Introduction

Themes and Issues

This volume is a collection of fifteen of my thematically related papers on phenomenology, logic, and the philosophy of mathematics. All of the essays are concerned with the interpretation, analysis, and development of ideas in Husserlian phenomenology in connection with recent and historical issues in the philosophy of mathematics and philosophy of logic.

Many of the interesting questions about phenomenology and the exact sciences that engaged such thinkers as Frege, Carnap, Schlick, and Weyl with the early phenomenologists have unfortunately been neglected in more recent times. One could speculate on why this has happened. On the one hand, it no doubt resulted from the development of certain trends in what has since been called ‘analytic’ philosophy. On the other hand, it resulted from the particular trajectory of Continental philosophy after Husserl. Husserl’s emphasis on science and the analysis of essence gave way almost immediately on the Continent to various philosophies of human existence under the influence of Husserl’s student Heidegger. In addition, there were long delays in the English translation of many of Husserl’s writings, and to complicate matters further, philosophy curricula at many universities came to be organized around the division between analytic and Continental philosophy.

In my view, this general division between the analytic and Continental traditions has not always been good for philosophy. Least of all should it be maintained in the case of philosophers such as Brentano, Husserl, and some others in the early phases of the phenomenological movement, for here there is still direct engagement with major figures in Anglo-American philosophy. Many of the essays in this book are concerned with issues and ideas that are common to both the Continental and the analytic traditions of philosophical thinking about logic and mathematics. I hope the book can be seen as adding to the growing literature that encourages communication between the traditions and puts some of these artificial divisions behind us.[1] The methods and ideas of analytic and Continental philosophy, where they are in fact different, can inform and enrich each other in many ways. There are of course significant disagreements on some of the issues, but I expect that by examining them we will only reach a deeper understanding.

§ 1

In order to appreciate the approach I take to phenomenology in this book it is necessary to realize that Husserl's own thinking about logic and mathematics went through several transformations. It will be useful to situate my work in a general way with respect to three main stages that can be discerned in Husserl's writings on these subjects. Roughly speaking, there is the early work of the *Philosophy of Arithmetic* (*PA*) and related writings (1891–1900), the middle period of the *Logical Investigations (LI)* (1900–1907), and the later period starting with the *Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy (Ideas I)* (1907–1938). (For much more detail see Tieszen 2004.) Some of the central changes that divide these periods from one another and that are relevant to my work in this volume are as follows: the *Philosophy of Arithmetic* consists of descriptive psychological and 'logical' investigations of arithmetic. Husserl's ontology at this point includes physical entities and processes and mental entities and processes. The 'ideal' or abstract objects that are clearly part of Husserl's ontology from 1900 onward are not to be found (in any obvious way) in *PA*. Husserl analyzed the origins of the natural number concept in *PA* by focusing on mental processes of abstraction, collection, and so on. As a result, some of the critics of the book, most notably Frege, thought they detected a form of psychologism in *PA*. Psychologism is the view that mathematics and logic are concerned with mental entities and processes, and that these sciences are in some sense branches of empirical psychology. Psychologism was a popular form of naturalism about logic and mathematics in the late nineteenth century.

[1] See, for example, Michael Dummett's *Origins of Analytic Philosophy*, Michael Friedman's *A Parting of the Ways: Carnap, Cassirer, and Heidegger* and *Reconsidering Logical Positivism*, and many of the writings of Dagfinn Føllesdal and J. N. Mohanty.

Another important point about *PA* is that the view of 'intuitive' or 'authentic' knowledge in arithmetic in the book is very limited. Almost all of our arithmetical knowledge is held to be 'symbolic' and in Part II of *PA* Husserl investigates this symbolic or 'logical' component of arithmetic. In this second half of *PA* Husserl develops a kind of formalism about arithmetic and some other parts of mathematics. It appears that parts of mathematics in which we cannot have 'authentic presentations' of objects are to be understood in terms of the inauthentic, merely symbolic representation of the objects. It is held that we cannot have authentic presentations of most of the natural numbers and that the same is true, for example, for the objects of classical real analysis and the objects of n-dimensional geometries with $n > 3$. In these cases, all we can do is to try to show, for example, that the formal systems that we take to be about such objects are consistent. Aspects of this kind of formalism are retained in the later writings but against a wider background that includes ideal meanings, essences, a retooled notion of intuition, and the like.

