COMPUTATIONAL ELECTROMAGNETICS FOR RF AND MICROWAVE ENGINEERING

The numerical approximation of Maxwell’s equations, computational electromagnetics (CEM), has emerged as a crucial enabling technology for radio-frequency, microwave, and wireless engineering. The three most popular “full-wave” methods – the Finite Difference Time Domain method, the Method of Moments, and the Finite Element Method – are introduced in this book by way of one- or two-dimensional problems. Commercial or public domain codes implementing these methods are then applied to complex, real-world engineering problems, and a careful analysis of the reliability of the results obtained is performed, along with a discussion of the many pitfalls that can result in inaccurate and misleading solutions. The book will empower readers to become discerning users of CEM software, with an understanding of the underlying methods, and confidence in the results obtained. It also introduces readers to the art of code development. This book has a dedicated website making available a number of MATLAB scripts, implementing much of the theory discussed, and including additional material on the practical applications of CEM. Suitable for senior undergraduate and graduate students taking courses on CEM, this would also be a valuable reference book for practicing engineers in the industry.

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COMPUTATIONAL ELECTROMAGNETICS FOR RF AND MICROWAVE ENGINEERING

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To Amor.
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On graduating twenty years back, in 1984, my first job was as a research engineer working on computational electromagnetics (CEM) at the National Institute for Aeronautical Systems Technology (as it was then called) of the Council for Scientific and Industrial Research (CSIR) in Pretoria, South Africa. It was an exciting time to be working in this field. Although a number of methods had already been successfully introduced, including the three which will be discussed in detail in this book, major advances were being made in all of these methods, and the power of desktop computers was growing in leaps and bounds. No commercial programs (or codes, as they are generally called) were then available for RF problems, but some US government-sponsored codes, in particular the NEC-2 code, were becoming available for general use.

The 1980s saw the final decade of the Cold War, which in some areas (such as Southern Africa) was far from cold. New military technologies, in particular stealth, were driving CEM to address progressively more electromagnetically complex problems. However, when the Cold War ended, far from CEM work coming to a halt, new commercial markets, such as the rapidly developing market in mobile telephony and personal communication systems, and the proliferation of electronic systems in motor vehicles, continued to drive the technology forward at breakneck speed throughout the 1990s. This was also due to the widespread availability of cheap and progressively more powerful personal computers as a crucial enabling technology.

CEM has now reached a modicum of maturity, with a number of powerful methods available, able to solve problems of real engineering interest at radio frequencies, and with a number of commercial codes available. This has brought a significant change in the profile of CEM practitioners, which has not been fully appreciated in the community at the time of writing. In addition to the traditional group of CEM users – largely academics, post-graduate students and research engineers at large corporations or research establishments – an entirely new generation of
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users has arisen. Their interest is typically in using an existing commercial packet to solve a particular problem as rapidly as possible. They may well not have any post-graduate exposure to CEM methods, and questions which may appear elementary to CEM researchers (such as which technique is most appropriate for the problem at hand) are actually far from obvious to the beginner in the field; furthermore, marketing can “hype” a particular implementation/technique to the point where it appears omnipotent. Commercial codes aside, even academic papers are not free of such bias.

This book aims to serve the interest of both “traditional” CEM users, primarily academics, researchers and research students, and also this new non-specialist user community in industry. The book aims to fill the gap between traditional undergraduate textbooks, which generally have at most a very cursory discussion of numerical methods; antenna texts, which concentrate only on the analysis of antennas using the methods; and the specialist books on each method which are frequently formidable reading for students, or unnecessarily detailed for engineers whose primary interest is in using the powerful CEM codes now available. In this book, the computational methods will generally be introduced using simple one-dimensional or two-dimensional examples, so that the core of the method can be appreciated without being overwhelmed by the problems of handling complex three-dimensional geometries. Following this, the extensions required to deal with the real three-dimensional world of RF engineering are outlined, so that one gains an appreciation for the operation of complex codes. Such is the complexity of general-purpose three-dimensional CEM codes that realistic applications cannot be undertaken with anything a post-graduate student can realistically be expected to develop during a typical course, and product cycles are too short in industry to make the development of general-purpose three-dimensional codes feasible, given that off-the-shelf codes are now available.

