Liquid Sloshing Dynamics

Theory and Applications

The problem of liquid sloshing in moving or stationary containers remains of great concern to aerospace, civil, and nuclear engineers; physicists; designers of road tankers and ship tankers; and mathematicians. Beginning with the fundamentals of liquid sloshing theory, this book takes the reader systematically from basic theory to advanced analytical and experimental results in a self-contained and coherent format. It presents liquid sloshing effects on space vehicles, storage tanks, road vehicle tanks and ships, and elevated water towers under ground motion.

The book is divided into four sections. Part I deals with the theory of linear liquid sloshing dynamics; Part II addresses the nonlinear theory of liquid sloshing dynamics, Faraday waves, and sloshing impacts; Part III presents the problem of linear and nonlinear interaction of liquid sloshing dynamics with elastic containers and supported structures; and Part IV considers the fluid dynamics in spinning containers and microgravity sloshing.

This book will be invaluable to researchers and graduate students in mechanical and aeronautical engineering, designers of liquid containers, and applied mathematicians.

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Foreword

When I was a very young boy I was enchanted by airplanes. The very idea that such a machine, with no apparent motions of its own – except, of course, for that tiny rotating thing at the front – could fly through the air was amazing. I could see that birds and insects could all fly with great dexterity, but that was because they could flap their wings, and thus support their weight as well as maneuver. And fish could even "fly" through water by motions of their body. How exciting it was then, to begin to learn something about how objects interact with the fluids surrounding them, and the useful consequences of those flows. That ultimately led, of course, to the broad study of fluid dynamics, with all of its wonderful manifestations.

There is hardly a single aspect of our daily lives, and indeed even of the entire universe in which we live, that is not in some way governed or described by fluid dynamics – from the locomotion of marine animals to the birth and death of distant galaxies. As a major field of technical and scientific knowledge, there are vast bodies of literature devoted to almost every facet of fluid behavior: laminar and turbulent flows, discontinuous (separated) flows, vortex flows, internal waves, free surface waves, compressible fluids and shock waves, multi-phase flows, and many, many others. With such a countless array of fluid phenomenon before us, what then leads to the focus of the present work?

Almost since the earliest philosophers began the development of what we now call "rational mechanics," curiosity about how liquids behave when contained in some form of vessel has been strong. Beginning with the oscillations of water in lakes and harbors occurring as the result of earthquakes, similar phenomena surround us at almost every turn, arising from all aspects of our modern technology and civilization as well as from nature herself. Thus, geophysicists and seismologists, engineers, mathematicians, and many other scientific workers have devoted themselves to the study of these fascinating subjects for many years. In terms of modern technology, the behavior of liquids in contained systems is of importance in the tankage of ocean-going ships and offshore platforms, in airplanes, automotive vehicles, railway cars, nuclear power plants, and, of course, rockets and spacecraft, and countless others as well. One would think, however, that the general question of the behavior of liquids in such systems is a rather simple one to answer – not so. Our everyday experience in carrying a cup of coffee or a bowl of soup may be frustrating unless we are very careful as to how we move, but may still deceive us into believing that the "sloshing" of the liquid is simple. Indeed, depending on the amplitudes and frequencies of the motions of the container, the responses of the liquid free surface can be almost bewildering in their complexity!

Interest in this subject came almost to a crescendo, of course, with the advent of large liquid propelled rockets nearly half a century ago. At that time almost no one imagined that so many forms of liquid behavior could manifest themselves or that such a tremendous body of technical literature would evolve as a consequence of trying to describe, understand, and predict these fascinating phenomena. Over the years, a fairly large number of monographs and literature summaries appeared in an effort to catalog what is known, and while these have each been useful in their own way, none has been exhaustive. In this work, the author has tried to do just that.

This is a monumental work in that it attempts to cover almost every aspect of liquid sloshing dynamics and cites approximately three thousand references. I would hope that its readers will find in it whatever they are looking for – if not, I do not know where else they should look and so they might have to discover it for themselves, perhaps using the methods and analytical tools described herein.