In the *LI* Husserl introduces ideal objects into his ontology and this in fact underwrites his own extended critique of psychologism in the "Prolegomena to Pure Logic." Husserl now recognizes physical entities and processes, psychical entities and processes, and ideal objects of different kinds (e.g., meanings, universals, natural numbers, sets). There are many claims in the *LI* about intentionality and the commitment of logic to objects such as ideal meanings and universals. It seems to me that much of what Husserl says about logic in the book would be best modeled in various higher-order intensional logics. (None of the presently existing systems of higher-order intensional logic, however, seems quite right.) The conception of intuition in the *LI* also changes in several ways. Now there is intuition not only of 'real' objects (physical or mental entities) but also of ideal objects. There is, in other words, ordinary sensory intuition or perception but also 'categorial' intuition of ideal objects. In relation to *PA*, the account of intuition is refined and extended in several ways. Intuition is now explicitly characterized in terms of the 'fulfillment' of intentions. The intentionality of logical and mathematical cognition comes to the fore in the *LI*. Husserl now holds that there can be static or dynamic fulfillment of intentions in either sensory intuition or categorial intuition. In dynamic fulfillment the intuition of an object is developed through sequences of acts in time. This raises the question of what could possibly be intuited through such sequences of acts, either in sensory or in categorial intuition. One can ask, for example, about the extent of the possibilities of dynamic intuition concerning particular mathematical objects such as natural numbers.

By the time of *Ideas I* there are other important changes. Phenomenology is now characterized as a transcendental idealism and is said to be an eidetic science, albeit a 'material' (as opposed to 'formal') eidetic science. The language of essences is now prominent and Husserl spells out some of the differences between material and formal eidetic sciences. We are still supposed to investigate mental activities of abstraction, collection, reflection, and the like, in connection with logic and mathematics, but this is now presented as an eidetic, descriptive, and epistemological (not psychological) undertaking. It is part of a transcendental philosophy. Although Husserl continues to recognize ideal objects such as meanings, essences, and natural numbers, along with ideal truths about these objects, the seemingly more robust realism about these objects and truths in the *LI* undergoes a shift. It is now taken up into Husserl's phenomenological idealism. The exact nature of this transition is still a subject of much discussion and debate among Husserl scholars. Husserl, however, now clearly speaks about how the 'ideality' or 'irreality' of such objects as meanings and essences is itself constituted, nonarbitrarily, in consciousness. It is not that ideal meanings or essences are mental objects. Indeed, they are constituted in consciousness as nonmental and as transcending consciousness. A logical truth such as the principle of noncontradiction, for example, is constituted as being true even if there were no conscious constituting beings. It is as if one takes a form of realism or platonism about logic and mathematics as a whole and places it inside a transcendental framework that is now meant to do justice to sensory experience and mathematical experience.

Husserl's mature views on formal and regional ontologies are also now in place. There is still an important place for purely formal logic and formal mathematics in Husserl's philosophy of the exact sciences and Husserl continues to speak about what he calls definite, formal axiom systems and their ontological correlates, definite manifolds. Roughly speaking, a 'definite' formal axiom system appears to be a consistent and complete axiom system, and a definite manifold is the system of formal objects, relations, and so on, to which a definite axiom system refers.

In connection with Husserl's views on formal systems, one can distinguish the meaning-intentions or ideal meanings that can be associated with signs in a formal system from the mere 'games meaning' that one can attach to signs in formal systems solely on the basis of manipulating sign configurations according to sets of rules (see *LI*, Investigation I,

§ 20). It is this 'games meaning' that takes center stage in strict formalist views of mathematics. In order to illustrate the distinction, let us create a miniature formal system right now. Suppose this formal system has only two rules of inference:

Rule 1: If there are sign configurations of the form $\Phi \sim \Psi$ and Φ then derive the sign configuration Ψ.
Rule 2: If there is a sign configuration of the form $\Phi \oplus \Psi$ then derive the sign configuration Φ.

