Impact Mechanics

Second Edition

Emphasizing nonpenetrating collisions, *Impact Mechanics: Second Edition* develops several different methodologies for analyzing collisions between structures – from rigid body theory for structures that are stiff and compact to vibration and wave analyses for flexible structures. A valuable reference for both professionals and advanced undergraduate and graduate students, the book builds upon foundation courses in dynamics and strength of materials. Worked examples and end-of-chapter homework problems are drawn both from industry and sports such as golf, baseball, soccer and billiards. New chapters present a generalized theory of multibody impact, as well as analyses of viscoelastic and viscoplastic impact. Effects of local compliance on impact dynamics are more generally described, and additional examples illustrating effects of friction during impact between bodies in either collinear or eccentric configurations are included.

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To Katerina and Jaime

Contents

Cambridge University Press & Assessment
978-0-521-84188-7 — Impact Mechanics
2nd Edition
W. J. Stronge
Frontmatter
More Information

Preface to Second Edition

Caminante, no hay comino.
Se hace camino al andar.

Traveller, there is no path
Paths are made by walking.
A. Machado, *Campos de Castilla*, 1964

Revision of this book for the second edition has preserved the basic organization and ethos of the first edition – it is organized as a graduate-level textbook which follows a path of increasing complexity. The book begins with the analysis of the direct impact of two rigid bodies, passes through considerations of impact between various types of deformable structures and finally leads to chaos theory for repeated collisions in metastable multibody systems. Along this path numerous solved examples, many drawn from the field of sport, help the reader assimilate the theory. Throughout, the aim is to present in a systematic way and explain the analysis of impact in various different systems.

The entire text has been re-examined and improvements were made throughout by rewriting for improved clarity as well as adding new material. In particular, there are new chapters that present a generalized theory of multibody impact, analyses of viscoelastic and viscoplastic impact, and impact on a variety of different types of sports balls. Major changes have been introduced to describe more generally the effects of local compliance on impact dynamics and to include additional examples illustrating effects of friction during impact between bodies in either collinear or eccentric configurations. Also, the chapter on state transitions in systems with multiple stable configurations (domino toppling or sequential collapse) has been revised to present a more lucid explanation of conditions required to support a wave of destabilization propagating through such a system. Furthermore, some new problems have been added at the end of most chapters in order to highlight essential points. This new edition has presented the opportunity to correct a few misprints that inevitably were present in the first edition. In this regard, it is my pleasure to gratefully acknowledge the helpful suggestions contributed by students and scholars using the first edition.

Since publication of the first edition in 2000, the field of impact mechanics has been an active area of both experimental and theoretical research. The analytical methods presented in this text are a foundation for much of this work. Writing this second edition has benefited from my research collaborations with Mont Hubbard, Khairul Ismail, and

Yunian Shen, who contributed to the ideas expressed as well as reading and commenting on sections of the text. It has been my pleasure to wrestle with problems in company with these inquisitive and thoughtful scholars. In addition I wish to thank my editors Steven Elliot and Mark Fox of Cambridge University Press who enthusiastically supported this project and provided valuable resources that encouraged its completion. Finally, I acknowledge the contribution of my wife, Katerina Chomenidou, whose patience and encouragement were essential in bringing the book to fruition.

Preface to First Edition

He who does not expect the unexpected will not find it.

Democritus, ~420 B.C.

When bodies collide, they come together with some relative velocity at an initial point of contact. If it were not for the contact force that develops between them, the normal component of relative velocity would result in overlap or interference near the contact point and this interference would increase with time. This reaction force deforms the bodies into a compatible configuration in a common contact surface that envelopes the initial point of contact. Ordinarily it is quite difficult and laborious to calculate deformations that are geometrically compatible, that satisfy equations of motion and that give equal but opposite reaction forces on the colliding bodies. To avoid this detail, several different approximations have been developed for analyzing impact; rigid body impact theory, Hertz contact theory, elastic wave theory, etc. This book presents a spectrum of different theories for collision and describes where each is applicable. The question of applicability largely depends on the materials of which the bodies are composed (their hardness in the contact region and whether or not they are rate-dependent), the geometric configuration of the bodies and the incident relative velocity of the collision. These factors affect the relative magnitude of deformations in the contact region in comparison with global deformations.