H. Norman Abramson
San Antonio, March 2004

Acknowledgment

The story of this book goes back to my first job as a Research Engineer at the Rocket Research Center, Cairo, Egypt, in 1963. I was involved in estimating the trajectories of liquid propellant rockets. This task required the knowledge of the actual values of mass moment of inertia and center of mass versus the time of rocket trajectory. The difficulty was mainly in the liquid part which constitutes almost 85% of the total weight of the rocket. I used to ignore the contribution of the liquid propellant in roll oscillations. This inability on my part motivated me to enroll for the Masters degree (by research) at Cairo University. Professor M. I. Rashid, my M.S. thesis advisor, encouraged me to conduct an extensive literature review on the entire subject of liquid propellant sloshing dynamics in addition to the main task of solving the Navier–Stokes equations to estimate the effective boundary layer thickness in roll due to tank walls and immersed pipes. Upon completion of my Masters degree, I was awarded a scholarship to study for my Ph.D. at the University of Edinburgh under the supervision of Professor A. D. S. Barr to work on the autoparametric interaction of liquid sloshing dynamics with supported structure dynamics. At that time I compiled over 900 references on liquid sloshing dynamics and related topics.

When I joined academia at Texas Tech University in 1981, I continued my research on liquid sloshing under random excitation with the help of my graduate students Drs Soundararajan, Hun Heo, Wenlung Li, J. Gau, and Messrs G. Lattore and R. Henriech. The work was supported by the National Research Foundation and dealt with the random response of liquid free surface under different types of random excitation. When I moved to Wayne State University, the work was extended jointly with Dr Valery Pilipchuk to consider the liquid sloshing impact supported by the National Science Foundation (NSF). Dr Mohamed El-Sayad joined me to conduct his Ph.D. on the same problem. During the same time, Professor Takashi Ikeda spent several weeks at Wayne State University to conduct Monte Carlo simulation on parametric random excitation of liquid containers supported on elastic structures in the presence of internal resonance. Both Professor Ikeda and Dr Pilipchuk joined me to write an extensive literature review on liquid sloshing dynamics citing over 1200 references. Upon completing the review article I was strongly motivated to write the present book to cover every aspect of the subject. However, as soon as I began, I found the task was formidable and beyond my own capacity. It was my wife, Sohair, who encouraged me to accept the challenge and to complete the task at my own pace. I thank her so deeply for her patience and longsuffering during the last four years. Indeed without her support I could not have completed the book.

Upon completion of the first draft of the manuscript, I asked Dr Norman Abramson, who I consider to be the father of liquid sloshing, to write a Foreword for the book. May God grant him health and many years. I extend my sincere thanks to my colleagues and friends who have helped me. In particular I thank Professors Victor Berdichevsky at Wayne State University, and Professor W. Eidel of the Universitat der Bundeswehr, Germany, who helped me prepare the proof of some results.

Last but not least, I would like to thank the editorial staff of Cambridge University Press. In particular, I thank Dr Phil Meyler who handled the editorial process, and Ms Emily Yossarian, Ms Carol Miller, and Mr Keith Westmoreland who have been working with me during the production process.

Introduction

Sloshing means any motion of the free liquid surface inside its container. It is caused by any disturbance to partially filled liquid containers. Depending on the type of disturbance and container shape, the free liquid surface can experience different types of motion including simple planar, nonplanar, rotational, irregular beating, symmetric, asymmetric, quasi-periodic and chaotic. When interacting with its elastic container, or its support structure, the free liquid surface can exhibit fascinating types of motion in the form of energy exchange between interacting modes. Modulated free surface occurs when the free-liquid-surface motion interacts with the elastic support structural dynamics in the neighborhood of internal resonance conditions. Under low gravity field, the surface tension is dominant and the liquid may be oriented randomly within the tank depending essentially upon the wetting characteristics of the tank wall.

The basic problem of liquid sloshing involves the estimation of hydrodynamic pressure distribution, forces, moments and natural frequencies of the free-liquid surface. These parameters have a direct effect on the dynamic stability and performance of moving containers.

Generally, the hydrodynamic pressure of liquids in moving rigid containers has two distinct components. One component is directly proportional to the acceleration of the tank. This component is caused by the part of the fluid moving with the same tank velocity. The second is known as "convective" pressure and represents the free-surface-liquid motion. Mechanical models such as mass-spring-dashpot or pendulum systems are usually used to model the sloshing part.