Suppose the alphabet of the formal system consists of the symbols P . . . Z, $P' \ldots Z'$, $P'' \ldots Z''$, the constants $\sim$, $\oplus$, and the signs (,), and that we have an appropriate definition of well-formed expressions.

Query: If we have the sign configurations $P \sim (Q \oplus R)$ and $P \oplus S$, then is the sign configuration Q derivable? Yes. Apply Rule 2 to $P \oplus S$ to obtain P. Apply Rule 1 to P and $P \sim (Q \oplus R)$ to obtain $Q \oplus R$. An application of Rule 2 to $Q \oplus R$ will give us Q as output.

We could try many other queries and play with this little formal system for a while. What, however, is this formal system *about*? Who knows, I just concocted it. It need not be about anything. In order to derive Q from the sign configurations that are given it is not necessary or even useful to know the meaning of the expression, if any, for which Q has been chosen. It is as though we are playing by the rules of a particular game and we need not be concerned with what the signs are about.

One can create indefinitely many formal systems at will. In the space of all possible formal systems, however, we find that some formal systems are correlated with existing parts of mathematics, are meaningful to us, interesting, or useful. If mathematics consists only of formal systems, then how could we single out those systems that actually correlate with existing parts of mathematics, that are meaningful, interesting, or useful?

Consider another example. Suppose I construct an axiomatic formal theory in the language of first-order logic with identity and function symbols. One can use standard rules of inference that are associated with such a theory. The theory has three axioms:

Axiom 1: $(\forall x)(\forall y)(\forall z)\ g(x, g(y, z)) = g(g(x, y), z)$
Axiom 2: $(\forall x)\ g(x, a) = x$
Axiom 3: $(\forall x)\ g(x, fx) = a$

Now would a sign configuration such as

$$(\forall x)(\forall y)(\forall z)(g(x, z) = g(y, z) \rightarrow x = y)$$

be derivable from these axioms on the basis of the standard rules of inference for quantification theory with '=' and function symbols? It turns out that it can be derived. One could again play with this system. What, however, is the point? What does it mean?

Suppose we interpret the system as follows: let the domain consist of the integers, 'g' be + (so that g(x, y) is just x + y), 'f' be negation (i.e., ¬), and 'a' name 0. It is now possible to see this theory as an axiomatization of group theory, and under this particular interpretation we are looking at the additive group of the integers. In deriving

$$(\forall x)(\forall y)(\forall z)(g(x, z) = g(y, z) \rightarrow x = y)$$

we are thus deriving a theorem of group theory. Notice how much our thinking about the formal system changes once we see it as an axiomatization of *group theory*. There is a change in the directedness of consciousness. Instead of being directed toward the signs and how they can be manipulated according to rules, we are, against this interpretive background, now directed in our thinking toward a rich mathematical theory that happens to have many applications.

On Husserl's view, it is by virtue of ideal meanings that we are referred to or directed toward certain kinds of objects or states of affairs. In mathematics and logic, in particular, we can be directed toward certain kinds of objects or states of affairs. In our mathematical thinking we may be directed, for example, to sets, natural numbers, groups, or spaces, where these are not to be understood as just more concrete signs. There is a kind of meaningfulness and directedness toward objects in many parts of mathematics that goes beyond the mere manipulation of sign configurations on the basis of rules. The signs are often interpreted in particular ways, and we are thereby directed in our thinking in a manner that would be absent short of such interpretation. I take this to be an important theme in the latter two stages of Husserl's thinking about logic and mathematics. It is a theme that attracted Gödel to Husserl's work even though critics have argued that Gödel's incompleteness theorems are incompatible with some of Husserl's claims about, or at least hopes for, definite axiom systems and definite manifolds. What Gödel was interested in, however, was the prospect offered by phenomenology for the (nonreductionistic) clarification of the meaning of basic terms in mathematics. I will have more to say about this later in this Introduction.

