

Cambridge University Press

0521842387 - A Guide to Monte Carlo Simulations in Statistical Physics, Second Edition

David P. Landau and Kurt Binder

Frontmatter

[More information](#)

A Guide to Monte Carlo Simulations in Statistical Physics, Second Edition

This new and updated deals with all aspects of Monte Carlo simulation of complex physical systems encountered in condensed-matter physics and statistical mechanics as well as in related fields, for example polymer science, lattice gauge theory and protein folding.

After briefly recalling essential background in statistical mechanics and probability theory, the authors give a succinct overview of simple sampling methods. The next several chapters develop the importance sampling method, both for lattice models and for systems in continuum space. The concepts behind the various simulation algorithms are explained in a comprehensive fashion, as are the techniques for efficient evaluation of system configurations generated by simulation (histogram extrapolation, multicanonical sampling, Wang-Landau sampling, thermodynamic integration and so forth). The fact that simulations deal with small systems is emphasized. The text incorporates various finite size scaling concepts to show how a careful analysis of finite size effects can be a useful tool for the analysis of simulation results. Other chapters also provide introductions to quantum Monte Carlo methods, aspects of simulations of growth phenomena and other systems far from equilibrium, and the Monte Carlo Renormalization Group approach to critical phenomena. A brief overview of other methods of computer simulation is given, as is an outlook for the use of Monte Carlo simulations in disciplines outside of physics. Many applications, examples and exercises are provided throughout the book. Furthermore, many new references have been added to highlight both the recent technical advances and the key applications that they now make possible.

This is an excellent guide for graduate students who have to deal with computer simulations in their research, as well as postdoctoral researchers, in both physics and physical chemistry. It can be used as a textbook for graduate courses on computer simulations in physics and related disciplines.

Cambridge University Press

0521842387 - A Guide to Monte Carlo Simulations in Statistical Physics, Second Edition

David P. Landau and Kurt Binder

Frontmatter

[More information](#)

DAVID P. LANDAU was born on June 22, 1941 in St. Louis, MO, USA. He received a BA in Physics from Princeton University in 1963 and a Ph.D. in Physics from Yale University in 1967. His Ph.D. research involved experimental studies of magnetic phase transitions as did his postdoctoral research at the CNRS in Grenoble, France. After teaching at Yale for a year he moved to the University of Georgia and initiated a research program of Monte Carlo studies in statistical physics. He is currently the Distinguished Research Professor of Physics and founding Director of the Center for Simulational Physics at the University of Georgia. He has been teaching graduate courses in computer simulations since 1982. David Landau has authored/co-authored more than 330 research publications and is editor/co-editor of more than 20 books. He is a Fellow of the American Physical Society and a past Chair of the Division of Computational Physics of the APS. He received the Jesse W. Beams award from SESAPS in 1987, and a Humboldt Fellowship and Humboldt Senior US Scientist award in 1975 and 1988 respectively. The University of Georgia named him a Senior Teaching Fellow in 1993. In 1998 he also became an Adjunct Professor at the Helsinki University of Technology. In 1999 he was named a Fellow of the Japan Society for the Promotion of Science. In 2002 he received the Aneesur Rahman Prize for Computational Physics from the APS, and in 2003 the Lamar Dodd Award for Creative Research from the University of Georgia. In 2004 he became the Senior Guangbiao Distinguished Professor (Visiting) at Zhajiang in China. He is currently a Principal Editor for the journal *Computer Physics Communications*.