Research students will find some features not often described in other books in this field, such as how to go about debugging and verifying a CEM code. Industrial users should find the discussions of the strengths and weaknesses of each method, as well as frequent modelling hints, comprehensive discussions of typical modelling errors, and the necessity of careful evaluation and verification of results, of great interest and utility. In short, the book discusses not only the science of CEM modelling, which can be gleaned from (much) reading, but also the art of developing and verifying reliable codes and computing reliable data, which is a skill generally derived from (sometimes bitter!) experience.

This book concentrates on the “big three” techniques in CEM – the Finite Difference Time Domain (FDTD) method, the Method of Moments (MoM) and the Finite Element Method (FEM). It was decided to focus on these three methods, since they are the most widely used in the field and all have been implemented
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in successful commercial codes; some other methods are very briefly discussed so that readers are at least aware of them, but this book makes no pretense of addressing these other methods in any detail. Furthermore, the discussion in this book is focussed exclusively on applications in RF engineering. Methods such as the FEM have been used with great success for magnetostatic problems, such as motor design, but this will not be discussed here at all. A feature not often found in other books at this level is a discussion of stratified media, using the Sommerfeld potentials. Although a theoretically advanced topic, the widespread use of integrated antennas, especially microstrip, has made an appreciation of at least the basics of this approach very important. Finally, the book does not pretend to be a comprehensive text on electromagnetic theory, high-frequency circuit theory, or antenna theory and design. There are a number of superb books addressing these topics and this book is designed to complement, not compete, with them. Frequent references are made to suitable books.

Readers will also note that the level of the material becomes increasingly sophisticated as the book progresses. This is by design. The FDTD method is the only method where one can realistically hope to develop useful code oneself in a reasonable timeframe, so the discussion of this method is rather more “nuts and bolts” than for the MoM or FEM. CEM methods can also be approached as essentially an exercise in applied mathematics; although interesting theoretical insights can be thus gained, it is the author’s experience that engineers do not readily take to this approach, certainly not for their initial introduction to the methods, so the introductory discussions of at least the FDTD method and MoM draw mainly on engineering physics, rather than applied mathematics. Some of the more theoretical approaches to CEM are introduced towards the end of the book, in the chapters on the MoM and FEM. (Perhaps because of the enormous amount of work on the FEM in applied mechanics, this is probably the method with the most well-developed mathematical background.) These include some elementary concepts from functional analysis, with the associated concepts of inner products and weighted residuals, as well as a brief mention of differential forms. A difficult decision was how much of the great volume of recent advances to reflect in the book. Topics such as the fast multipole method have revitalized the MoM in particular, and cannot be ignored, but the treatment of this and some other “research frontier” material is of necessity cursory.

A highly problematic issue was the selection of which commercial CEM codes to use to illustrate complex real-world implementations. One factor influencing this was the availability of a no-cost limited feature version of the software, as in the case of the MoM code FEKO; however, the FDTD and FEM codes discussed are unfortunately not available in such a format. The discussion tries to highlight generic features which a code should offer, and how users can exploit these.
User-manual style descriptions of how to use particular codes have been avoided as far as possible, so that discussions of one particular code should extend to other commercial codes implementing the same method, at least to a degree. At the time of writing, FEKO supported a type of scripting language, which has been used in places to automate the generation of complex geometries for MoM analysis; the constructs (FOR loops, IF-THEN-ELSE conditionals) are felt to be sufficiently generic to be useful in other codes supporting similar features.