A collision between hard bodies occurs in a very brief period of time. The duration of contact between a ball and bat, a hammer and nail, or an automobile and lamp post is no more than a few milliseconds. This brief period has been used to justify rigid body impact theory in which bodies instantaneously change velocity when they collide. As a consequence of the instantaneous period of contact, the bodies have negligible displacement during the collision. For any analysis of changes in momentum occurring during impact, the approximation that displacements are negligibly small greatly simplifies the analysis. With this approximation, the changes in velocity can be calculated without integrating accelerations over the contact period. Along with this simplification, however, there is a hazard associated with loss of information about the contact forces that cause these changes in velocity – without forces the changes in velocity cannot be directly associated with deformability of the bodies.

In order to solve more complex problems, particularly those involving friction, we develop a method that spreads out the changes in velocity by considering that they are a continuous function of impulse rather than time. With this approach, the approximation of negligible displacement during a very brief period of contact results in an equation of motion with constant coefficients; this equation is trivially integrable to obtain changes in momentum of each body as a function of the impulse of contact force. This permits the analyst to follow the evolution of contact and variation in relative velocity across the contact patch as a function of impulse.

For rigid body impact theory the equations of dynamics are not sufficient to solve for the changes in velocity – an additional relation is required. Commonly this relation is provided by the *coefficient of restitution*. Most books on mechanics treat the coefficient of restitution as an impact law; i.e., for the contact points of colliding bodies, they consider the coefficient of restitution to be an empirical relationship for the normal component of relative velocity at incidence and separation. This has been satisfactory for collisions between smooth bodies but for bodies with rough surfaces where friction opposes sliding during impact, the usual kinematic (or Newton) coefficient of restitution has a serious deficiency. In the technical or scientific literature, the topic of rigid body impact was reopened in 1984 largely as a consequence of some problems where calculations employing the kinematic coefficient gave solutions which were patently unrealistic – for collisions in which friction opposes small initial slip, calculations that employed the kinematic coefficient of restitution predicted an increase in kinetic energy as a consequence of the collision. In order to rectify this problem and clearly separate dissipation due to friction from that due to irreversible internal deformations near the contact point of colliding bodies, a different definition of the coefficient of restitution (termed the energetic coefficient of restitution) was proposed and is used throughout this book. In those problems where friction is negligible or where slip is unidirectional during contact, the energetic and kinematic coefficients are equal – if slip changes in direction during contact however, these coefficients are distinct.

These methods will be illustrated by analyses of practical examples. Many of these are taken from sport; e.g. the bounce of a hockey puck, the spin (and consequent hook or slice) resulting from mis-hitting a golf shot and an analysis of batting for maximum range.

While rigid body impact theory is effective for analyzing the response of hard bodies, more complex analytical descriptions are required if a colliding body is soft or deformable; i.e., if the collision generates significant structural deformations far from the contact region. This occurs if the impact occurs near a slender section of a colliding body or if the body is hollow as in the case of an inflated ball. To calculate the response of deformable bodies, a time-dependent analysis is required since the contact force depends on local deformation of the body. In this case the response depends on the compliance of the respective contact regions in addition to the inertia properties and initial relative velocities that determine the outcome for rigid body impacts. The reward for the additional complexity of time-dependent impact analysis for deformable bodies is that an empirically determined coefficient of restitution is no longer required to relate the final and initial states of the system. This relationship can be calculated for any particular material and structural properties. For colliding bodies that are compact in shape and composed of hard materials, the contact stresses rapidly diffuse so that substantial deformations occur only local to the point of initial contact; in this case the change of state resulting from impact can be calculated on the basis of quasi-static continuum mechanics. On the other hand, for impacts that are transverse to some slender member where the collision generates vibratory motion far from the site of impact, the calculation must be based on structural dynamics of beams, plates, or shells. Examples are provided for impact between elastic-plastic solids and for collisions against slender elastic plates and beams.

This textbook has evolved from lecture notes prepared for an upper division course presented originally at the University of California, Davis. A later version of the course was tested on students at the National University of Singapore. Those notes have been expanded by additional material developed subsequently by my students, colleagues and me. Our interest has been in developing more physically based analytical models in order to improve the accuracy of calculations of impact response and to increase the range of applicability for any measurement of collision properties of a system. In these respects this book is complementary to the neo-classical treatise *Impact: the Theory and Physical Behaviour of Colliding Solids*, by W. Goldsmith – a monograph which provides a wealth of experimental data on collision behavior of metals, glasses and natural materials. The present text has stepped off from this base to incorporate the physically based knowledge of mechanics of collision that has been developed in the last fifty years. The background required in order to appreciate the analytical methods described here are undergraduate engineering courses in dynamics, strength of materials, and vibrations.