A liquid's motion inside its container has an infinite number of natural frequencies, but it is the lowest few modes that are most likely to be excited by the motion of a vehicle. Most studies have therefore concentrated on investigating forced harmonic oscillations near the lowest natural frequencies, predicted by the fluid field linear equations. However, nonlinear effects result in the frequency of maximum response being slightly different from the linear natural frequency and dependent on amplitude. Nonlinear effects include amplitude jump, parametric resonance, chaotic liquid surface motion, and nonlinear sloshing mode interaction due to the occurrence of internal resonance among the liquid sloshing modes. The nonlinearities associated with free-surface motion inside moving containers are different from those nonlinear water waves in ocean and canals. The theory of nonlinear dispersive waves originated by Stokes (1947) and solitons[1] observed by Russell (1844) is well documented in Debnath (1994) and will not be addressed in this book.

[1] The solitary wave represents not a periodic wave, but the propagation of a single isolated symmetrical hump of unchanged form.

Analytical solutions are limited to regular geometric tank shapes such as cylindrical, and rectangular. The nature of sloshing dynamics in cylindrical tanks is better understood than for prismatic tanks. However, analytical techniques for predicting large-amplitude sloshing are still not fully developed. Such loads are extremely important in the design stage of supporting structures and internal components of vehicle tanks. In addition, much of the sloshing technology developed for space applications is not applicable to road tankers. The reason is that more emphasis has been placed on frequencies and total forces as they relate to control system requirements. Accordingly, the effects of local peak impact pressure on structural requirements have not been studied to any extent. Further, the excitation amplitudes considered in space applications are too small for road vehicle simulation. To avoid catastrophic sloshing in space vehicles, the control system frequencies, the vehicle elastic structure frequencies, and the fluid-slosh frequencies must be fairly widely separated.

Sloshing phenomena in moving rectangular tanks can usually be described by considering only two-dimensional fluid flow if the tank width is much smaller than its breadth. Sloshing in spherical or cylindrical tanks, however, is usually described by three-dimensional flow. Tanks with two-dimensional flow are divided into two classes: low and high liquid fill depths. The low fill depth case is characterized by the formation of hydraulic jumps and traveling waves for excitation periods around resonance. At higher fill depths, large standing waves are usually formed in the resonance frequency range. When hydraulic jumps or traveling waves are present, extremely high impact pressures can occur on the tank walls. Impact pressures are only measured experimentally and cannot be estimated theoretically or numerically.

Early attempts to analyze liquid waves in oscillating containers include those of Hough (1895), Honda and Matsushita (1913), Jeffries (1924), Sen (1927), Goldsborough (1930), Westergaard (1933), Binnie (1941, 1955), K. W. Smith (1947, 1956), C. Smith (1948), Taylor (1950, 1954), Luskin and Lapin (1952), Schy (1952), Moiseev (1952a,b,c,d, 1953, 1954, 1956), Senda and Nakagawa (1954), Nakagawa (1955, 1956), Birkhoff (1956), Narimanov (1956, 1957c), Okhotsimski (1956), Heinrich and Kaufman (1956), Sretanskii (1956, 1957), Housner (1957), Krein and Moiseev (1957), and Shved (1959). The fundamental theory of liquid surface waves is documented in several references (see, e.g., Lamb, 1945, Stoker, 1957, Kochin, *et al.*, 1964, Thomson, 1965, Brodkey, 1967 and Barber and Ghey, 1969). Different aspects of the subject were reviewed by Sloane (1960), Cooper (1960), Abramson (1961a, 1963b, 1966b, 1968), Ring (1964), Abramson and Kana (1967), Brown (1982b), Rammerstofer, *et al.* (1990), and Ibrahim, *et al.* (2001). Some research monographs entirely devoted to different space problems of liquid sloshing include Abramson (1966a), Moiseev (1968), Moiseev and Rumyantsev (1968), Babskii, *et al.* (1976a), Narimanov, *et al.* (1977), Myshkis, *et al.* (1987, 1992), Walter (1987), and Monti (2001).

