our eyes: we can look at a polygon drawn on a board and say at a glance whether it is a square or a rectangle (a proportional judgement, and readily definable as such). But the gap between visual and aural judgement seems unbridgeable, because there are no sounds which exhibit proportion in the way we know it from sight. Sounds can differ in timbre, in pitch, in volume, in duration – all of which are perceived as qualities except the last. The quantitative differences of duration constitute musical rhythm, which is the only proportionality in music directly accessible to perception. Since the same verse can be either spoken or sung in identical rhythm, however, and since only the latter would be counted fully musical by most people, the study of rhythm alone does not answer the question of what makes sounds musical. So we require a new starting point.

In spring, before the leaves come out in the hardwood forests of south-western Ontario, the ground is blanketed with the white

2 Ptol. *Harm.* 5.19–24, 93.11–94.1.
3 For example, of fingers to hand, hand to forearm, forearm to arm, and so on: Polyclitus ap. Gal. *De plac. Hipp. et Plat.* V.3 (= DK 40 A 3).

three-petalled wildflower called the trillium. It is a beautiful sight, but saying precisely why can be as difficult as explaining why a melody is beautiful. We could perhaps list some of the visible aspects of trilliums that must contribute to their beauty: their shape, their colour, their texture, the curl of their petals. But it is hard to get further, partly because the beauty we assess with our eyes, like the beauty we assess with our ears, seems to arise from many factors in combination. One approach, then, might be to narrow the focus and consider one factor in isolation. Suppose for the moment that we select a single trillium, and ignore all the things that make it beautiful that have nothing to do with its shape. Doing this requires restricting our vision: we might contemplate a black-and-white photograph of a pressed trillium, and find that it is still beautiful. Why? Ptolemy would probably say because of its various manifestations of *symmetria* (commensurability, proportion): literally, the ways in which its parts measure each other.[4] The idealised trillium (more perfect than any individual specimen we might obtain) has three petals of identical size and shape, whose points trace the outline of an equilateral triangle, the simplest and most symmetrical polygon. Polyclitus would probably say that this sort of basic mathematical structure is what makes works of art beautiful too. The hunch was that music, when it is beautiful, is so because it participates similarly in the mathematics of nature. But because proportion is a visual concept, sounds must somehow be rendered visible in order for us to investigate their proportionality. This can only be done indirectly, by assessing the dimensions of the physical objects that resonate when sounds occur. Some of these objects do not immediately appear to possess proportionality (the human vocal organs, for instance); others do (panpipes). The key, then, is to remove all the factors which cannot lead to an investigation of proportionality, like colour and texture for the trillium: we need an instrument that will provide a black-and-white photograph of pressed sounds. This is the monochord.

In Greek the instrument was called, simply, the 'measuring-rod' (*kanōn*). It consisted of a single string stretched over a soundbox

[4] See e.g. Ptol. *Harm.* 92.27–30.

whose surface could be marked with measurements, like a ruler. Fixed bridges at either end raised the string above the ruler, and a movable bridge allowed the string to be divided at any point in between. As long as the string was uniform, the only factor which could now contribute to its pitch was the length of the plucked section: thickness, tension and linear density were controlled. Divide the string in half by placing a movable bridge at its mid-point, and the half-length will sound a note an octave above that produced by the whole length. Here are proportion and musical beauty in one place, for the octave is (and was also for the Greeks) a privileged interval in music, and no matter where we construct it on the string, the lower note will always take twice as much length as the upper one. Thus we could say that the ratio of the octave is 2:1. Likewise, a division into two thirds generates the fifth (3:2), and one into three quarters produces the fourth (4:3). Musical relationships are now quantifiable, and just as the musician can say that a fourth and a fifth together make an octave, the mathematician can say that $4{:}3 \times 3{:}2 = 2{:}1$. The proportions which appear to underlie these three fundamental concords are all to be found in the first four numbers, 4:3:2:1. The special character of certain musical intervals, otherwise accessible to perception only as qualities, now opens itself to enquiry in the realm of arithmetic: one can attempt to define what makes musical concords concordant on the basis of their mathematical properties alone. This was the approach taken by a number of Greek musical writers, the earliest of whom were associated with the Pythagoreans.