KURT BINDER was born on February 10, 1944 in Korneuburg, Austria, and then lived in Vienna, where he received his Ph.D. in 1969 at the Technical University of Vienna. Even then his thesis dealt with Monte Carlo simulations of Ising and Heisenberg magnets, and since then he has pioneered the development of Monte Carlo simulation methods in statistical physics. From 1969 until 1974 Kurt Binder worked at the Technical University in Munich, where he defended his Habilitation thesis in 1973 after a stay as IBM postdoctoral fellow in Zurich in 1972/73. Further key times in his career were spent at Bell Laboratories, Murray Hill, NJ (1974), and a first appointment as Professor of Theoretical Physics at the University of Saarbrücken back in Germany (1974–1977), followed by a joint appointment as full professor at the University of Cologne and the position as one of the directors of the Institute of Solid State Research at Jülich (1977–1983). He has held his present position as Professor of Theoretical Physics at the University of Mainz, Germany, since 1983, and since 1989 he has also been an external member of the Max-Planck-Institut for Polymer Research at Mainz. Kurt Binder has written more than 800 research publications and edited 5 books dealing with computer simulation. His book (with Dieter W. Heermann) *Monte Carlo Simulation in Statistical Physics: An Introduction*, first published in 1988, is in its fourth edition. Kurt Binder has been a corresponding member of the Austrian Academy of Sciences in Vienna since 1992 and received the Max Planck Medal of the German Physical Society in 1993. He also acts as Editorial Board member of several journals and has served as Chairman of the IUPAP Commission on Statistical Physics. In 2001 he was awarded the Berni Alder CECAM prize from the European Physical Society.

Cambridge University Press

0521842387 - A Guide to Monte Carlo Simulations in Statistical Physics, Second Edition

David P. Landau and Kurt Binder

Frontmatter

[More information](#)

A Guide to Monte Carlo Simulations in Statistical Physics

Second Edition

David P. Landau

Center for Simulational Physics, The University of Georgia

Kurt Binder

Institut für Physik, Johannes-Gutenberg-Universität Mainz



Cambridge University Press
0521842387 - A Guide to Monte Carlo Simulations in Statistical Physics, Second Edition
David P. Landau and Kurt Binder
Frontmatter
[More information](#)

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521842389

© David P. Landau and Kurt Binder 2000

This book is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without
the written permission of Cambridge University Press.

First published 2000
Second edition published 2005

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this book is available from the British Library

Library of Congress Cataloging in Publication data

ISBN-13 978-0-521-84238-9 hardback
ISBN-10 0-521-84238-7 hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this book, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Contents

	<i>page</i>
Preface	xii
1 Introduction	1
1.1 What is a Monte Carlo simulation?	1
1.2 What problems can we solve with it?	2
1.3 What difficulties will we encounter?	3
1.3.1 Limited computer time and memory	3
1.3.2 Statistical and other errors	3
1.4 What strategy should we follow in approaching a problem?	4
1.5 How do simulations relate to theory and experiment?	4
1.6 Perspective	6
2 Some necessary background	7
2.1 Thermodynamics and statistical mechanics: a quick reminder	7
2.1.1 Basic notions	7
2.1.2 Phase transitions	13
2.1.3 Ergodicity and broken symmetry	24
2.1.4 Fluctuations and the Ginzburg criterion	25
2.1.5 A standard exercise: the ferromagnetic Ising model	25
2.2 Probability theory	27
2.2.1 Basic notions	27
2.2.2 Special probability distributions and the central limit theorem	29
2.2.3 Statistical errors	30
2.2.4 Markov chains and master equations	31
2.2.5 The ‘art’ of random number generation	32
2.3 Non-equilibrium and dynamics: some introductory comments	39
2.3.1 Physical applications of master equations	39
2.3.2 Conservation laws and their consequences	40
2.3.3 Critical slowing down at phase transitions	43
2.3.4 Transport coefficients	45
2.3.5 Concluding comments	45
References	45
3 Simple sampling Monte Carlo methods	48
3.1 Introduction	48
3.2 Comparisons of methods for numerical integration of given functions	48