In the essays included in this volume it should be clear that I favor the middle and late stages of Husserl's phenomenology. If pressed, I would choose the third stage over the work at the other stages. Of course, there are also some themes (e.g., the concern for the 'origins' of concepts of logic and mathematics) that remain more or less constant throughout the stages. In the essays that follow I am always writing from the point of view of the later two stages in Husserl's work, even when I write about Husserl's earliest work. It will not be possible to understand what I am doing in the chapters unless this is kept in mind. Husserl himself did not have the opportunity to rewrite such works as the *Philosophy of Arithmetic* from the perspective of his more mature philosophy. It is left to his readers to try to rethink the early work on the philosophy of arithmetic and the philosophy of geometry in terms of the later ideas and methods.

My earlier book, *Mathematical Intuition: Phenomenology and Mathematical Knowledge*, was also written from the perspective of ideas in transcendental phenomenology. The book was focused on mathematical intuition in the case of natural numbers and finite sets, and the idea was to see what kind of account one could develop in this case, on the basis of some of the recent literature on mathematical intuition but also on Husserl's later ideas on intentionality, meaning, ideal objects, possibilities of dynamic fulfillment of mathematical intentions, the analysis of the origins of mathematical concepts in everyday experience, and related views about acts of abstraction, reflection, formalization, and so on. Some of the essays in the present volume, especially Chapters 11 through 15, need to be understood from this perspective. I am concerned with the matter of how far we can push the idea of the fulfillment or fulfillability of intentions that are directed toward particular mathematical objects. What is the best way, for example, to understand talk about the (potential) presence or absence to human consciousness of particular natural numbers? Are there limits on the intuition of particular mathematical objects? This can be seen as an effort to understand the relationship between the more *intuitive* parts of mathematics that have their origins in everyday experience (for example, arithmetic and elementary geometry) and the more *conceptual* and rarefied parts of mathematics (such as higher set theory) where it appears that we cannot have complete or fully determinate intuitions of particular mathematical objects (even if we can engage in 'objective', meaningful, eidetic thinking that appears to be directed toward such objects).

Because I have compared some of Husserl's ideas in the elementary parts of mathematics that have their origins in everyday experience with ideas in intuitionism (see Part III) one might form the impression that I think Husserl himself was an intuitionist. As should be obvious in many of the following chapters, this would amount to a misunderstanding of what I am doing. Husserl was not an intuitionist in the style of Brouwer or

Heyting. There are many differences between his views and those of the traditional intuitionists. One could list the differences. One of the central differences, for example, is that from 1900 onward Husserl has ideal objects such as meanings in his ontology and he speaks about these objects in a way that is more platonistic than would be pleasing to Brouwer or Heyting. Mental entities and processes are required for knowing about natural numbers, for example, but the objects known about – natural numbers – are not themselves mental entities. Rather, they are ideal objects. There are also some important differences concerning solipsism, the explicit recognition of the intentionality of human consciousness, meaning theory, the place of formalization, the views of what can be intuited, and the like. Husserl holds, for example, that there is intuition of meanings or of essences, and of the relations of essences to one another, but it is not clear that traditional intuitionism would have a place for this kind of intuition.

At the same time, it is very interesting to compare Husserl's explorations of logic and mathematics with those of the intuitionists. Husserl never tired of arguing that in order to do justice to logic and mathematics we must investigate the subjective side as well as the objective side of these sciences. We need to consider not only the ideal objects and truths of logic and mathematics but also the subjective acts and processes by virtue of which we come to know about objects and truths. Such acts and processes include carrying out sequences of acts in time, abstracting, collecting, reflecting, and various forms of memory and imagination. No one in recent times has done more to investigate the subjective side of logic and mathematics than Husserl and people such as Brouwer and Heyting. There are bound to be interesting and important points of contact. Some of these points of contact were in fact already being explored in Husserl's time by such individuals as Oskar Becker (who was one of Husserl's research assistants), Hermann Weyl (who was influenced in some ways by Husserl), Felix Kaufmann (also influenced by Husserl), and Heyting himself. Weyl, for example, comes close to identifying the phenomenology of mathematics with intuitionism in some of his comments. As I have indicated, I think this identification goes too far. It does seem to me, however, that there are ideas about subjectivity, intersubjectivity, meaning, intuition, internal time, and other topics in transcendental phenomenology that can be used to support and develop some ideas in intuitionism. Indeed, this is the tack I take in some of the chapters in Part III. In any case, I think it is not a good idea to act as if this period in the development of phenomenological ideas about mathematics did not exist.

