

## SPACE-TIME CODING: THEORY AND PRACTICE

This book covers the fundamental principles of space-time coding for wireless communications over multiple-input multiple-output (MIMO) channels, and sets out practical coding methods for achieving the performance improvements predicted by the theory.

Starting with background material on wireless communications and the capacity of MIMO channels, the book then reviews design criteria for space-time codes. A detailed treatment of the theory behind space-time block codes then leads on to an in-depth discussion of space-time trellis codes. The book continues with discussion of differential space-time modulation, BLAST and some other space-time processing methods. The final chapter addresses additional topics in space-time coding.

Written by one of the inventors of space-time block coding, this book is ideal for a graduate student familiar with the basics of digital communications, and for engineers implementing the theory in real systems.

The theory and practice sections can be used independently of each other. Exercises can be found at the end of the chapters.

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# SPACE-TIME CODING THEORY AND PRACTICE

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## Preface

The use of multiple antennas in most future wireless communication systems seems to be inevitable. Today, the main question is how to include multiple antennas and what are the appropriate methods for specific applications. The academic interest in space-time coding and multiple-input multiple-output (MIMO) systems has been growing for the last few years. Recently, the industry has shown a lot of interest as well. It is amazing how fast the topic has emerged from a theoretical curiosity to the practice of every engineer in the field. It was just a few years ago, when I started working at AT&T Labs – Research, that many would ask “who would use more than one antenna in a real system?” Today, such skepticism is gone.

The fast growth of the interest and activities on space-time coding has resulted in a spectrum of people who actively follow the field. The range spans from mathematicians who are only curious about the interesting mathematics behind space-time coding to engineers who want to build it. There is a need for covering both the theory and practice of space-time coding in depth. This book hopes to fulfill this need. The book has been written as a textbook for first-year graduate students and as a reference for the engineers who want to learn the subject from scratch. An early version of the book has been used as a textbook to teach a course in space-time coding at the University of California, Irvine. The goal of such a course is the introduction of space-time coding to anyone with some basic knowledge of digital communications. In most cases, we start with common ideas for single-input single-output (SISO) channels and extend them to MIMO channels. Therefore, students or engineers with no knowledge of MIMO systems should be able to learn all the concepts. While graduate students might be interested in all the details and the proofs of theorems and lemmas, engineers may skip the proofs and concentrate on the results without sacrificing the continuity of the presentation.

A typical course on space-time coding may start with some background material on wireless communications and capacity of MIMO channels as covered in Chapters 1 and 2. A review of design criteria for space-time codes is covered in Chapter 3.

Chapters 4 and 5 provide a detailed treatment of the theory behind space-time block codes. A practitioner who is only interested in the structure of the codes may bypass all the proofs in these chapters and concentrate only on the examples. Chapters 6 and 7 discuss space-time trellis codes in depth. Each chapter includes discussions on the performance analysis of the codes and simulation results. For those who are more interested in the practical aspects of the topic, simulation results are sufficient and the sections on performance analysis may be skipped. The practitioners may continue with Chapter 11 and its discussion on MIMO-OFDM and Chapter 9 on receiver design. On the other hand, those who are more interested in the theory of space-time codes can follow with Chapter 8 and its treatment of differential space-time modulation. Finally, for the sake of completeness, we discuss BLAST and some other space-time processing methods in Chapters 9 and 10. Homework problems have been included at the end of each chapter.

The book includes the contribution of many researchers. I am grateful to all of them for generating excitement in the field of space-time coding. My special thanks goes to my very good friend and former colleague Professor Vahid Tarokh who introduced space-time coding to me. Also, I should thank my other colleagues at AT&T Labs – Research who initiated most of the basic concepts and ideas in space-time coding. Without the support of my department head at AT&T Labs – Research, Dr. Behzad Shahraray, I would not be able to contribute to the topic and I am thankful to him for providing the opportunity. Also, Professor Rob Calderbank has been a big supporter of the effort.

The early versions of this book have been read and reviewed by my students and others. Their comments and suggestions have improved the quality of the presentation. Especially, comments from Professor John Proakis, Professor Syed Jafar, Dr. Masoud Olfat, and Hooman Honary have resulted in many improvements. My Ph.D. students, Li Liu, Javad Kazemitabar, and Yun Zhu, have helped with the proofreading of a few chapters. Many of the presented simulation results have been double checked by Yun Zhu. I also thank the National Science Foundation (NSF) for giving me an NSF Career Award to support my research and educational goals related to this book.

Last but not least, I would like to thank my wife, Paniz, for her support and love.