3.2.1 Simple methods	48
3.2.2 Intelligent methods	50
3.3 Boundary value problems	51
3.4 Simulation of radioactive decay	53
3.5 Simulation of transport properties	54
3.5.1 Neutron transport	54
3.5.2 Fluid flow	55
3.6 The percolation problem	56
3.6.1 Site percolation	56
3.6.2 Cluster counting: the Hoshen–Kopelman algorithm	59
3.6.3 Other percolation models	60
3.7 Finding the groundstate of a Hamiltonian	60
3.8 Generation of ‘random’ walks	61
3.8.1 Introduction	61
3.8.2 Random walks	62
3.8.3 Self-avoiding walks	63
3.8.4 Growing walks and other models	65
3.9 Final remarks	66
References	66
4 Importance sampling Monte Carlo methods	68
4.1 Introduction	68
4.2 The simplest case: single spin-flip sampling for the simple Ising model	69
4.2.1 Algorithm	70
4.2.2 Boundary conditions	74
4.2.3 Finite size effects	77
4.2.4 Finite sampling time effects	90
4.2.5 Critical relaxation	98
4.3 Other discrete variable models	105
4.3.1 Ising models with competing interactions	105
4.3.2 q -state Potts models	109
4.3.3 Baxter and Baxter–Wu models	110
4.3.4 Clock models	111
4.3.5 Ising spin glass models	113
4.3.6 Complex fluid models	114
4.4 Spin-exchange sampling	115
4.4.1 Constant magnetization simulations	115
4.4.2 Phase separation	115
4.4.3 Diffusion	117
4.4.4 Hydrodynamic slowing down	120
4.5 Microcanonical methods	120
4.5.1 Demon algorithm	120
4.5.2 Dynamic ensemble	121
4.5.3 Q2R	121
4.6 General remarks, choice of ensemble	122

Cambridge University Press

0521842387 - A Guide to Monte Carlo Simulations in Statistical Physics, Second Edition

David P. Landau and Kurt Binder

Frontmatter

[More information](#)

	Contents	vii
4.7 Statics and dynamics of polymer models on lattices		122
4.7.1 Background		122
4.7.2 Fixed bond length methods		123
4.7.3 Bond fluctuation method		124
4.7.4 Enhanced sampling using a fourth dimension		125
4.7.5 The ‘wormhole algorithm’ – another method to equilibrate dense polymeric systems		127
4.7.6 Polymers in solutions of variable quality: θ -point, collapse transition, unmixing		127
4.7.7 Equilibrium polymers: a case study		130
4.8 Some advice		133
References		134
5 More on importance sampling Monte Carlo methods for lattice systems		137
5.1 Cluster flipping methods		137
5.1.1 Fortuin–Kasteleyn theorem		137
5.1.2 Swendsen–Wang method		138
5.1.3 Wolff method		141
5.1.4 ‘Improved estimators’		142
5.1.5 Invaded cluster algorithm		142
5.1.6 Probability changing cluster algorithm		143
5.2 Specialized computational techniques		144
5.2.1 Expanded ensemble methods		144
5.2.2 Multispin coding		144
5.2.3 N -fold way and extensions		145
5.2.4 Hybrid algorithms		148
5.2.5 Multigrid algorithms		148
5.2.6 Monte Carlo on vector computers		148
5.2.7 Monte Carlo on parallel computers		149
5.3 Classical spin models		150
5.3.1 Introduction		150
5.3.2 Simple spin-flip method		151
5.3.3 Heatbath method		153
5.3.4 Low temperature techniques		153
5.3.5 Over-relaxation methods		154
5.3.6 Wolff embedding trick and cluster flipping		154
5.3.7 Hybrid methods		155
5.3.8 Monte Carlo dynamics vs. equation of motion dynamics		156
5.3.9 Topological excitations and solitons		156
5.4 Systems with quenched randomness		160
5.4.1 General comments: averaging in random systems		160
5.4.2 Parallel tempering: a general method to better equilibrate systems with complex energy landscapes		163
5.4.3 Random fields and random bonds		164
5.4.4 Spin glasses and optimization by simulated annealing		165