In order to investigate music by means of ratio and proportion, therefore, the scientist needs to make sacrifices. Questions about how timbre and volume contribute to what is beautiful in music cannot be addressed; the causes of these attributes will be puzzled out in the science of acoustics. Questions of rhythm, too, will constitute a separate branch of investigation. The field of enquiry is narrowed to questions about the relationships between pitches in music, and because the questions have been framed in terms of proportion, these relationships will be further limited to those which can be expressed as a ratio of numbers. Indeed, those which cannot be so expressed will be considered unmusical: proportionality itself then becomes a condition of the musical. One of those who took this view was Adrastus of Aphrodisias, who wrote on music in the second century AD:

ὑπὸ μὲν οὖν τῶν ἀλόγων ἄλογοι καὶ ἐκμελεῖς γίνονται ψόφοι, οὓς οὐδὲ φθόγγους χρὴ καλεῖν κυρίως, ἤχους δὲ μόνον, ὑπὸ δὲ τῶν ἐν λόγοις τισὶ πρὸς ἀλλήλους πολλαπλασίοις ἢ ἐπιμορίοις ἢ ἁπλῶς ἀριθμοῦ πρὸς ἀριθμὸν ἐμμελεῖς καὶ κυρίως καὶ ἰδίως φθόγγοι.

Under irrational relations noises are irrational and unmelodic, and should not strictly even be called notes, but only sounds; but under relations that place them in certain ratios to one another, the multiple or the epimoric or simply that of number to number,[5] they are melodic, and are strictly and properly notes.[6]

Multiple ratios are those in which one term is a multiple of the other (mn:n); the ratios of the octave (2:1), octave plus fifth (3:1) and double octave (4:1) are of this form.[7] Epimoric ratios are those which have a 'part (*morion*) in addition': that is, the greater term exceeds the smaller by a simple part of the smaller ($(n + 1)$:n); the ratios of the fifth (3:2), fourth (4:3) and tone (9:8) are of this form.[8] 'Number to number' ratios are those whose terms have no special relationship; the most common example is the interval left over when two tones are taken away from a fourth. This is the so-called *leimma* ('leftover'), an interval slightly smaller than half of a tone, whose ratio is 256:243.[9]

Adrastus calls this last category 'number to number' because this is the way such ratios are referred to in Greek: the *leimma* is 'the ⟨ratio⟩ of 256 to 243' (ὁ τῶν σνϛʹ πρὸς τὰ σμγʹ ⟨λόγος⟩).[10]

5 An *arithmos* is something slightly different from what we mean by the term 'number': in Greek terms, it is 'a plurality (*plēthos*) composed of units' (Euc. *El.* VII def. 2); in our terms, this means a positive integer greater than one. Thus 'ratio of numbers' in the Greek sense excludes a quantity such as π (the ratio of the circumference of a circle to its diameter, two incommensurable magnitudes).

6 Adrastus ap. Theo. Sm. 50.14–19, trans. Barker 1989: 214.

7 Octave plus fifth: $2{:}1 \times 3{:}2 = 3{:}1$. Double octave: $(2{:}1)^2 = 4{:}1$. These are often referred to as compound ratios. Here and throughout the book I avoid the modern names for intervals greater than the octave in favour of those used by Greek authors (I write 'octave plus fifth', for example, rather than 'twelfth').

8 Just as the tone is the interval by which a fifth exceeds a fourth, so too $9{:}8 = 3{:}2 \div 4{:}3$. Note that the definition is more exact than the expression $(n + 1)$:n in that it excludes 2:1, which is not an epimoric ratio but a multiple: see Theo. Sm. 76.21–77.2. The Latin writers translate the term *superparticularis*, 'superparticular'.

9 $4{:}3 \div (9{:}8)^2 = 256{:}243$.

