INTERNAL GRAVITY WAVES

The study of internal gravity waves provides many challenges: they move along interfaces as well as in fully three-dimensional space, and they do so at relatively fast temporal and small spatial scales – making them difficult to observe and resolve in weather and climate models. The equations describing their evolution are well established but their solution poses various mathematical challenges associated with singular boundary value problems and large amplitude dynamics.

This book provides the first comprehensive treatment of the theory for small- and large-amplitude internal gravity waves, whether existing as interfacial waves in a layered fluid or as internal waves in continuously stratified fluid. Over 120 schematics, numerical simulations and images from laboratory experiments illustrate the theory and mathematical techniques, while the 130 exercises allow the reader to test their understanding of the theory and its applications. This is an invaluable single resource for academic researchers and graduate students studying the motion of waves within the atmosphere and oceans, and more generally for mathematicians, physicists and engineers interested in the peculiar properties of propagating, growing and breaking waves.

Bruce R. Sutherland holds a B.Math. from the University of Waterloo and a Ph.D. in Physics from the University of Toronto where, under the supervision of W. R. Peltier, he ran numerical simulations of internal waves generated by shear instability. As a Research Associate working with P. F. Linden at the University of Cambridge, he helped develop the laboratory method known as the synthetic schlieren technique, which was applied to the examination of internal waves generated by turbulence. Now a Professor in the departments of Physics and of Earth and Atmospheric Sciences, and an Adjunct Professor in Mathematical and Statistical Sciences, at the University of Alberta, he continues to develop theories and to run laboratory experiments and numerical simulations that examine the generation, propagation and breaking of internal gravity waves.

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More information

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To Brendan, Cameron and Samantha

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Preface

Why write a book on internal gravity waves when so many other books cover the subject already? The textbooks listed in the appendix include at least some discussion of internal gravity waves. Some focus upon interfacial waves, which are internal gravity waves at interfaces; some focus upon internal waves, which exist in continuously stratified fluid. Different books emphasize different dynamics such as mechanisms for generation, propagation in non-uniform media, nonlinear evolution and stability. Textbooks on geophysical fluid dynamics (e.g. Gill (1982), Vallis (2006)) understandably devote only a chapter to the subject because, although internal waves are non-negligible in their influence upon global weather and ocean circulation patterns, they are by no means dominant. Internal waves are noise, if sometimes irritatingly loud. Textbooks on the theory of waves and instability (e.g. Whitham (1974), Lighthill (1978), Drazin and Reid (1981), Craik (1985)) examine how non-uniform media and nonlinearity affect the evolution of interfacial and internal waves. But these books can be daunting to graduate students lacking strong mathematical backgrounds. Textbooks on stratified fluid dynamics (e.g. Turner (1973), Baines (1995)) help to provide physical insight into the dynamics of internal gravity waves through a combination of theory and laboratory experiments, though sometimes without providing the mathematical details. Some textbooks are devoted to the subject of internal gravity waves (e.g. Miropol'sky (2001), Nappo (2002), Vlasenko *et al.* (2005)), but these focus either on atmospheric or oceanic waves.

The approach taken here is to provide the physics and mathematics describing internal gravity waves in a way that is accessible to students who have been exposed to multivariable calculus and ordinary differential equations. An understanding of partial differential equations, though useful, is not necessary. A background in atmosphere–ocean science and fluid dynamics is not assumed. Chapter 1 covers this material at an introductory level, presenting only those details that are necessary for modelling internal gravity waves and the environment in which they exist. This chapter also introduces the mathematical description of waves and their properties.

Chapter 2 describes the structure and evolution of periodic, small-amplitude interfacial waves, beginning with a detailed description of surface waves. Although surface waves are not internal gravity waves, they are part of everyone's common experience thus making it easier to draw the link between mathematical theory and reality. We will find that surface waves are a special case of internal gravity waves at the interface between two fluids. They occur in the limit where the upper layer density (that of air) is much smaller than the lower layer density (that of water). The discussion goes on to describe waves at the interface between fresh and salty water or between hot and cold fluid, whether a gas or liquid. In the presence of shear an otherwise flat interface may become unstable to undular disturbances. The influence of interfacial waves upon the growth and structure of the instability is also discussed in this chapter.

Whereas interfacial waves occur where the density decreases rapidly with height over a negligibly small distance, internal waves move vertically through a fluid whose (effective) density decreases continuously with height. The rate of this decrease determines a fundamental quantity used in the description of internal waves known as the buoyancy frequency. This is derived for liquids and gases at the start of Chapter 3. Thereafter, the equations for periodic, small-amplitude internal waves in uniformly stratified fluid are derived and solved. This chapter includes a discussion of the peculiar behaviour of internal waves near sloping boundaries and describes how their structure is affected by rotation and relatively rapid density changes with height.

Chapter 4 introduces the mathematics necessary to model waves of non-negligibly small amplitude. The changes in frequency and structure of finite-amplitude interfacial and internal waves are examined. Special attention is drawn to the case of finite-amplitude interfacial waves in shallow water which can take the form of hump-shaped, solitary waves. The chapter also describes the various forms of instability associated with waves including modulational instability, parametric subharmonic instability, overturning and shear instability.

Internal gravity waves are generated by flow over topography, convective storms, imbalance of large-scale circulations, thunderstorm outflows, river plumes and so on. Of these, the first generation mechanism is best understood theoretically and is the focus of Chapter 5. This begins with the classic problem of internal waves generated by an oscillating cylinder. The mathematics of this section is more advanced than elsewhere but is included in part to illustrate how this conceptually simple problem is challenging to model mathematically in a way that gives meaningful physical results. The rest of the chapter discusses the generation of interfacial and internal waves by steady and oscillatory (tidal) flow over hills. The generation of internal waves by non-rigid sources such as plumes, gravity currents and turbulence is becoming better understood as a result of high-resolution numerical simulations and laboratory experiments. But it is beyond the scope of this book to discuss such recent and on-going research.

In Chapter 6 the propagation of waves in non-uniform media is described. This includes the description of interfacial waves approaching a slope and of internal waves in non-uniformly stratified shear flows. In parts of the ocean and atmosphere, internal waves exhibit a somewhat universal relationship between their amplitude, frequency and spatial scale. The chapter closes with an empirical description of these waves.

Although references are not included in the text, the appendix lists other textbooks and articles that the reader can use to follow-up on various topics. The journal articles are organized by subject matter, more or less following the order of presentation in the book. It is hoped that this style will help the reader follow the history of research into each subject up until the time of writing. In some cases, this organization also serves to emphasize links between the theory of internal gravity waves in both layered and continuously stratified fluids.

Many colleagues have helped guide the structure and content of this book. In particular, I would like to thank Joan Alexander, Eric D'Asaro, Oliver Buhler, Colm-cille Caulfield, Kathleen Dohan, Morris Flynn, David Fritts, Jody Klymak, Eric Kunze, Jennifer MacKinnon and Rob Pinkel for their illuminating insights and stimulating discussions. I am particularly grateful to Joseph Ansong, Geoffrey Brown, Heather Clark, Hayley Dosser, Kate Gregory, Amber Holdsworth, Justine McMillan, James Munroe and Joshua Nault for their constructive criticism and support. Finally, I wish to acknowledge the hard work and ingenuity of undergraduate students Kyle Holland and Cara Kozack who helped prepare many of the figures.