5.4.5 Ageing in spin glasses and related systems	169
5.4.6 Vector spin glasses: developments and surprises	170
5.5 Models with mixed degrees of freedom: Si/Ge alloys, a case study	171
5.6 Sampling the free energy and entropy	172
5.6.1 Thermodynamic integration	172
5.6.2 Groundstate free energy determination	174
5.6.3 Estimation of intensive variables: the chemical potential	174
5.6.4 Lee–Kosterlitz method	175
5.6.5 Free energy from finite size dependence at T_c	175
5.7 Miscellaneous topics	176
5.7.1 Inhomogeneous systems: surfaces, interfaces, etc.	176
5.7.2 Other Monte Carlo schemes	182
5.7.3 Inverse Monte Carlo methods	184
5.7.4 Finite size effects: a review and summary	185
5.7.5 More about error estimation	186
5.7.6 Random number generators revisited	187
5.8 Summary and perspective	189
References	190
6 Off-lattice models	194
6.1 Fluids	194
6.1.1 NVT ensemble and the virial theorem	194
6.1.2 NpT ensemble	197
6.1.3 Grand canonical ensemble	201
6.1.4 Near critical coexistence: a case study	205
6.1.5 Subsystems: a case study	207
6.1.6 Gibbs ensemble	212
6.1.7 Widom particle insertion method and variants	215
6.1.8 Monte Carlo Phase Switch	217
6.1.9 Cluster algorithm for fluids	220
6.2 ‘Short range’ interactions	222
6.2.1 Cutoffs	222
6.2.2 Verlet tables and cell structure	222
6.2.3 Minimum image convention	222
6.2.4 Mixed degrees of freedom reconsidered	223
6.3 Treatment of long range forces	223
6.3.1 Reaction field method	223
6.3.2 Ewald method	224
6.3.3 Fast multipole method	225
6.4 Adsorbed monolayers	226
6.4.1 Smooth substrates	226
6.4.2 Periodic substrate potentials	226
6.5 Complex fluids	227
6.5.1 Application of the Liu–Luijten algorithm to a binary fluid mixture	230

	Contents	ix
6.6 Polymers: an introduction		231
6.6.1 Length scales and models		231
6.6.2 Asymmetric polymer mixtures: a case study		237
6.6.3 Applications: dynamics of polymer melts; thin adsorbed polymeric films		240
6.7 Configurational bias and ‘smart Monte Carlo’		245
References		248
7 Reweighting methods		251
7.1 Background		251
7.1.1 Distribution functions		251
7.1.2 Umbrella sampling		251
7.2 Single histogram method: the Ising model as a case study		254
7.3 Multi-histogram method		261
7.4 Broad histogram method		262
7.5 Transition matrix Monte Carlo		262
7.6 Multicanonical sampling		263
7.6.1 The multicanonical approach and its relationship to canonical sampling		263
7.6.2 Near first order transitions		264
7.6.3 Groundstates in complicated energy landscapes		266
7.6.4 Interface free energy estimation		267
7.7 A case study: the Casimir effect in critical systems		268
7.8 ‘Wang-Landau sampling’		270
7.9 A case study: evaporation/condensation transition of droplets		273
References		274
8 Quantum Monte Carlo methods		277
8.1 Introduction		277
8.2 Feynman path integral formulation		279
8.2.1 Off-lattice problems: low-temperature properties of crystals		279
8.2.2 Bose statistics and superfluidity		285
8.2.3 Path integral formulation for rotational degrees of freedom		286
8.3 Lattice problems		288
8.3.1 The Ising model in a transverse field		288
8.3.2 Anisotropic Heisenberg chain		290
8.3.3 Fermions on a lattice		293
8.3.4 An intermezzo: the minus sign problem		296
8.3.5 Spinless fermions revisited		298
8.3.6 Cluster methods for quantum lattice models		301
8.3.7 Continuous time simulations		302
8.3.8 Decoupled cell method		302
8.3.9 Handscomb’s method		303
8.3.10 Wang-Landau sampling for quantum models		304
8.3.11 Fermion determinants		306
8.4 Monte Carlo methods for the study of groundstate properties		307
8.4.1 Variational Monte Carlo (VMC)		308