10 Number to number ratios are also called 'epimeric' (ἐπιμερής): see e.g. Theo. Sm. 78.6. The Latin equivalent is *superpartiens*, 'superpartient'.

This marks an important difference between Greek usage and ours, and it is a linguistic difference with ideological consequences. The privileged status of multiple and epimoric ratios is reflected in the fact that they can be expressed in a single word. Just as we can say 'duple' for 2:1, 'triple' for 3:1 and 'quadruple' for 4:1, Greek authors could also say 'hemiolic' (literally 'half-and-whole') for 3:2, 'epitritic' (literally 'a quarter in addition') for 4:3, 'epogdoic' for 9:8. This usage was not limited to ratios with smaller terms: the 27:1 ratio, for example, is ἑπτακαιεικοσιπλάσιος; the 17:16 ratio is ἐφεκκαιδέκατος.[11] The fact that the proportions found in the first four numbers (4:3:2:1) correspond to intervals which Greek musicians unanimously identified as concords was thus taken as an indication that the special status of these intervals reflected a broader principle which could be seen in the simplicity of the ratios and of the form of their expression.

Investigating music within these parameters means studying the different combinations and arrangements of intervals which arise in music with the ratios always in view. The science which pursued this investigation was concerned with the mathematical 'fitting-together' (*harmonia*) of the constituent notes and intervals of music. Aristotle called it 'mathematical harmonics', to distinguish it from 'hearing-based harmonics',[12] for the thesis that musical intervals acquire their particular qualities through the ratios to which they seem to correspond was not uncontroversial in antiquity. Those who had investigated music before Aristotle had done so in a variety of ways, not all of which are clear to us now from the surviving remnants of their work. But for Aristotle this variety could be condensed into a single dichotomy: some prioritised the mathematical aspects of the study of *harmonia*, and others prioritised the audible aspects. Aristotle called the former 'mathematical harmonicists', or more literally, 'those who investigate harmonics

[11] ἑπτακαιεικοσιπλάσιος: Nicom. *Harm.* 260.17; ἐφεκκαιδέκατος: Adrastus ap. Theo. Sm. 69.15. The more common way of writing epimorics with larger terms is just as brief: e.g. ὁ ἐπὶ ιϛʹ for 17:16 (Ptol. *Harm.* 24.12). Readers familiar with Boethius and the Latin tradition will recognise 'hemiolic' as 'sesquialter', 'epitritic' as 'sesquitertian' (*sesquitertius*) and 'epogdoic' as 'sesquioctave' (*sesquioctauus*).

[12] ἁρμονική ἥ τε μαθηματικὴ καὶ ἡ κατὰ τὴν ἀκοήν, *An. post.* 79a1–2; with the former cf. *Metaph.* 997b21.

according to numbers'.[13] The hearing-based harmonicists, he says, know 'the fact that' (τὸ ὅτι), but the mathematical harmonicists know 'the reason why' (τὸ διότι), for the mathematical scientists 'are in possession of the demonstrations of the causes'.[14]

Later in the same text (the *Posterior Analytics*), Aristotle lists some of the concerns of mathematical harmonics:

τί ἐστι συμφωνία; λόγος ἀριθμῶν ἐν ὀξεῖ καὶ βαρεῖ. διὰ τί συμφωνεῖ τὸ ὀξὺ τῷ βαρεῖ; διὰ τὸ λόγον ἔχειν ἀριθμῶν τὸ ὀξὺ καὶ τὸ βαρύ. ἆρ᾽ ἔστι συμφωνεῖν τὸ ὀξὺ καὶ τὸ βαρύ; ἆρ᾽ ἐστὶν ἐν ἀριθμοῖς ὁ λόγος αὐτῶν; λαβόντες δ᾽ ὅτι ἔστι, τίς οὖν ἐστιν ὁ λόγος;

What is concord? – a ratio of numbers between the high-pitched and the low-pitched. Why does the high-pitched form a concord with the low-pitched? – because the high-pitched and the low-pitched stand in a ratio of numbers. Does there exist a concord between the high-pitched and the low-pitched? – Is their ratio in numbers? Granted that it is, what then is the ratio?[15]