The problem of liquid sloshing in moving or stationary containers remains of great concern to aerospace, civil, and nuclear engineers, physicists, designers of road tankers and ship tankers, and mathematicians. Civil engineers and seismologists have been studying liquid sloshing effects on large dams, oil tanks and elevated water towers under ground motion. They also mounted liquid tanks on the roofs of multistory buildings as a means to control building oscillations due to earthquakes. Since the early 1960s, the problem of liquid sloshing dynamics has been of major concern to aerospace engineers studying the influence of liquid-propellant sloshing on the flight performance of jet vehicles, and new areas of research activities have emerged. The modern theory of nonlinear dynamics has indeed promoted further studies and uncovered complex nonlinear phenomena. These include rotary sloshing, Faraday waves, nonlinear liquid sloshing interaction with elastic structures, internal resonance effects, stochastic sloshing dynamics, hydrodynamic sloshing impact dynamics, g-jitter under microgravity field, dynamics of liquid bridges, cross-waves, and spatial resonance. The dynamic stability of liquefied natural gas tankers and ship cargo tankers, and liquid hydrodynamic impact loading are problems of current interest to the designers of such systems. In populated cities, gasoline and other flammable liquid tankers are prone to rollover accidents while entering and exiting highways. This is a difficult mathematical problem to solve analytically, since the dynamic boundary condition at the free surface is nonlinear and the position of the free surface varies with time in a manner not known a priori. A liquid free surface in partially filled containers can experience a wide spectrum of motions such as planar, nonplanar, rotational, quasi-periodic, chaotic, and disintegration. Other important contributions include the development of digital computer codes to solve complex problems that were difficult to handle in the past.

The purpose of this book is to present the basic theories and results developed in different applications in a coherent format. It highlights the major achievements and results reported in the literature. It begins with the fundamentals of liquid sloshing theory to take the reader systematically from the basic theory to advanced analytical and experimental results. Each chapter includes an extensive literature review for its major sections, followed by analytical description and main results. The book is supported by an extensive bibliography of technical journal papers, NASA reports, research monographs, M.S. and Ph.D. theses, and conference proceedings.

The book can be divided into four major parts. Part I deals with the theory of linear liquid sloshing dynamics (Chapters 1 to 3). Part II addresses the nonlinear theory of liquid sloshing dynamics (Chapters 4, 6, and 7). Note that Chapter 5 is in Part II but links linear and nonlinear equivalent mechanical models, and thus it also belongs to Part I. Part III presents the problem of linear and nonlinear interaction of liquid sloshing dynamics with elastic containers and supported structures (Chapters 8 to 10). Part IV considers the fluid dynamics in spinning containers and microgravity sloshing (Chapters 11 and 12). The book is closed by an extensive list of references that exceeds 2600.

The fluid field equations are developed in the first chapter with the purpose of determining the natural frequencies and the corresponding mode shapes in different container geometries. The analytical results are compared with those measured experimentally. Within the framework of the linear theory, the hydrodynamic loads and moments generated under harmonic excitations are evaluated in Chapter 2. A unique feature in this chapter is the study of forced excitation of magnetic fluids. Analytical treatments of some nontraditional tank geometries that were regarded as difficult in the past are presented.

Chapter 3 presents the influence of viscous damping and sloshing suppression devices in closed containers. The damping and modal analysis in a circular cylindrical container is analytically treated. The influence of sloshing suppression devices is addressed based on reported experimental results. The basic concept of Stokes boundary layers over an oscillating flat plate is introduced as a foundation to the estimation of inertia and damping parameters. It introduces the contribution of periodic boundary layers of fluids, in upright circular containers experiencing roll oscillations, to the system inertia and damping parameters. The analysis is extended to determine the contribution of immersed rods and pipes.

