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Cambridge University Press
978-0-521-84006-4 - Particle Detectors, Second Edition
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Interactions of particles and radiation with matter

When the intervals, passages, connections, weights, impulses, collisions, movement, order, and position of the atoms interchange, so also must the things formed by them change.

Lucretius

Particles and radiation can be detected only through their interactions with matter. There are specific interactions for charged particles which are different from those of neutral particles, e.g. of photons. One can say that every interaction process can be used as a basis for a detector concept. The variety of these processes is quite rich and, as a consequence, a large number of detection devices for particles and radiation exist. In addition, for one and the same particle, different interaction processes at different energies may be relevant.

In this chapter, the main interaction mechanisms will be presented in a comprehensive fashion. Special effects will be dealt with when the individual detectors are being presented. The interaction processes and their cross sections will not be derived from basic principles but are presented only in their results, as they are used for particle detectors.

The main interactions of charged particles with matter are *ionisation* and *excitation*. For relativistic particles, *bremsstrahlung* energy losses must also be considered. Neutral particles must produce charged particles in an interaction that are then detected via their characteristic interaction processes. In the case of photons, these processes are the photoelectric effect, Compton scattering and pair production of electrons. The electrons produced in these *photon interactions* can be observed through their ionisation in the sensitive volume of the detector.

1.1 Interactions of charged particles

Charged particles passing through matter lose kinetic energy by *excitation* of bound electrons and by *ionisation*. Excitation processes like

$$\begin{aligned} e^- + \text{atom} \rightarrow \text{atom}^* + e^- & \\ \hookrightarrow \text{atom} + \gamma & \end{aligned} \tag{1.1}$$

lead to low-energy photons and are therefore useful for particle detectors which can record this luminescence. Of greater importance are pure scattering processes in which incident particles transfer a certain amount of their energy to atomic electrons so that they are liberated from the atom.

The *maximum transferable kinetic energy* to an electron depends on the mass m_0 and the momentum of the incident particle. Given the momentum of the incident particle

$$p = \gamma m_0 \beta c \;, \tag{1.2}$$

where γ is the Lorentz factor $(= E/m_0c^2)$, $\beta c = v$ the velocity, and m_0 the rest mass, the maximum energy that may be transferred to an electron (mass m_e) is given by [1] (see also Problem 1.6)

$$E_{\text{kin}}^{\max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e/m_0 + (m_e/m_0)^2} = \frac{2m_e p^2}{m_0^2 + m_e^2 + 2m_e E/c^2} \;. \tag{1.3}$$

In this case, it makes sense to give the kinetic energy, rather than total energy, since the electron is already there and does not have to be produced. The kinetic energy E_{kin} is related to the total energy E according to

$$E_{\text{kin}} = E - m_0c^2 = c\sqrt{p^2 + m_0^2c^2} - m_0c^2 \;. \tag{1.4}$$

For low energies

$$2\gamma m_e/m_0 \ll 1 \tag{1.5}$$

and under the assumption that the incident particles are heavier than electrons $(m_0 > m_e)$ Eq. (1.3) can be approximated by

$$E_{\text{kin}}^{\max} \approx 2m_e c^2 \beta^2 \gamma^2 \;. \tag{1.6}$$

A particle (e.g. a muon, $m_\mu c^2 = 106\,\text{MeV}$) with a Lorentz factor of $\gamma = E/m_0c^2 = 10$ corresponding to $E = 1.06\,\text{GeV}$ can transfer approximately $100\,\text{MeV}$ to an electron (mass $m_ec^2 = 0.511\,\text{MeV}$).

If one neglects the quadratic term in the denominator of Eq. (1.3), $(m_e/m_0)^2 \ll 1$, which is a good assumption for all incident particles except for electrons, it follows that

$$E_{\mathrm{kin}}^{\mathrm{max}} = \frac{p^2}{\gamma m_0 + m_0^2/2m_e} \quad . \tag{1.7}$$

For relativistic particles $E_{\mathrm{kin}} \approx E$ and $pc \approx E$ holds. Consequently, the maximum transferable energy is

$$E^{\mathrm{max}} \approx \frac{E^2}{E + m_0^2c^2/2m_e} \tag{1.8}$$

which for muons gives

$$E^{\mathrm{max}} = \frac{E^2}{E + 11\,\mathrm{GeV}} \quad . \tag{1.9}$$

In the extreme relativistic case $\left(E \gg m_0^2c^2/2m_e\right)$, the total energy can be transferred to the electron.