Where appropriate, references are provided for further reading. In general, only those readily available in the English language archival open literature have been listed. On one or two occasions, internal reports have been included. The engineering community is divided on the use of such references; authors in the USA in particular often reference such reports in journal papers, which often prove frustratingly difficult to locate, sometimes being limited to US distribution only. In consequence, this has only been done when there is no other published version of the material. A similar problem can be encountered with theses; here, however some significant recent research has necessitated limited reference to recent dissertations, since these results are yet to appear in the archival literature.

The book draws primarily on the literature of Western science. Much work was done on computational electromagnetics in especially the former Soviet Union, but unfortunately little has been translated, and what has been is very difficult reading for electronic engineers trained in the Western tradition; it also tends to be at a much higher theoretical level than the main thrust of this book.

This book is an outgrowth of notes developed over a fifteen year period for a post-graduate course taught by the author at the University of Stellenbosch, South Africa, as well as a short course for industry taught by the author and several colleagues in 1999. Extensive integration of the material was undertaken during the author’s sabbatical visit as a Guest Professor at the Delft University of Technology during 2003, where the course was also taught. Chapter 2 is adapted and extended from notes originally prepared by James T. Aberle at Arizona State University, Tempe, AZ, USA, and he is credited accordingly, but the rest of the authorship is that of DBD.
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Stimulating careers are frequently the result of interactions with interesting people, and I would like to acknowledge a number of exceptional engineering scientists who have either mentored me, worked with me, or studied under me. My late father, an electronic engineer, spent much of his career working in the microwave and telecommunications industry in the UK and South Africa and sparked my early interest in electronic engineering; he started his career during the Second World War, working on some of the first radar sets deployed in South Africa (and later North Africa and Italy). John Cloete, Wynand Louw, Derek McNamara, and Jan Malherbe gave inspiring undergraduate and post-graduate courses at the University of Pretoria from 1981–1983, which originally fired my interest in this specific field. John and Derek continued as research supervisors for my M.Sc. and Ph.D. research on the MoM from 1986–1991. Dirk Baker gave me my first job at the CSIR in 1984; he is an outstanding antenna engineer and his scepticism of computed results was an invaluable baptism of fire. John Cloete offered me the opportunity to join the University of Stellenbosch in 1988 and we have continued to interact most fruitfully throughout my career.

Rick Ziolkowski taught me the power of the FDTD method during my postdoctoral stay at the University of Arizona in 1993. (Rick made significant contributions to the method and its applications, especially in complex material modelling.) Ron Ferrari and Ricky Metaxas kindly hosted me at Cambridge University during a sabbatical visit in 1997, where I had the opportunity to enrich greatly my knowledge of the FEM during frequent discussions with them and their students. Jim Aberle (Arizona State University) brought novel ideas to the teaching of the FDTD as well as spectral domain MoM methods, during a short course we taught in 1999; his ideas are reflected in places in this book. Leo Ligthart and Alex Yarovoy hosted me during my 2003 sabbatical at Delft University of Technology, during which time I initiated the actual writing of this book; their enthusiasm was very supportive.
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Of my research students: in particular, the work of a number of my doctoral students is reflected in places in this volume, especially Frans Meyer – who went on to co-found Electromagnetic Software and Systems (Pty) Ltd., turning research ideas in CEM into commercially successful products – Marianne Bingle, Matthys Botha, Pierre Steyn, and Riana Geschke, and I would like to acknowledge their dedication to research excellence here. Frans and Matthys’s work in particular is described in some detail in the final chapter. I would also like to thank Matthys for his proofreading and detailed comments on, and suggestions for, the final two chapters, which were most useful. Very useful interactions with a number of engineers (some of them previously my graduate students) at Electromagnetic Software and Systems are also reflected in this book, including Ulrich Jakobus (the original author of FEKO), Johann van Tonder, Isak Theron, Gronum Smith, Danie le Roux, and Sam Clarke. Many years of continuing discussions on electromagnetics with my colleagues at the University of Stellenbosch, in particular John Cloete, Petrie Meyer, Howard Reader, and Keith Palmer have also influenced the development of this book, as have those with colleagues in electronic engineering in general, in particular Dave Weber and the late David Frost.