Chapter 4 deals with weakly nonlinear sloshing dynamics under lateral excitation. The associated phenomena include nonplanar unstable motion of the free surface accompanied by rotation of the nodal diameter (rotary sloshing) and chaotic sloshing. These phenomena can be uncovered using the theory of weakly nonlinear oscillations for quantitative analysis and the modern theory of nonlinear dynamics for stability analysis. In circular containers, the problem is strongly analogous to the dynamics of a spherical pendulum whose support point experiences lateral sinusoidal excitation. Some nonlinear theorems have been developed to predict not only rotary sloshing, but also other complex motions such as period doubling and chaos. The main sources of nonlinearity in the fluid field equations are the free-surface boundary conditions. The study is devoted to circular cylinders and rectangular containers. There is a critical fluid depth, which separates two regimes of liquid nonlinearity, hard and soft. A unique feature in this chapter is the random excitation of rigid containers, which has a direct application to liquid containers subjected to earthquakes. The chapter includes the problem of self-induced sloshing usually encountered in nuclear reactors. Self-induced sloshing implies the interaction between the liquid free-surface dynamics and submerged fluid jet dynamics.

Chapter 5 introduces the concept of equivalent mechanical models. A realistic representation of the liquid dynamics inside closed containers can be approximated by an equivalent mechanical system. The equivalence is taken in the sense of equal resulting forces and moments acting on the tank wall. By properly accounting for the equivalent mechanical system, the problem of overall dynamic system behavior can be formulated more simply. For small oscillations in which the fluid free surface remains planar without rotation of its nodal diameter the equivalent mechanical model can be represented by a set of pendulums or mass-spring-dashpot systems. For relatively large-amplitude oscillations, in which the free liquid surface experiences nonplanar motion, the equivalent mechanical model is a simple pendulum describing relatively large motion. To model nonplanar and rotational sloshing one has to replace the simple pendulum by a compound or spherical pendulum. The theory of the spherical pendulum will be presented for promoting our understanding of rotary sloshing. For strongly nonlinear motion associated with hydrodynamic pressure impacts, the modeling can be achieved by an equivalent pendulum of high restoring force. This modeling will be considered in details in Chapter 7.

Chapter 6 deals with the problem of parametric sloshing or Faraday's waves. Parametric sloshing refers to the motion of the liquid free surface due to an excitation applied perpendicular to the plane of the undisturbed free surface. Generally, parametric oscillation occurs in dynamical systems as a result of time-dependent variation of such parameters as inertia, damping, or stiffness. The early research efforts focused on the estimation of instability boundaries of liquid in the presence of viscous and surface tension forces or in their absence. While the damping forces raise the threshold excitation amplitude, they do not bring the free-surface amplitude into a bounded value. It is the inherent nonlinearity of the surface-wave amplitude that can limit the growth of the amplitude. Both nonlinear theory and experimental tests revealed "soft" or "hard" spring characteristics depending on the liquid depth ratio. With the advent of the modern theory of nonlinear dynamics, many complex surface dynamic phenomena have been uncovered. These include modal competition, quasi-periodic motion, chaotic motion, and mixed mode motion in the presence of internal resonance. These phenomena are presented for circular and rectangular tanks and the corresponding motion was identified in bifurcation diagrams under slow variations of certain control parameters such as excitation amplitude and frequency. Most of these phenomena may take place under random parametric excitation in terms of excitation bandwidth and spectral density function.

Chapter 7 presents the basic concepts of the theory of hydrodynamic slamming and impacting. The material of this chapter has not been addressed in other research monographs on liquid sloshing dynamics. It begins with the analogy of shock waves that take place in a gas column. Analytical expressions of hydrodynamic impact forces and moments acting on the tank walls, due to sinusoidal pitching excitation, are given in closed approximate forms and compared with experimental measurements. Under lateral excitation, the modal interaction is presented. Hydrodynamic pressure impacts due to sudden acceleration of the tank are displayed according to experimental measurements. A phenomenological modeling of impact forces and damping forces is proposed and used in the analytical modeling of elastic structures carrying liquid containers. The impact forces are replaced by a high-power force, which covers both elastic and rigid impact cases. Three nontraditional analytical techniques, namely saw-tooth time transformation, nonsmooth coordinate transformation, and Lie group transformations are briefly outlined. An application of a mechanical system simulating an elastic structure carrying a liquid container that allows high-power impact forces will be adopted. The response will be obtained using the method of multiple scales and the results will be compared with those obtained by nontraditional techniques. The analysis involves the simultaneous occurrence of parametric resonance and internal resonance conditions. Sloshing impact in ship tankers and road tankers are addressed together with a brief account of numerical algorithms used for large surface motion.