x	Contents	
	8.4.2 Green's function Monte Carlo methods (GFMC)	309
	8.5 Concluding remarks	311
	References	312
9	Monte Carlo renormalization group methods	315
	9.1 Introduction to renormalization group theory	315
	9.2 Real space renormalization group	319
	9.3 Monte Carlo renormalization group	320
	9.3.1 Large cell renormalization	320
	9.3.2 Ma's method: finding critical exponents and the fixed point Hamiltonian	322
	9.3.3 Swendsen's method	323
	9.3.4 Location of phase boundaries	325
	9.3.5 Dynamic problems: matching time-dependent correlation functions	326
	9.3.6 Inverse Monte Carlo renormalization group transformations	327
	References	327
10	Non-equilibrium and irreversible processes	328
	10.1 Introduction and perspective	328
	10.2 Driven diffusive systems (driven lattice gases)	328
	10.3 Crystal growth	331
	10.4 Domain growth	333
	10.5 Polymer growth	336
	10.5.1 Linear polymers	336
	10.5.2 Gelation	336
	10.6 Growth of structures and patterns	337
	10.6.1 Eden model of cluster growth	337
	10.6.2 Diffusion limited aggregation	338
	10.6.3 Cluster-cluster aggregation	340
	10.6.4 Cellular automata	340
	10.7 Models for film growth	342
	10.7.1 Background	342
	10.7.2 Ballistic deposition	343
	10.7.3 Sedimentation	343
	10.7.4 Kinetic Monte Carlo and MBE growth	344
	10.8 Transition path sampling	347
	10.9 Outlook: variations on a theme	348
	References	348
11	Lattice gauge models: a brief introduction	350
	11.1 Introduction: gauge invariance and lattice gauge theory	350
	11.2 Some technical matters	352
	11.3 Results for $Z(N)$ lattice gauge models	352
	11.4 Compact $U(1)$ gauge theory	353
	11.5 $SU(2)$ lattice gauge theory	354

Cambridge University Press

0521842387 - A Guide to Monte Carlo Simulations in Statistical Physics, Second Edition

David P. Landau and Kurt Binder

Frontmatter

[More information](#)

	Contents	xi
11.6 Introduction: quantum chromodynamics (QCD) and phase transitions of nuclear matter	355	
11.7 The deconfinement transition of QCD	357	
11.8 Where are we now?	360	
References	362	
12 A brief review of other methods of computer simulation	363	
12.1 Introduction	363	
12.2 Molecular dynamics	363	
12.2.1 Integration methods (microcanonical ensemble)	363	
12.2.2 Other ensembles (constant temperature, constant pressure, etc.)	367	
12.2.3 Non-equilibrium molecular dynamics	370	
12.2.4 Hybrid methods (MD + MC)	370	
12.2.5 <i>Ab initio</i> molecular dynamics	371	
12.3 Quasi-classical spin dynamics	372	
12.4 Langevin equations and variations (cell dynamics)	375	
12.5 Micromagnetics	376	
12.6 Dissipative particle dynamics (DPPD)	377	
12.7 Lattice gas cellular automata	378	
12.8 Lattice Boltzmann Equation	379	
12.9 Multiscale simulation	379	
References	381	
13 Monte Carlo methods outside of physics	383	
13.1 Commentary	383	
13.2 Protein folding	383	
13.2.1 Introduction	383	
13.2.2 Generalized ensemble methods	384	
13.2.3 Globular proteins: a case study	386	
13.3 ‘Biologically inspired physics’	387	
13.4 Mathematics/statistics	388	
13.5 Sociophysics	388	
13.6 Econophysics	388	
13.7 ‘Traffic’ simulations	389	
13.8 Medicine	391	
References	392	
14 Outlook	393	
Appendix: listing of programs mentioned in the text	395	
Index	427	

Preface

Historically physics was first known as ‘natural philosophy’ and research was carried out by purely theoretical (or philosophical) investigation. True progress was obviously limited by the lack of real knowledge of whether or not a given theory really applied to nature. Eventually experimental investigation became an accepted form of research although it was always limited by the physicist’s ability to prepare a sample for study or to devise techniques to probe for the desired properties. With the advent of computers it became possible to carry out simulations of models which were intractable using ‘classical’ theoretical techniques. In many cases computers have, for the first time in history, enabled physicists not only to invent new models for various aspects of nature but also to solve those same models without substantial simplification. In recent years computer power has increased quite dramatically, with access to computers becoming both easier and more common (e.g. with personal computers and workstations), and computer simulation methods have also been steadily refined. As a result computer simulations have become another way of doing physics research. They provide another perspective; in some cases simulations provide a theoretical basis for understanding experimental results, and in other instances simulations provide ‘experimental’ data with which theory may be compared. There are numerous situations in which direct comparison between analytical theory and experiment is inconclusive. For example, the theory of phase transitions in condensed matter must begin with the choice of a Hamiltonian, and it is seldom clear to what extent a particular model actually represents a real material on which experiments are done. Since analytical treatments also usually require mathematical approximations whose accuracy is difficult to assess or control, one does not know whether discrepancies between theory and experiment should be attributed to shortcomings of the model, the approximations, or both. The goal of this text is to provide a basic understanding of the methods and philosophy of computer simulations research with an emphasis on problems in statistical thermodynamics as applied to condensed matter physics or materials science. There exist many other simulation problems in physics (e.g. simulating the spectral intensity reaching a detector in a scattering experiment) which are more straightforward and which will only occasionally be mentioned. We shall use many specific examples and, in some cases, give explicit computer programs, but we wish to