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Electromagnetic Software and Systems and Computer Simulation Technology kindly provided evaluation copies of FEKO and CST MICROWAVE STUDIO™ respectively. The former also provided the image on which the cover art was based. My thanks to Vanessa Weber for the graphic design she produced from this for the cover.

The love and forebearance of my wife Amor, who was bearing our first child Bruce during much of the period when this book was in preparation, was essential.

Finally, the support of the Cambridge University Press team is much appreciated.
To the reader

This book is designed to serve as an introduction to computational electromagnetics for radio-frequency applications. It assumes the reader has completed typical undergraduate courses in electromagnetic field theory, and has some basic knowledge of antenna design and microwave systems.

For readers in a hurry, who already know which of the techniques discussed they would like to learn more about, it is possible to go directly to the relevant chapters, but it would nonetheless be useful first to read the introductory chapter. For those in a hurry, but who need first to find out which method (or methods) to use, this chapter is essential reading.

For readers who intend working through most of the book, it would be best to work through it in the sequence presented, although the chapters on the Sommerfeld formulation and practical applications thereof could be omitted without interrupting the sequence of presentation. A more detailed outline of the book may be found in Section 1.11; this will also assist readers to locate rapidly the parts of the book of interest to them.

At the end of each chapter, a list of references linked to the chapter topic is presented, for further reading and study.
Throughout this book, the following notation is used. Spatial vectors are indicated as \( \vec{E} \) (in this case, the electric field). Vectors in the linear algebra sense are indicated as \( \{x\} \), and matrices as \( [A] \). The individual elements of a vector or matrix are of course indicated as \( x_i \) or \( A_{ij} \) respectively. Otherwise, the notation is as generally encountered in engineering books on this topic. A summary is presented below.

The time convention used for phasor quantities is \( e^{j\omega t} \), hence, an \( e^{-jkr} \) plane wave propagates in the direction of increasing \( r \). (Note that physics books often adopt the \( e^{-i\omega t} \) convention, in which case the sign also changes in the plane wave exponential factor.)

\[
\begin{align*}
\nabla \times & \quad \text{the curl operation} \\
\nabla \cdot & \quad \text{the divergence operation} \\
x & \quad \text{the vector cross product of two vectors} \\
\vec{E} & \quad \text{the (field) vector} \ E \\
\epsilon_0 & \quad \text{the permittivity of free space (} \approx 8.854 \times 10^{-12} \text{ F/m)} \\
\epsilon_r & \quad \text{relative permittivity of a dielectric material (dimensionless)} \\
\mu_0 & \quad \text{the permeability of free space (} 4\pi \times 10^{-7} \text{ H/m)} \\
\mu_r & \quad \text{relative permeability of a magnetic material (dimensionless)} \\
c & \quad \text{the speed of light in free space (} \approx 2.9979 \times 10^8 \text{ m/s)} \\
\lambda & \quad \text{wavelength [m/s] or real part of spectral variable} \ k \rho \\
(\text{the meaning will be clear from the context}) \\
\lambda_i & \quad \text{simplex coordinate} \ i \\
\mathcal{O}(M^n) & \quad \text{of the order of} \ M^n, \text{ formally,} \\
\mathcal{N} = \mathcal{O}(M^n) & \Rightarrow \lim_{M \to \infty} \log \mathcal{N} / \log M = n \\
[A] & \quad \text{the matrix} \ A \\
a_{ij} & \quad \text{the} \ i,j^{th} \text{element of matrix} \ A \\
\{x\} & \quad \text{the (algebraic) vector} \ x
\end{align*}
\]
List of notation

$x_i$  the $i$th element of vector $\{x\}$

$\|\{x\}\|$  the Euclidean norm of the vector $\{x\}$ of length $n$,

$$\|\{x\}\| \equiv \sqrt{\sum_{i=1}^{n} |x_i|^2}$$

$\forall$  is defined as

$\forall$  for all

$|z|$  absolute value of $z$

$\Rightarrow$  implies