If the incident particle is an electron, these approximations are no longer valid. In this case, one gets, compare Eq. (1.3),

$$E_{\mathrm{kin}}^{\mathrm{max}} = \frac{p^2}{m_e + E/c^2} = \frac{E^2 - m_e^2c^4}{E + m_ec^2} = E - m_ec^2 \quad , \tag{1.10}$$

which is also expected in classical non-relativistic kinematics for particles of equal mass for a central collision.

1.1.1 Energy loss by ionisation and excitation

The treatment of the maximum transferable energy has already shown that incident electrons, in contrast to heavy particles ($m_0 \gg m_e$), play a special rôle. Therefore, to begin with, we give the energy loss for 'heavy' particles. Following Bethe and Bloch [2–8]*, the average energy loss dE per length dx is given by

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = 4\pi N_{\mathrm{A}} r_e^2 m_e c^2 z^2 \frac{Z}{A} \frac{1}{\beta^2} \left(\ln \frac{2m_ec^2\gamma^2\beta^2}{I} - \beta^2 - \frac{\delta}{2} \right) \quad , \tag{1.11}$$

* For the following considerations and formulae, not only the original literature but also secondary literature was used, mainly [1, 4–12] and references therein.

where

- z – charge of the incident particle in units of the elementary charge
- Z, A – atomic number and atomic weight of the absorber
- m_e – electron mass
- r_e – classical electron radius ($r_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{m_e c^2}$ with ε_0 – permittivity of free space)
- N_A – Avogadro number (= number of atoms per gram atom) $= 6.022 \cdot 10^{23}\,\mathrm{mol}^{-1}$
- I – mean excitation energy, characteristic of the absorber material, which can be approximated by

$$I = 16\ Z^{0.9}\,\mathrm{eV} \quad \text{for } Z > 1\ .$$

To a certain extent, I also depends on the molecular state of the absorber atoms, e.g. $I = 15\,\mathrm{eV}$ for atomic and 19.2 eV for molecular hydrogen. For liquid hydrogen, I is 21.8 eV.

- δ – is a parameter which describes how much the extended transverse electric field of incident relativistic particles is screened by the charge density of the atomic electrons. In this way, the energy loss is reduced (*density effect*, 'Fermi plateau' of the energy loss). As already indicated by the name, this density effect is important in dense absorber materials. For gases under normal pressure and for not too high energies, it can be neglected.
For energetic particles, δ can be approximated by

$$\delta = 2 \ln \gamma + \zeta\ ,$$

where ζ is a material-dependent constant.
Various approximations for δ and material dependences for parameters, which describe the density effect, are discussed extensively in the literature [9]. At very high energies

$$\delta/2 = \ln(\hbar\omega_\mathrm{p}/I) + \ln \beta\gamma - 1/2\ ,$$

where $\hbar\omega_\mathrm{p} = \sqrt{4\pi N_e r_e^3}\, m_e c^2/\alpha = 28.8\sqrt{\varrho\,\langle Z/A\rangle}\,\mathrm{eV}$ is the plasma energy (ϱ in g/cm^3), N_e the electron density, and α the fine-structure constant.

A useful constant appearing in Eq. (1.11) is

$$4\pi N_{\mathrm{A}} r_e^2 m_e c^2 = 0.3071 \, \frac{\mathrm{MeV}}{\mathrm{g/cm^2}} \quad . \tag{1.12}$$

In the logarithmic term of Eq. (1.11), the quantity $2m_e c^2 \gamma^2 \beta^2$ occurs in the numerator, which, according to Eq. (1.6), is identical to the maximum transferable energy. The average energy of electrons produced in the ionisation process in gases equals approximately the ionisation energy [2, 3].

If one uses the approximation for the maximum transferable energy, Eq. (1.6), and the shorthand

$$\kappa = 2\pi N_{\mathrm{A}} r_e^2 m_e c^2 z^2 \cdot \frac{Z}{A} \cdot \frac{1}{\beta^2} \quad , \tag{1.13}$$

the *Bethe–Bloch formula* can be written as

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = 2\kappa \left(\ln \frac{E_{\mathrm{kin}}^{\mathrm{max}}}{I} - \beta^2 - \frac{\delta}{2} \right) \quad . \tag{1.14}$$

The energy loss $-\mathrm{d}E/\mathrm{d}x$ is usually given in units of $\mathrm{MeV/(g/cm^2)}$. The length unit $\mathrm{d}x$ (in $\mathrm{g/cm^2}$) is commonly used, because the energy loss per area density

$$\mathrm{d}x = \varrho \cdot \mathrm{d}s \tag{1.15}$$

with ϱ density (in $\mathrm{g/cm^3}$) and $\mathrm{d}s$ length (in cm) is largely independent of the properties of the material. This length unit $\mathrm{d}x$ consequently gives the area density of the material.

