BOSE-CONDENSED GASES AT FINITE TEMPERATURES

The discovery of Bose–Einstein condensation (BEC) in trapped ultracold atomic gases in 1995 has led to an explosion of theoretical and experimental research on the properties of Bose-condensed dilute gases.

The first treatment of BEC at finite temperatures, this book presents a thorough account of the theory of two-component dynamics and non-equilibrium behaviour in superfluid Bose gases. It uses a simplified microscopic model to give a clear, explicit account of collective modes in both the collisionless and collision-dominated regions. Major topics such as kinetic equations, local equilibrium and two-fluid hydrodynamics are introduced at an elementary level, before more detailed treatments and microscopic derivations of the underlying equations are given. Explicit predictions are worked out and linked to experiments. Providing a platform for future experimental and theoretical studies on the finite temperature dynamics of trapped Bose gases, this book is ideal for researchers and graduate students in ultracold atom physics, atomic, molecular and optical physics, and condensed matter physics.

Allan Griffin is Professor Emeritus of Physics at the University of Toronto, Canada. His research has been on superfluid helium, superconductivity and the theory of ultracold matter and quantum gases. He is co-editor of *Bose–Einstein Condensation*, and the author of *Excitations in a Bose-condensed Liquid* (1996, 2005, both from Cambridge University Press).

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Preface

Since the creation of Bose–Einstein condensation (BEC) in trapped atomic gases in 1995, there has been an enormous amount of research on ultracold quantum gases. However, most theoretical studies have ignored the dynamical effect of the thermally excited atoms. In this book, we try to give a clear development of the key ideas and theoretical techniques needed to deal with the dynamics and nonequilibrium behaviour of trapped Bose gases at finite temperatures. By limiting ourselves from the beginning to a relatively simple microscopic model, we can concentrate on the new physics which arises when dealing with the correlated motions of both the condensate and noncondensate degrees of freedom. This book also sets the stage for the future generalizations that will be needed to understand the coupled dynamics of the superfluid and normal fluid components in strongly interacting Bose gases, where there is significant depletion of the condensate even at $T = 0$.

The core of this book is based on a long paper published by the authors (Zaremba, Nikuni and Griffin, 1999). In the last decade, together with our coworkers, we have extended and applied this work in many additional papers. The starting point of our approach is not original, in that it consists of combining the Gross–Pitaevskii equation for the condensate with a Boltzmann equation for the noncondensate atoms. The kinetic equation for trapped superfluid Bose gases we use was first developed and studied in a pioneering series of papers by Kirkpatrick and Dorfman in 1985 on a uniform Bose gas at finite temperatures. In the initial phase of writing this book, we intended to refer to our coupled equations for the condensate and the thermal cloud by some name other than ZNG, but nothing we came up with was particularly descriptive or natural and we finally gave up the attempt. Therefore, we simply refer to them as the ZNG coupled equations, following the custom in the current literature of the last few years.

In the introductory chapter we discuss in more detail how our starting equations and general approach for dealing with a Bose-condensed gas at finite temperature are built on the work of previous theorists over the last five decades. On a longer time scale, our book makes use of methods for solving kinetic equations for Bose gases that were originally developed for classical gases by Boltzmann, Enskog and Chapman over a century ago.

This book should be of interest to all graduate students and researchers working in the area of ultracold dilute quantum gases and BEC. We hope that it will be particularly useful to experimental researchers, who may find the original theoretical literature difficult to understand, not to mention widely dispersed among many separate papers. It should also appeal to those in the wider theoretical and mathematical physics community who are interested in nonequilibrium problems in general, in quantum transport theory, in many body theory or in quantum field theory at finite temperatures. We have made no attempt to repeat the material on BEC in trapped atomic gases discussed in the two excellent books by Pethick and Smith (2008) and by Pitaevskii and Stringari (2003), and we hope these books will be referred to as needed by the reader.

The book includes a detailed treatment of the collision-dominated two-fluid region. In a graduate course on ultracold atoms, an introduction to the two-fluid hydrodynamics in trapped superfluid gases could be based on Chapters 2, 3, 14 and 15 as well as Section 8.1 of Chapter 8.

Originally our plan was to discuss the connections with other theoretical approaches used to deal with trapped Bose gases at finite temperatures. However, once we found that a clear exposition and development of our approach and the physics underlying it already filled 450 pages, our plan to review other formalisms in any depth was not feasible. We give references to other approaches at various places in the book. In addition, excellent reviews given at a recent workshop are available online (see footnote 2 in Chapter 1).

Allan Griffin would like to acknowledge the work of his students on the theory of interacting Bose gases, both before and after the discovery of BEC in 1995. In particular, Edward Taylor has been helpful in editing Chapters 14 and 16. Tetsuro Nikuni thanks Satoru Konabe and Takashi Inoue for valuable discussions and useful suggestions.

We thank our colleague Alexander (Sandy) Fetter for a critical reading of Chapter 9 on vortices, and our postdoctoral associates Brian Jackson and Jamie Williams for their important contributions to some of the topics covered in this book. We specifically acknowledge Brian Jackson for his suggestions on Chapters 11 and 12 and for a careful reading of the earlier chapters.

We owe special thanks to Helen Iyer at Toronto for her work over several years in preparing this book for publication. Helen did much of the manuscript preparation using the Cambridge LaTex macro and also coordinated the endless changes to various chapter files on our book's webpage.

We are extremely grateful to Edward Taylor for his crucial assistance in checking and revising the tex file and figures, as well as preparing the index. We thank Hui Hu for help with preparing the cover illustration. The English in several chapters has been greatly improved by the editorial work of Christine McClymont.

It is a pleasure to thank Simon Capelin at Cambridge University Press for his continuing support over many years. We still are amazed by the care Susan Parkinson took in copy-editing our difficult manuscript. Susan's detailed thoughtful analysis of every line of text, equation and figure eliminated many errors and ambiguities; this has led to a much more readable book.

The research on ultracold quantum gases of Allan Griffin and Eugene Zaremba has been supported by grants from NSERC of Canada over the last decade. Tetsuro Nikuni has been supported by grants from JSPS of Japan.