This book is about the monochord and its use within the tradition of mathematical harmonics, from the instrument's first appearance to the *Harmonics* of Claudius Ptolemy. The chronological scope of the book is defined in two ways. Firstly, it is in Ptolemy's work that the monochord receives its fullest, most detailed, most creative and methodologically rigorous treatment in antiquity. There are a number of important ancient witnesses who followed him: his earliest commentator, Porphyry, for one, and Boethius, whose *De institutione musica* transmitted the instrument and its use to the Latin West. But Porphyry is in some ways more helpful for his testimony about those whose monochord-informed harmonics preceded Ptolemy, and Boethius contributes little to the subject that is new.[16] Secondly, the history of the monochord in the Middle Ages has been written by others. Boethius' treatise has been studied both from the point of view of mediaeval music theory and from that of its Neoplatonic background,[17] and the uses of the

13 οἱ κατὰ τοὺς ἀριθμοὺς ἁρμονικοί, *Top.* 107a15–16.
14 *An. post.* 79a2–4.
15 *An. post.* 90a18–23, trans. Barker 1989: 70–1, following Ross' reading (ἐν ὀξεῖ καὶ βαρεῖ) rather than Bekker's (ἐν ὀξεῖ ἢ βαρεῖ) in 90a19.
16 The instrument also appears in the work of Aristides Quintilianus (possibly third century AD), Gaudentius (possibly fourth century AD) and many Latin authors of late antiquity.
17 See especially Bower 1989, Heilmann 2007. Mathiesen (1999: 629–36) gives an overview of the work. Barbera (1991) examines Boethius' transmission of a version

monochord in the Latin West from Boethius to 1500 have been exhaustively treated.[18] An area which is still in need of further work is the Arabic tradition. The most important items among the Greek literature on the monochord were known to Arabic music theorists of the ninth and tenth centuries, who added to the tradition by adapting the instrument to a new theoretical context, but this important aspect of its mediaeval legacy has been little studied.[19]

The aim of the book is to contextualise the monochord and its use within this chronological scope on four levels. The first, and narrowest, is mathematical harmonics: I shall attempt to establish when the instrument first came into use (toward the end of the fourth century BC, I shall suggest, although we cannot be certain); who among early mathematical harmonicists used it, and who did not; what mathematical harmonics could be done without it; and what it contributed to the science when it first appeared.

The second level is Greek harmonics more broadly. Aristotle's statement that it is the mathematical scientists who are in possession of the demonstrations of the causes, and the implied superiority of the mathematical approach to harmonics which follows from this, was challenged by two of his students, Aristoxenus and Theophrastus. Aristoxenus did not go so far as to deny that certain ratios can be found in the physical dimensions of instruments when they produce certain intervals, but he denied absolutely the value of such observations for the study of musical theory. We do not perceive music quantitatively, he argued, and so our science must be carried out in the realm of what we do perceive. Furthermore, pursuing the study of music on the authority of perception does not require relegating reason to a negligible role, nor does

of a short Hellenistic treatise on monochord division (the *Sectio canonis*); related issues (including Boethius' transmission of Ptolemy) are considered by Bowen and Bowen 1997.

18 The first strides forward were made by Wantzloeben (1911); more work was done by Adkins (1963, 1967). Other contributions followed: e.g. Hughes 1969, Lindley 1980, Brockett 1981, Herlinger 1987, Pesce 1999 and especially Meyer 1996. See Herlinger 2002 for a summary of the literature on the monochord in the Middle Ages.