Chapter 8 presents the linear problem of liquid interaction with its elastic container. The first step in studying this interaction is to estimate the linear eigenvalue problem and the linear response to external excitations. The coupling may take place between the liquid free-surface dynamics and either the tank bending oscillations or the breathing modes (or shell modes). Breathing vibrations of the tank are essentially radial, such that both flexure and stretching deformations of the wall occur while the longitudinal axis of the tank remains straight. The mathematical formulation of interacting forces is formulated for a general motion of the liquid containers. The linear analysis deals mainly with the estimation of the coupled natural frequencies of liquid–structure systems. Specific cases are addressed. These include: (i) rigid tank walls with an elastic bottom modeled as either a stretched membrane or elastic plate, (ii) a rigid bottom with elastic walls experiencing either bending deformation or breathing vibrations, and (iii) interaction with a completely elastic container. The nonlinear interaction under external and parametric excitations is treated in Chapter 9. The nonlinear interaction covers three different groups: (1) nonlinear free-surface interaction with linear shell deformation, (2) linear free-surface interaction with nonlinear shell deformation, and (3) nonlinear surface interaction with nonlinear shell deformation. Storage tanks interacting with their liquid and the numerical algorithms developed to solve for the liquid–tank interaction are reviewed. Chapter 10 deals with the interaction of elastic support structure and tuned liquid sloshing absorbers. This problem was not addressed in other books. This type of interaction takes place between the free-liquid-surface motion and the supported elastic structure dynamics based on the assumption that the liquid container is rigid. Under the base motion of the supporting structure, the fluid container experiences motion in a certain trajectory governed by the excitation and the liquid response. The free-liquid-surface motion will result in hydrodynamic forces that are fed back to the supporting structure. Civil engineers took advantage of this by using liquid tanks placed on the top of large buildings to act as vibration absorbers. A detailed treatment of sloshing absorbers such as tuned liquid-sloshing absorbers, and liquid-column vibration absorbers is presented. Nonlinear interaction of liquid free surface with support structure is then treated in the presence of parametric excitation, internal resonance, and combination resonance, under deterministic and random excitations.

Chapter 11 presents the problem of liquid dynamics in spinning tanks. This problem is usually encountered in the study of stability and control of rockets, space vehicles, liquid-cooled gas turbines, centrifuges, earth with its oceans and fluid core. The spinning and unavoidable wobbling and precession of spacecraft vehicles cause the contained liquid to oscillate, and thus generate dynamic forces, which can destabilize the spacecraft. Note that the kinetic energy of the spinning spacecraft is dissipated by the liquid motion and thus destabilizes the vehicle and causes it to tumble. Spacecraft vehicles are designed to spin in order to gain gyroscopic stiffness during the transfer from low earth orbit. This spin helps to control the location of liquid propellant in its container. Another class of problems deals with the dynamics and stability of rotating rigid bodies as they are applied to the evolution of celestial bodies and astronavigation control. Stabilization is achieved when the spacecraft spins about its axis of minimum moment of inertia. A satellite that spins about its axis of minimum moment of inertia may experience instability if energy is dissipated. This is similar to a spinning top on a rough surface and as a result of friction the top's nutation angle increases as it seeks to conserve angular momentum. Other topics addressed in this chapter include the dynamics of fluid-filled spinning containers, inviscid fluid in a partially filled upright cylinder, inertia waves in rotating fluid, and rotating liquids in microgravity.

The last chapter deals with the physics and dynamics of liquid sloshing in microgravity. In view of the diversity of microgravity fluid physics, this chapter will address selected topics supported by literature reviews of the main problems. It begins with the mechanics of free liquid surface under microgravity, stability of static and dynamic free liquid surface, and contact line and contact angle. The modal analysis includes the estimation of free-surface natural frequencies and associated surface shapes under micro- and zero-gravity environments. The problem of forcing sloshing for slipping and anchored contact lines will be addressed for simple cases. The influence of g-jitter and liquid handling are discussed briefly. Capillary systems including Marangoni flow and liquid bridges will be treated for static and dynamic cases. The chapter also includes the problem of the thermocapillary effects of fluid flow and liquid bridges, and the sloshing of cryogenics, and briefly discusses hydroelastic oscillations under microgravity.

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