Acknowledgments

In writing this second edition I have drawn heavily on my papers previously published in technical journals; sometimes these were written in company with graduate students and/or colleagues. I wish to thank Calloway Golf Company for providing photographs in Chapter 12. Furthermore, I wish to acknowledge that permission to use copyrighted material has been granted by Elsevier Publications, International Sports Engineering Association, American Society of Mechanical Engineers, and Royal Society of London. At the back of this book specific permissions are listed in a Permissions section. The cover illustration was drawn by Ben Yip.

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W. J. Stronge
Frontmatter
More Information

Symbols

Man is not a circle with a single center; he is an ellipse with two foci. Facts are one, ideas are the other.

– Victor Hugo, *Les Miserables*

a	radius of cylinder or sphere; radius of contact area
$\bar{a}$	$= a_c/a_Y$, nondimensional maximum contact radius
b	width, thickness
c	dashpot force coefficient (also see μ_0)
c_0	longitudinal wave speed, uniaxial stress (thin bars)
c_g	$= d\omega/dk$, group velocity of propagating waves
c_p	$= \omega/k$, phase velocity of propagating waves
e_N, e_P, e_*	kinematic, kinetic, energetic coefficient of restitution
f, g, h	functions
h_O	moment of momentum about O
g	$= 9.81$ ms^{-2}, gravitational constant
i	$= \sqrt{-1}$ imaginary unit; typical number in series
k	$= 2\pi/\lambda$, wave number
k_r	area radius of gyration for cross-section of bar about centroid
$\hat{k}_r$	mass radius of gyration of body B for center of mass
m	$= \left(M^{-1}+M'^{-1}\right)^{-1}$, effective mass
m_{ij}	inertia matrix for contact point C
m	generalized inertia matrix (r x r where r = number generalized coordinates)
n_3	unit vector normal to common tangent plane
p	$= p_3$, normal component of reaction impulse at point of contact
p_c	normal impulse at transition from period of compression to restitution
p_f	normal impulse at termination of restitution period
p_s	normal impulse at termination of initial period of sliding
q	transverse force per unit length
q_r	generalized coordinate
r	radial coordinate
r_i, r'_i	position vectors from centers of mass G & G$'$ to point of contact C.

s	$= \sqrt{v_1^2 + v_2^2}$, sliding speed at any normal impulse
$\hat{s}$	$= \mathrm{sgn}(v_1)$, direction of sliding (planar changes of velocity)
t	time
t_1	time of transition from initial stick to sliding
t_2	time of transition from sliding to stick
u_i	components of displacement
$\dot{u}_I$	particle velocity in wave incoming to interface
$\dot{u}_R$	particle velocity in wave reflected from interface
$\dot{u}_T$	particle velocity in wave transmitted through interface
v	$= v_3$, normal component of relative velocity of coincident contact points
v_0	normal component of relative velocity of contact points at incidence
v_f	normal component of relative velocity at termination of restitution
x	axial coordinate
y	transverse coordinate, nondimensional indentation
z	depth coordinate
A	area of cross-section
A_i	constant
D_i	dissipation of energy from work of ith component of force
E, E'	Young's moduli of material in bodies B, B$'$
E_*	$= E_1E_2/(E_1 + E_2)$, effective Young's modulus at contact
F	$= F_3$, normal force at contact point
F_i	components of contact force
G	shear modulus of material
I_{ij}, I'_{ij}	moments, products of inertia for bodies B, B$'$ about respective center of mass
L	length
M	bending moment at section of beam
M, M'	mass of rigid bodies B, B$'$, respectively
P, P'	normal component of impulse acting on bodies B, B$'$, respectively
R	radius of cylinder, sphere
R_*	$= R_1R_2/(R_1 + R_2)$ effective radius of contact curvature
$\bar{R}_*$	effective radius of contact curvature after plastic deformation
S	shear force at section of beam
T	kinetic energy of system of colliding bodies
T_0	incident kinetic energy of system
T_f	final kinetic energy of system at termination of period of restitution
U	potential energy (e.g., gravitational potential)
V_i, V'_i	components of velocity at contact points C & C$'$
$\hat{V}_i$, $\hat{V}'_i$	components of velocity at centers of mass of bodies B & B$'$
W_n, W_3	work of normal component of reaction force at C
W_c	$= W_3(p_c)$ work of normal force during compression
W_f	$= W_3(p_f)$ final work of normal force

