

## Fundamentals of Atmospheric Modeling

### Second Edition

This well-received and comprehensive textbook on atmospheric processes and numerical methods has been thoroughly revised and updated. The new edition includes a wide range of new numerical techniques for solving problems in areas such as cloud microphysics, ocean-atmosphere exchange processes, and atmospheric radiative properties. It also contains improved descriptions of atmospheric physics, dynamics, radiation, aerosol, and cloud processes. Numerous examples and problems are included, with answers available to lecturers at <http://www.cambridge.org/0521548659>

*Fundamentals of Atmospheric Modeling* is essential reading for researchers and advanced students of atmospheric science, meteorology, and environmental science.

MARK Z. JACOBSON is an associate professor of civil and environmental engineering at Stanford University. Goals of his research are to improve our understanding of physical, chemical, and dynamical processes in the atmosphere through numerical modeling and to improve the simulation of air pollution, weather, and climate. He is the author of two textbooks: *Fundamentals of Atmospheric Modeling* and *Atmospheric Pollution: History, Science, and Regulation*.

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Mark Z. Jacobson  
Frontmatter  
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# Fundamentals of Atmospheric Modeling

Second Edition

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MARK Z. JACOBSON

*Stanford University*



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*To Dionna and Daniel*

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## Preface

Modern atmospheric science is a field that combines meteorology, physics, mathematics, chemistry, computer sciences, and to a lesser extent geology, biology, microbiology, and oceanographic sciences. Until the late 1940s scientific studies of the atmosphere were limited primarily to studies of the weather. At that time, heightened concern about air pollution caused an increase in studies of atmospheric chemistry. With the invention of the computer, modeling of weather and air pollution commenced. Since the late 1940s, the number of meteorological and air-pollution studies has increased rapidly, and many meteorological and air-pollution models have merged.

The purposes of this book are to provide (1) a physical understanding of dynamical meteorology, land- and water-surface processes, radiation, gas chemistry, aerosol microphysics and chemistry, and cloud processes, (2) a description of numerical methods and computational techniques used to simulate these processes, and (3) a catalog of steps required to construct, apply, and test a numerical model.

The first chapter of this book gives an overview of model processes and time scales. Chapter 2 describes atmospheric structure, composition, and thermodynamics. In Chapters 3–5, basic equations describing dynamical meteorology are derived. In Chapter 6, numerical methods of solving partial differential equations are discussed. A technique of solving dynamical meteorological equations is provided in Chapter 7. In Chapter 8, boundary-layer and ground processes are described. Chapter 9 introduces radiation. Chapters 10–12 focus on photochemistry and numerical methods of solving chemical equations. Chapters 13–17 describe aerosol physical and chemical processes. Chapter 18 discusses cloud thermodynamics and microphysics. Chapter 19 discusses aqueous chemistry in aerosol particles and clouds. Chapter 20 describes sedimentation and dry deposition. Chapter 21 outlines computer model development, application, and testing.

The book is designed as an upper-level undergraduate, graduate, and research text. The text assumes students have a basic physical science, mathematical, and computational background. Both *Système Internationale* (SI) and centimeter-gram-second (CGS) units are used. Dynamical meteorologists often use SI units, and atmospheric chemists often use CGS units. Thus, both unit systems are retained. Unit and variable conversions are given in Appendix A.

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