Equation (1.11) represents only an approximation for the energy loss of charged particles by ionisation and excitation in matter which is, however, precise at the level of a few per cent up to energies of several hundred GeV. However, Eq. (1.11) cannot be used for slow particles, i.e., for particles which move with velocities which are comparable to those of atomic electrons or slower. For these velocities ($\alpha z \gg \beta \geq 10^{-3}$, $\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$: fine-structure constant) the energy loss is proportional to β. The energy loss of slow protons, e.g. in silicon, can be described by [10–12]

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = 61.2 \, \beta \, \frac{\mathrm{GeV}}{\mathrm{g/cm^2}} \quad , \quad \beta < 5 \cdot 10^{-3} \quad . \tag{1.16}$$

Equation (1.11) is valid for all velocities

$$\beta \gg \alpha z \quad . \tag{1.17}$$

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Table 1.1. *Average energy loss of minimum-ionising particles in various materials [10–12]; gases for standard pressure and temperature*

Absorber	$\frac{dE}{dx}\big\vert_{\min}\left[\frac{\mathrm{MeV}}{\mathrm{g/cm^2}}\right]$	$\frac{dE}{dx}\big\vert_{\min}\left[\frac{\mathrm{MeV}}{\mathrm{cm}}\right]$
Hydrogen (H_2)	4.10	$0.37 \cdot 10^{-3}$
Helium	1.94	$0.35 \cdot 10^{-3}$
Lithium	1.64	0.87
Beryllium	1.59	2.94
Carbon (Graphite)	1.75	3.96
Nitrogen	1.82	$2.28 \cdot 10^{-3}$
Oxygen	1.80	$2.57 \cdot 10^{-3}$
Air	1.82	$2.35 \cdot 10^{-3}$
Carbon dioxide	1.82	$3.60 \cdot 10^{-3}$
Neon	1.73	$1.56 \cdot 10^{-3}$
Aluminium	1.62	4.37
Silicon	1.66	3.87
Argon	1.52	$2.71 \cdot 10^{-3}$
Titanium	1.48	6.72
Iron	1.45	11.41
Copper	1.40	12.54
Germanium	1.37	7.29
Tin	1.26	9.21
Xenon	1.25	$7.32 \cdot 10^{-3}$
Tungsten	1.15	22.20
Platinum	1.13	24.24
Lead	1.13	12.83
Uranium	1.09	20.66
Water	1.99	1.99
Lucite	1.95	2.30
Shielding concrete	1.70	4.25
Quartz (SiO_2)	1.70	3.74

Given this condition, the energy loss decreases like $1/\beta^2$ in the low-energy domain and reaches a broad minimum of ionisation near $\beta\gamma \approx 4$. Relativistic particles ($\beta \approx 1$), which have an energy loss corresponding to this minimum, are called *minimum-ionising particles* (MIPs). In light absorber materials, where the ratio $Z/A \approx 0.5$, the energy loss of minimum-ionising particles can be roughly represented by

$$-\left.\frac{dE}{dx}\right|_{\min} \approx 2\,\frac{\mathrm{MeV}}{\mathrm{g/cm^2}} \ . \tag{1.18}$$

In Table 1.1, the energy losses of minimum-ionising particles in different materials are given; for further values, see [10–12].

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The energy loss increases again for $\gamma > 4$ (*logarithmic rise* or *relativistic rise*) because of the logarithmic term in the bracket of Eq. (1.11). The increase follows approximately a dependence like $2 \ln \gamma$.

The decrease of the energy loss at the ionisation minimum with increasing atomic number of the absorber originates mainly from the Z/A term in Eq. (1.11). A large fraction of the logarithmic rise relates to large energy transfers to few electrons in the medium (δ *rays* or knock-on electrons). Because of the density effect, the logarithmic rise of the energy loss saturates at high energies.

For heavy projectiles (e.g. like copper nuclei), the energy loss of slow particles is modified because, while being slowed down, electrons get attached to the incident nuclei, thereby decreasing their effective charge.

The energy loss by ionisation and excitation for muons in iron is shown in Fig. 1.1 [10, 11, 13].

The energy loss according to Eq. (1.11) describes only energy losses due to ionisation and excitation. At high energies, radiation losses become more and more important (see Sect. 1.1.5).

Figure 1.2 shows the ionisation energy loss for electrons, muons, pions, protons, deuterons and α particles in air [14].

Equation (1.11) gives only the average energy loss of charged particles by ionisation and excitation. For thin absorbers (in the sense of Eq. (1.15), average energy loss $\langle \Delta E \rangle \ll E_{\max}$), in particular, strong fluctuations around the average energy loss exist. The energy-loss distribution for thin absorbers is strongly asymmetric [2, 3].