$\bar{W}_f$	$= W_1(p_f) + W_2(p_f) + W_3(p_f)$, final total work of contact reaction
X	nondimensional displacement
Y	yield stress
Z	$= dX/d(\omega t)$, nondimensional velocity
α	$= M/\rho AL$, mass ratio
$\beta_1, \beta_2, \beta_3$	inertia coefficients (planar changes in velocity)
γ	$EI/\rho A$
γ	$= \beta_2^2/\beta_1\beta_2$, inertia parameter
γ_R	$= (\Gamma - 1)/(\Gamma + 1)$, reflection coefficient
γ_T	$= 2(A_1\Gamma/A_2)/(\Gamma + 1)$, transmission coefficient
γ_1	$= \left[p(t_f) - p(t_1)\right]/p_c$, ratio of impulse during final slip to p_c
γ_0	shear warping at neutral axis
$\bar{\gamma}$	$= \Xi\gamma_0$, shear rotation of cross-section
δ	relative indentation at contact point
ε_{ijk}	permutation tensor
ε_{ij}	components of strain
ζ	$= c$ / c_{cr} damping ratio
η	local coordinate
η	$= x - ct$, Galilean coordinate
η	square root of ratio of tangential to normal compliance
θ	$= dw/dx$, rotation of section; inclination of body
ϑ	ratio of kinetic energy of toppling group to that of leading element
ϑ_Y	ratio of mean fully plastic indentation pressure to uniaxial yield stress
κ	stiffness coefficient of spring element
λ	wave length of propagating disturbance
μ_0	dashpot force coefficient
μ	Amontons–Coulomb coefficient of limiting friction (dry friction)
$\bar{\mu}$	friction coefficient for stick
ν	Poisson’s ratio
ξ	local coordinate
ξ	$= x + ct$, galilean coordinate
$\bar{\xi}$	$= 2d/a$, characteristic depth for plane strain deformation field
ρ	mass density
σ_{ij}	components of stress
τ	nondimensional time; characteristic time
φ	$\omega_{i+1}(-)/\omega_{i+1}(+)$, ratio angular speeds before and after impact
ϕ	$=$ $\tan^{-1}(v_2/v_1)$, sliding direction in tangent plane
$\hat{\phi}$	isoclinic direction of slip
$\hat{\phi}_*$	separatrix direction of slip
χ	stiffness ratio
ψ_0	angle of incidence for relative velocity at contact point
ψ_f	angle of rebound for relative velocity at contact point

ω	tangential resonant frequency
ω_0	initial angular velocity
ω_0	$=\kappa/m$, characteristic frequency of oscillation
ω_c	cut-off frequency for propagation
ω_d	damped resonant frequency
ω_i, ω_i'	angular velocity vectors for bodies B & B′
ω_i	angular speed for contact at point i
Γ	$=A_2\rho_2c_2/A_1\rho_1c_1$, impedance ratio
Ξ	Timoshenko beam coefficient
Σ_0	$=F_0/A$, negative pressure
Φ	internal volume
Ψ	geometry of polygonal solid
Ω	normal resonant frequency
$\mathbf{e}$	unit vector parallel to common tangent plane
$\mathbf{h}_O$	moment of momentum about point O
$\hat{\mathbf{h}}$	moment of momentum about center of mass
$\mathbf{n}$	unit vector normal to common tangent plane
$\mathbf{r}_i$	position vector of ith particle relative to center of mass
$\hat{\mathbf{s}}$	$\equiv \mathrm{v_e}/\lvert\mathrm{v_e}\rvert$, direction of sliding (3D)
$\mathbf{v}$	= V – V′ relative velocity across contact point
$\mathbf{F}_i$	force on ith particle
$\hat{\mathbf{I}}$	moment of inertia for center of mass
$\mathbf{P}_i$	impulse on ith particle
$\mathbf{V}_i$	velocity of ith particle
$\hat{\mathbf{V}}$	velocity of center of mass
$\boldsymbol{\rho}_i$	position vector of ith particle relative to inertial reference frame
$\boldsymbol{\Phi}$	matrix
$\boldsymbol{\Omega}$	angular velocity of rigid body