QUANTUM FINANCE

Path Integrals and Hamiltonians for Options and Interest Rates

This book applies the mathematics and concepts of quantum mechanics and quantum field theory to the modelling of interest rates and the theory of options. Particular emphasis is placed on path integrals and Hamiltonians.

Financial mathematics at present is almost completely dominated by stochastic calculus. This book is unique in that it offers a formulation that is completely independent of that approach. As such many new results emerge from the ideas developed by the author.

This pioneering work will be of interest to physicists and mathematicians working in the field of finance, to quantitative analysts in banks and finance firms, and to practitioners in the field of fixed income securities and foreign exchange. The book can also be used as a graduate text for courses in financial physics and financial mathematics.

BELAL E. BAAQUIE earned his B.Sc. from Caltech and Ph.D. in theoretical physics from Cornell University. He has published over 50 papers in leading international journals on quantum field theory and related topics, and since 1997 has regularly published papers on applying quantum field theory to both the theoretical and empirical aspects of finance. He helped to launch the *International Journal of Theoretical and Applied Finance* in 1998 and continues to be one of the editors.

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I dedicate this book to my father Mohammad Abdul Baaquie and to the memory of my mother Begum Ajmeri Roanaq Ara Baaquie, for their precious lifelong support and encouragement.

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Foreword

After a few early isolated cases in the 1980s, since the mid-1990s hundreds of papers dealing with economics and finance have invaded the physics preprint server xxx.lanl.gov/cond-mat, initially devoted to condensed matter physics, and now covering subjects as different as computer science, biology or probability theory. The flow of paper posted on this server is still increasing – roughly one per day – addressing a range of problems, from financial data analysis to analytical option-pricing methods, agent-based models of financial markets and statistical models of wealth distribution or company growth. Some papers are genuinely beautiful, others are rediscoveries of results known by economists, and unfortunately some are simply crazy.

A natural temptation is to apply the tools one masters to other fields. In the case of physics and finance, this temptation is extremely strong. The sophisticated tools developed in the last 50 years to deal with statistical mechanics and quantum mechanics problems are often of immediate interest in finance and in economics. Perturbation theory, path integral (Feynman–Kac) methods, random matrix and spin-glass theory are useful for option pricing, portfolio optimization and game theoretical situations, and many new insights have followed from such transfers of knowledge.

Within theoretical physics, quantum field theory has a special status and is regarded by many as the queen of disciplines, that has allowed one to unravel the most intimate intricacies of nature, from quantum electrodynamics to critical phenomena. In the present book, Belal Baaquie tells us how these methods can be applied to finance problems, and in particular to the modelling of interest rates. The interest rate curve can be seen as a string of numbers, one for each maturity, fluctuating in time. The 'one-dimensional' nature of these randomly fluctuating rates imposes subtle correlations between different maturities, that are most naturally described using quantum field theory, which was indeed created to deal with nontrivial correlations between fluctuating fields. The level of complexity of the bond market (reflecting the structure of the interest rate curve) and its derivatives (swaps, caps, floors, swaptions) requires a set of efficient and adapted techniques. My feeling is that the methods of quantum field theory, which naturally grasp complex structures, are particularly well suited for this type of problems. Belal Baaquie's book, based on his original work on the subject, is particularly useful for those who want to learn techniques which will become, in a few years, unavoidable. Many new ideas and results improving our understanding of interest rate markets will undoubtedly follow from an in-depth exploration of the paths suggested in this fascinating (albeit sometimes demanding) opus.

Jean-Philippe Bouchaud
Capital Fund Management and CEA-Saclay

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Preface

Financial markets have undergone tremendous growth and dramatic changes in the past two decades, with the volume of daily trading in currency markets hitting over a trillion US dollars and hundreds of billions of dollars in bond and stock markets. Deregulation and globalization have led to large-scale capital flows; this has raised new problems for finance as well as has further spurred competition among banks and financial institutions.

The resulting booms, bubbles and busts of the global financial markets now directly affect the lives of hundreds of millions of people, as was witnessed during the 1998 East Asian financial crisis.

The principles of banking and finance are fairly well established [16, 76, 87] and the challenge is to apply these principles in an increasingly complicated environment. The immense growth of financial markets, the existence of vast quantities of financial data and the growing complexity of the market, both in volume and sophistication, has made the use of powerful mathematical and computational tools in finance a necessity. In order to meet the needs of customers, complex financial instruments have been created; these instruments demand advanced valuation and risk assessment models and systems that quantify the returns and risks for investors and financial institutions [63, 100].

The widespread use in finance of stochastic calculus and of partial differential equations reflects the traditional presence of probabilists and applied mathematicians in this field. The last few years has seen an increasing interest of theoretical physicists in the problems of applied and theoretical finance. In addition to the vast corpus of literature on the application of stochastic calculus to finance, concepts from theoretical physics have been finding increasing application in both theoretical and applied finance. The influx of ideas from theoretical physics, as expressed for example in [18] and [69], has added a whole collection of new mathematical and computational techniques to finance, from the methods of classical and quantum physics to the use of path integration, statistical mechanics and so on. This book is part of the on-going process of applying ideas from physics to finance.

The long-term goal of this book is to contribute to a quantum theory of finance; towards this end the theoretical tools of quantum physics are applied to problems in finance. The larger question of applying the formalism and insights of (quantum) physics to economics, and which forms a part of the larger subject of econophysics [88, 89], is left for future research.

The mathematical background required of the readership is the following:

- A good grasp of calculus
- Familiarity with linear algebra
- Working knowledge of probability theory

The material covered in this book is primarily meant for physicists and mathematicians conducting research in the field of finance, as well as professional theorists working in the finance industry. Specialists working in the field of derivative instruments, corporate and Treasury Bonds and foreign currencies will hopefully find that the theoretical tools and mathematical ideas introduced in this book broadens their repertoire of quantitative approaches to finance.

This book could also be of interest to researchers from the theoretical sciences who are thinking of pursuing research in the field of finance as well as graduates students with the required mathematical training. An earlier draft of this book was taught as an advanced graduate course to a group of students from financial engineering, physics and mathematics.

Given the diverse nature of the potential audience, fundamental concepts of finance have been reviewed to motivate readers new to the field. The chapters on 'Introduction to finance' and on 'Derivative securities' are meant for physicists and mathematicians unfamiliar with concepts of finance. On the other hand, discussions on quantum mechanics and quantum field theory are meant to introduce specialists working in finance and in mathematics to concepts from quantum theory.

Acknowledgments

I am deeply grateful to Lawrence Ma for introducing me to the subject of theoretical finance; most of my initial interest in mathematical finance is a result of the patient explanations of Lawrence.

I thank Jean-Philippe Bouchaud for instructive and enjoyable discussions, and for making valuable suggestions that have shaped my thinking on finance; the insights that Jean-Philippe brings to the interface of physics and finance have been particularly enlightening.

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