## Standard notation

$\ \cdot\ $	Euclidean norm
$\ \cdot\ _F$	Frobenius norm
$\otimes$	tensor product
$*$	conjugate
$+$	Moore-Penrose generalized inverse (pseudo-inverse)
$\det$	determinant of a matrix
$E$	expectation
$f_X(x)$	pdf of $X$
$H$	Hermetian
$\Im$	imaginary part
$I$	imaginary part
$I_N$	$N \times N$ identity matrix
$j$	$\sqrt{-1}$
$K_X$	covariance of vector $X$
$\Re$	real part
$R$	real part
$T$	transpose
$\text{Tr}$	trace of a matrix
$\text{Var}$	variance

## Space-time coding notation

$\alpha_{n,m}$	path gain from transmit antenna $n$ to receive antenna $m$
$\chi_k$	a chi-square RV with $2k$ degrees of freedom
$\eta$	noise
$\phi$	rotation parameter for constellations
$\gamma$	SNR
$\rho(N)$	Radon function
$\theta$	rotation parameter for STBCs
$b$	number of transmit bits per channel use
$C$	capacity
$C_{\text{out}}$	outage capacity
$\mathcal{C}$	set of complex numbers
$\mathbf{C}$	$T \times N$ transmitted codeword
$\mathcal{C}$	set of super-orthogonal codes
$d_{\text{min}}$	minimum distance
$E_s$	average power of transmitted symbols
$f_d$	Doppler shift
$G_c$	coding gain
$G_d$	diversity gain
$\mathbf{G}$	generating matrix for a STTC
$\mathcal{G}$	generator matrix for a STBC
$\mathbf{H}$	$N \times M$ channel matrix
$I$	number of states in a trellis
$J$	number of delta functions in the impulse response of a frequency selective fading channel
$J$	number of groups in a combined spatial multiplexing and space-time coding system
$2^l$	number of branches leaving each state of a trellis
$K$	number of transmitted symbols per block

$L$	number of orthogonal (data) blocks in a SOSTTC
$L$	size of IFFT and FFT blocks in OFDM
$L$ -PSK	a PSK constellation with $L = 2^b$ symbols
$M$	number of receive antennas
$N$	number of transmit antennas
$\mathcal{N}$	$T \times M$ noise matrix
$N_0$	noise samples have a variance of $N_0/2$ per complex dimension
$P$	number of trellis transitions (two trellis paths differ in $P$ transitions)
$P_{\text{out}}$	outage probability
$Q$	the memory of a trellis
$r$	transmission rate in bits/(s Hz)
$r$	received signal
$\mathbf{r}$	$T \times M$ received matrix
$R$	rate of a STC
$\mathbb{R}$	set of real numbers
$s$	transmitted signal
$S_t$	the state of the encoder at time $t$
$x$	indeterminant variable
$\mathbb{Z}$	set of integers

## Abbreviations

ADC	Analog to Digital Converter
AGC	Automatic Gain Control
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BLAST	Bell Labs Layered Space-Time
BPSK	Binary Phase Shift Keying
BSC	Binary Symmetric Channel
CCDF	Complementary Cumulative Distribution Function
CDF	Cumulative Distribution Function
CDMA	Code Division Multiple Access
CSI	Channel State Information
CT	Cordless Telephone
DAST	Diagonal Algebraic Space-Time
DASTBC	Diagonal Algebraic Space-Time Block Code
D-BLAST	Diagonal BLAST
DECT	Digital Cordless Telephone
DFE	Decision Feedback Equalization
DPSK	Differential Phase Shift Keying
EDGE	Enhanced Data for Global Evolution
FER	Frame Error Rate
FFT	Fast Fourier Transform
FIR	Finite Impulse Response
GSM	Global System for Mobile
IFFT	Inverse Fast Fourier Transform
iid	independent identically distributed
IMT	International Mobile Telephone
ISI	Intersymbol Interference

*Abbreviations*

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LAN	Local Area Network
LDSTBC	Linear Dispersion Space-Time Block Code
LOS	Line of Sight
MGF	Moment Generating Function
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
MMAC	Multimedia Mobile Access Communication
MMSE	Minimum Mean Squared Error
MRC	Maximum Ratio Combining
ML	Maximum-Likelihood
MTCM	Multiple Trellis Coded Modulation
OFDM	Orthogonal Frequency Division Multiplexing
OSTBC	Orthogonal Space-Time Block Code
PAM	Pulse Amplitude Modulation
PAN	Personal Area Network
PAPR	Peak-to-Average Power Ratio
PDA	Personal Digital Assistant
PDC	Personal Digital Cellular
pdf	probability density function
PEP	Pairwise Error Probability
PHS	Personal Handyphone System
PSK	Phase Shift Keying
QAM	Quadrature Amplitude Modulation
QOSTBC	Quasi-Orthogonal Space-Time Block Code
QPSK	Quadrature Phase Shift Keying
RF	Radio Frequency
RLST	Random Layered Space-Time
RV	Random Variable
SER	Symbol Error Rate
SISO	Single-Input Single-Output
SIMO	Single-Input Multiple-Output
SM	Spatial Multiplexing
SNR	Signal to Noise Ratio
SOSTTC	Super-Orthogonal Space-Time Trellis Code
SQOSTTC	Super-Quasi-Orthogonal Space-Time Trellis Code
STBC	Space-Time Block Code
STTC	Space-Time Trellis Code
TAST	Threaded Algebraic Space-Time
TASTBC	Threaded Algebraic Space-Time Block Code

TCM	Trellis Coded Modulation
TDD	Time Division Duplexing
TDMA	Time Division Multiple Access
V-BLAST	Vertical BLAST
ZF	Zero Forcing