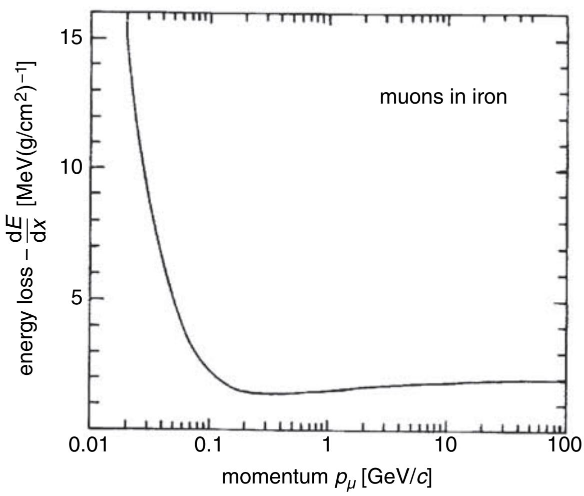

Fig. 1.1. Energy loss by ionisation and excitation for muons in iron and its dependence on the muon momentum.

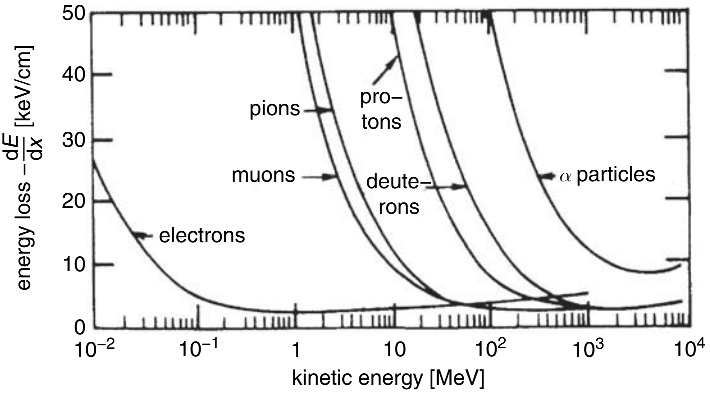

Fig. 1.2. Energy loss for electrons, muons, pions, protons, deuterons and α particles in air [14].

This behaviour can be parametrised by a *Landau distribution*. The Landau distribution is described by the inverse Laplace transform of the function s^s [15–18]. A reasonable approximation of the Landau distribution is given by [19–21]

$$L(\lambda) = \frac{1}{\sqrt{2\pi}} \cdot \exp\left[-\frac{1}{2}(\lambda + \mathrm{e}^{-\lambda})\right] , \tag{1.19}$$

where λ characterises the deviation from the *most probable energy loss*,

$$\lambda = \frac{\Delta E - \Delta E^{\mathrm{W}}}{\xi} , \tag{1.20}$$

ΔE – actual energy loss in a layer of thickness x,
ΔE^{W} – most probable energy loss in a layer of thickness x,

$$\xi = 2\pi N_{\mathrm{A}} r_e^2 m_e c^2 z^2 \frac{Z}{A} \cdot \frac{1}{\beta^2} \varrho x = \kappa \varrho x \tag{1.21}$$

(ϱ – density in g/cm^3, x – absorber thickness in cm).

The general formula for the most probable energy loss is [12]

$$\Delta E^{\mathrm{W}} = \xi \left[\ln\left(\frac{2m_e c^2 \gamma^2 \beta^2}{I}\right) + \ln\frac{\xi}{I} + 0.2 - \beta^2 - \delta(\beta\gamma)\right] . \tag{1.22}$$

For example, for argon and electrons of energies up to 3.54 MeV from a ^{106}Rh source the most probable energy loss is [19]

$$\Delta E^{\mathrm{W}} = \xi \left[\ln \left(\frac{2m_e c^2 \gamma^2 \beta^2}{I^2} \xi \right) - \beta^2 + 0.423 \right] . \tag{1.23}$$

The most probable energy loss for minimum-ionising particles ($\beta\gamma = 4$) in 1 cm argon is $\Delta E^{\mathrm{W}} = 1.2$ keV, which is significantly smaller than the average energy loss of 2.71 keV [2, 3, 19, 22]. Figure 1.3 shows the energy-loss distribution of 3 GeV electrons in a thin-gap drift chamber filled with Ar/CH_4 (80:20) [23].

Experimentally, one finds that the actual energy-loss distribution is frequently broader than represented by the Landau distribution.