There are also interesting points of contact with some of Hilbert's philosophical ideas about mathematics, especially in Husserl's early work. Some of the connections were already being discussed by people around Husserl, for instance, Dietrich Mahnke and Oskar Becker. Just as Husserl was no Brouwerian intuitionist, however, he was also not what we might nowadays think of as a Hilbertian formalist or finitist. One could again list some significant differences in their general philosophical views, especially if one considers Husserl's work from 1900 onward. The kind of 'games meaning' that one can associate with signs in formal systems plays a central role in Hilbert's formalism, and Husserl certainly favors the axiomatization and formalization of mathematical theories. One should try to show that formal axiom systems are consistent and complete, and that they possess other desirable properties. This is all a legitimate and important part of formal logic and formal mathematics. The meaningfulness of mathematics, however, is not everywhere exhausted by the games meaning associated with symbols. There are also ideal meanings that can be expressed by symbols, and it is by virtue of these meanings that there is reference to ideal objects. The meaning theory is different from what we find in Hilbert. It is a bit more like what we find in Frege, although here too there are differences. There is also intuition not only of finite sign configurations (as tokens or as types) but of natural numbers, universals, and meanings themselves. There is an explicit concern with the intentionality of mathematical thinking and with the 'origins' of mathematical concepts of a sort that is not to be found in Hilbert. There are also other interesting points of comparison.

In a similar manner, one could compare and contrast Husserl's views on logic and mathematics with those of many of his contemporaries. It is useful and interesting to look at Husserl's views in connection with those of Cantor, Frege, Carnap, Russell, and others. I think that all of this needs to be explored. Husserl's views, however, cannot be straightforwardly identified with any of these other positions. One needs to sort through the details.

One way in which Husserl's general approach stands out from that of many other philosophers of mathematics is that he combines an investigation of the central feature of human consciousness – intentionality – with an investigation of fundamental issues in the philosophy of mathematics and logic. In the philosophy of mind it is usually taken as a basic fact that human consciousness, especially in scientific modes of thinking, exhibits intentionality. If we recognize this, then we will not be able to ignore the fact that the objectivity of such sciences as logic and mathematics must always be correlated somehow with the subjectivity (and intersubjectivity) of the scientists engaged in acquiring scientific knowledge. Husserl's work is especially interesting for its claim that there are subjective and objective sides to mathematics and logic and for the way it investigates both sides of these sciences in an effort to mesh their epistemologies with their ontological claims.

§ 2

It will be useful to include some specific comments on each of the chapters in this collection in order to put them into proper perspective and to indicate some connections between them. The essays are grouped into three categories. Part I of the volume contains three essays on reason, science, and mathematics from the viewpoint of phenomenology. The first chapter provides an overview of Husserl's claims about reason and about logic as a theory of science. It takes up the idea of logic as the science of all possible sciences (as *mathesis universalis*), including Husserl's mature view (in *Formal and Transcendental Logic*) of the three levels of what he calls 'objective formal logic', along with some of his ideas on manifold theory and formal ontology. The essay follows Husserl's thinking about science through his late work on the crisis of the modern sciences. That crisis stems from misapplications of various forms of naturalism and objectivism. The scientism that comes in for criticism in Husserl's late work is a view that has abandoned philosophical rationalism. Just as there can be science within reason, there can also be science without reason. The second chapter in Part I focuses on some problems in the philosophy of mathematics in particular and on ways they might be approached from the perspective of transcendental phenomenology. It should be read as a general overview that is filled out in various ways in subsequent chapters. The third essay focuses in more detail on geometry. It discusses Husserlian views about 'ideation' or the intuition of essences in connection with modern pure geometry, the idea that there are different formal systems of geometry and different spatial ontologies, and some Husserlian themes about the origins of geometry. It considers the idea of creating new variants of geometry based on the formalization of Euclidean geometry. The essay also discusses the concern for the consistency of the resulting systems, especially when we have left behind the more familiar and intuitive domains of two and three dimensions.

Part II of the volume is centered on a series of essays on Kurt Gödel's interest in Husserl's phenomenology. Gödel had turned his attention