19 The most important theorists in this tradition are Al-Kindī (ninth century) and Al-Fārābī (tenth century), who knew many of the important Greek harmonic texts which have survived to modern times, including Ptolemy's *Harmonics*. See Barbera 1991: 7–8 and Mathiesen 1999: 610–11 for brief accounts; the sources are listed by Shiloah (1979 and 2003).

it necessarily fail to demonstrate the causes of things to do with music: the third book of what survives as his *Elementa harmonica* is concerned almost exclusively with giving demonstrations, in Aristotle's sense of the word, of things to do with melodic succession.[20] The reason Aristoxenus' non-rational approach to harmonics (by which I mean merely 'not thinking about intervals as ratios', rather than 'unreasoned') is so important to the history of the monochord is that it provoked a counter-attack: this came in the form of a very short treatise called *The Division of the Monochord* (usually cited by its Latin title, *Sectio canonis*), in which the instrument made its first appearance in Greek literature. It is attributed, insecurely, to Euclid, but is probably to be dated to his generation (c. 300 BC).[21] It consists almost entirely of demonstrations of the primary theorems of mathematical harmonics, some of which are formulated in such a way as to refute specific rival arguments of Aristoxenus. That the vindication of the mathematical approach brought the monochord into the literature of music theory for the first time is significant: from its first appearance it was a polemical instrument as much as a musical one. And yet in later centuries, as harmonic theorists sought in different ways to bring the rival traditions together and combine their inheritances, the monochord also appears in a mediating role.

The third level is Greek mathematics. The arguments of the *Sectio canonis* are framed in the formulaic language of Euclidean arithmetic: they rely on theorems demonstrated in the arithmetical books of Euclid's *Elements* (V, VII–IX), and they employ a similar style of presentation. More specifically, they prove propositions in arithmetic through constructions in geometry: simple constructions, in which numbers are represented as line segments, and ratios as the relationships between their lengths. The monochord comes to participate in this presentation, for it makes the

20 *El. harm.* III contains twenty-three theorems set out as 'proofs' or 'demonstrations' (*apodeixeis*). It is now generally accepted that the text of the *Elementa harmonica* as we have it is a combination of more than one original work: see Mathiesen 1999: 294–334, Gibson 2005: 39–75. For a recent attempt to delineate its ancient components and their relationships, see Barker 2007, ch. 5; for the unitarian position, see Bélis 1986, ch. 1.

21 Besides the authorship of the treatise, its date and unity have been the subject of significant debate. These issues will be discussed in chapter 3. The monochord is attested independently in a fragment of Duris of Samos (c. 300 BC); this will be examined in chapter 2.

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Excerpt
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connection between ratios and intervals by means of simple geometrical constructions. A ratio can only be heard as an interval if it can first be represented as a relationship between two segments of string. Viewed from the level of mathematics, the instrument is a kind of extension of the diagrams used in Euclidean arithmetic, and its use in mathematical harmonics is consequently limited in ways that also limit the arithmetical use of geometrical diagrams.

The fourth level is Greek science more broadly. Here I have had to be selective. The third level raises questions about how instruments are incorporated into the methods of sciences other than harmonics, and what such a comparison can tell us about the role of the monochord as a scientific instrument. What, if anything, is unique about the way it mediates between the sensory and intelligible realms? If there is anything unique about it, what effect does this have on the way it was used by Greek harmonicists, and on the development of their scientific methods? To frame preliminary answers to these questions (the eligible material is vast, and what I offer here is only a start), I have chosen a pair of astronomical instruments discussed by Ptolemy in his *Almagest*, and have compared his introduction of them with his introduction of the monochord in the *Harmonics*.

When a geared calendrical device was found in an ancient shipwreck off the Greek island of Antikythera in 1901, it was a scientific instrument without a literature. The complexity of the Antikythera mechanism came as a surprise to students of ancient science and technology, because such a level of mechanical sophistication could not have been inferred from surviving Greek literature.[22] The monochord, by contrast, is an instrument without an archaeology. Because its history must be written entirely from books, there are a number of things we cannot know about its earliest incarnations: their dimensions, the materials of their construction, and so on. But what we can be more certain of are the uses to which it was put, so far as these are described for us by Greek authors. Precisely how the Antikythera mechanism functioned, and what exactly it was designed to do, are matters of

[22] On the Antikythera mechanism, see especially Price 1974; Bromley 1986; Freeth, Bitsakis *et al*. 2006; Freeth, Jones *et al*. 2008.