emphasize that these methods are applicable to a wide variety of systems including those which are not treated here at all. As computer architecture changes the methods presented here will in some cases require relatively minor reprogramming and in other instances will require new algorithm development in order to be truly efficient. We hope that this material will prepare the reader for studying new and different problems using both existing as well as new computers.

At this juncture we wish to emphasize that it is important that the simulation algorithm and conditions be chosen with the physics problem at hand in mind. The *interpretation* of the resultant output is critical to the success of any simulational project, and we thus include substantial information about various aspects of thermodynamics and statistical physics to help strengthen this connection. We also wish to draw the reader's attention to the rapid development of scientific visualization and the important role that it can play in producing *understanding* of the results of some simulations.

This book is intended to serve as an introduction to Monte Carlo methods for graduate students, and advanced undergraduates, as well as more senior researchers who are not yet experienced in computer simulations. The book is divided up in such a way that it will be useful for courses which only wish to deal with a restricted number of topics. Some of the later chapters may simply be skipped without affecting the understanding of the chapters which follow. Because of the immensity of the subject, as well as the existence of a number of very good monographs and articles on advanced topics which have become quite technical, we will limit our discussion in certain areas, e.g. polymers, to an introductory level. The examples which are given are in FORTRAN, not because it is necessarily the best scientific computer language, but because it is certainly the most widespread. Many existing Monte Carlo programs and related subprograms are in FORTRAN and will be available to the student from libraries, journals, etc. A number of sample problems are suggested in the various chapters; these may be assigned by course instructors or worked out by students on their own. Our experience in assigning problems to students taking a graduate course in simulations at the University of Georgia over a 20-year period suggests that for maximum pedagogical benefit, students should be required to prepare cogent reports after completing each assigned simulational problem. Students were required to complete seven 'projects' in the course of the quarter for which they needed to write and debug programs, take and analyze data, and prepare a report. Each report should briefly describe the algorithm used, provide sample data and data analysis, draw conclusions and add comments. (A sample program/output should be included.) In this way, the students obtain practice in the summary and presentation of simulational results, a skill which will prove to be valuable later in their careers. For convenience, the case studies that are described have been simply taken from the research of the authors of this book – the reader should be aware that this is by no means meant as a negative statement on the quality of the research of numerous other groups in the field. Similarly, selected references are given to aid the reader in finding

Cambridge University Press

0521842387 - A Guide to Monte Carlo Simulations in Statistical Physics, Second Edition

David P. Landau and Kurt Binder

Frontmatter

[More information](#)

more detailed information, but because of length restrictions it is simply not possible to provide a complete list of relevant literature. Many coworkers have been involved in the work which is mentioned here, and it is a pleasure to thank them for their fruitful collaboration. We have also benefited from the stimulating comments of many of our colleagues and we wish to express our thanks to them as well.

The pace of advances in computer simulations continues unabated. This Second Edition of our 'guide' to Monte Carlo simulations updates some of the references and includes numerous additions. New text describes algorithmic developments that appeared too late for the first edition or, in some cases, were excluded for fear that the volume would become too thick. Because of advances in computer technology and algorithmic developments, new results often have much higher statistical precision than some of the older examples in the text. Nonetheless, the older work often provides valuable pedagogical information for the student and may also be more readable than more recent, and more compact, papers. An additional advantage is that the reader can easily reproduce some of the older results with only a modest investment of modern computer resources. Of course, newer, higher resolution studies that are cited often permit yet additional information to be extracted from simulational data, so striving for higher precision should not be viewed as 'busy work'. We have also added a brief new chapter that provides an overview of some areas outside of physics where traditional Monte Carlo methods have made an impact. Lastly, a few misprints have been corrected, and we thank our colleagues for pointing them out.