For thick absorber layers, the tail of the Landau distribution originating from high energy transfers, however, is reduced [24]. For very thick absorbers $\left(\frac{\mathrm{d}E}{\mathrm{d}x} \cdot x \gg 2m_e c^2 \beta^2 \gamma^2\right)$, the energy-loss distribution can be approximated by a Gaussian distribution.

The energy loss $\mathrm{d}E/\mathrm{d}x$ in a compound of various elements i is given by

$$\frac{\mathrm{d}E}{\mathrm{d}x} \approx \sum_i f_i \left. \frac{\mathrm{d}E}{\mathrm{d}x} \right|_i , \tag{1.24}$$

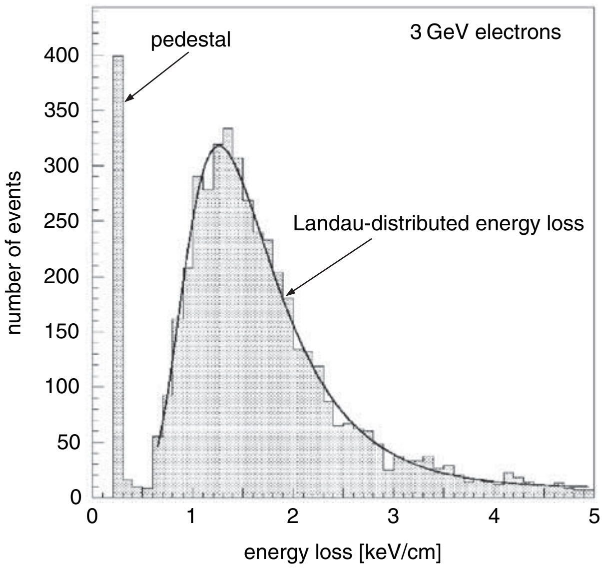

Fig. 1.3. Energy-loss distribution of 3 GeV electrons in a thin-gap drift chamber filled with Ar/CH_4 (80:20) [23].

where f_i is the mass fraction of the ith element and $\frac{\mathrm{d}E}{\mathrm{d}x}\big|_i$, the average energy loss in this element. Corrections to this relation because of the dependence of the ionisation constant on the molecular structure can be safely neglected.

The energy transfers to ionisation electrons can be so large that these electrons can cause further ionisation. These electrons are called δ rays or knock-on electrons. The energy spectrum of knock-on electrons is given by [1, 10–12, 25]

$$\frac{\mathrm{d}N}{\mathrm{d}E_{\mathrm{kin}}} = \xi \cdot \frac{F}{E_{\mathrm{kin}}^2} \tag{1.25}$$

for $I \ll E_{\mathrm{kin}} \leq E_{\mathrm{kin}}^{\mathrm{max}}$.

F is a spin-dependent factor of order unity, if $E_{\mathrm{kin}} \ll E_{\mathrm{kin}}^{\mathrm{max}}$ [12]. Of course, the energy spectrum of knock-on electrons falls to zero if the maximum transferable energy is reached. This kinematic limit also constrains the factor F [1, 25]. The spin dependence of the spectrum of the knock-on electrons only manifests itself close to the maximum transferable energy [1, 25].

The strong fluctuations of the energy loss in thin absorber layers are quite frequently not observed by a detector. Detectors only measure the energy which is actually deposited in their sensitive volume, and this energy may not be the same as the energy lost by the particle. For example, the energy which is transferred to knock-on electrons may only be partially deposited in the detector because the knock-on electrons can leave the sensitive volume of the detector.

Therefore, quite frequently it is of practical interest to consider only that part of the energy loss with energy transfers E smaller than a given cut value E_{cut}. This *truncated energy loss* is given by [10–12, 26]

$$-\frac{\mathrm{d}E}{\mathrm{d}x}\bigg|_{\leq E_{\mathrm{cut}}} = \kappa \left(\ln \frac{2m_e c^2 \beta^2 \gamma^2 E_{\mathrm{cut}}}{I^2} - \beta^2 - \delta \right) , \tag{1.26}$$

where κ is defined by Eq. (1.13). Equation (1.26) is similar, but not identical, to Eq. (1.11). Distributions of the truncated energy loss do not show a pronounced Landau tail as the distributions (1.19) for the mean value (1.11). Because of the density effect – expressed by δ in Eqs. (1.11) or (1.26), respectively – the truncated energy loss approaches a constant at high energies, which is given by the Fermi plateau.

So far, the energy loss by ionisation and excitation has been described for heavy particles. Electrons as incident particles, however, play a special rôle in the treatment of the energy loss. On the one hand, the total energy loss of electrons even at low energies (MeV range) is influenced by bremsstrahlung processes. On the other hand, the ionisation loss requires