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The Philosophy of Gottlob Frege

This book is an analysis of Frege's views on language and metaphysics raised in "On Sense and Reference," arguably one of the most important philosophical essays of the past hundred years. It provides a thorough introduction to the function/argument analysis and applies Frege's technique to the central notions of predication, identity, existence, and truth. Of particular interest is the analysis of the Paradox of Identity and a discussion of three solutions: the little-known *Begriffsschrift* solution, the sense/reference solution, and Russell's "On Denoting" solution. Russell's views wend their way through the work, serving as a foil to Frege. Appendixes give the proofs of the first sixty-eight propositions of *Begriffsschrift* in modern notation.

This book will be of interest to students and professionals in philosophy and linguistics.

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For Marsha, Robin, and Josh

With Love

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Preface

Gottlob Frege is celebrated for his distinction between the *Sinn* and *Bedeutung* – the sense and reference – of a term. The distinction is readily understood. The reference of the name 'Plato' is the bearer of the name, that most famous and widely revered philosopher, who lived more than two thousand years ago in ancient Greece. The sense of the name 'Plato', on the other hand, corresponds to what we would ordinarily recognize as belonging to its meaning: what speakers and hearers understand by the word that enables them to identify what they are talking about and to use the word intelligently. Why is Frege celebrated for this distinction? After all, just a generation or two before, Mill (1843) expounded his distinction between the connotation and denotation of a name. In *The Port Royal Logic*, Arnauld (1662) drew a kindred distinction between an idea and its extension. In his *Summa Logicae*, William of Ockham (c. 1323) distinguished between the term in mental language associated with a word and what it supposits. Earlier still, in ancient times, the Stoic logicians distinguished between an utterance, its signification, and the name-bearer.[1] This is a very natural distinction, and we find variations on its theme reappearing throughout philosophical history. What makes *Frege's* distinction so noteworthy? The answer lies with his compositionality principles, one for reference and the other for sense. These represent a genuine advance. Frege conceived of the semantic value of a complex construction in language as being determined by the simpler ones from which it is built in a mathematically rule-governed manner. These rules provided him with a framework within which rationally to connect and unify the semantic story posited for various linguistic entities. At the very same time, it generated an explanation for the creativity of language. This last insight, which

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came into clearer focus only late in Frege's intellectual life, has proved compelling and invigorating to the logical, psychological, linguistic, and philosophical investigation of language in the twentieth and twenty-first centuries.

Although the rudiments of the function/argument analysis were in place in *Begriffsschrift*, the fundamental semantic notion of the content [*Inhalt*] of a sentence was unstable. Frege was assuming a classic philosophical picture of a level of thoughts and another level of a reality that was represented by these thoughts. But it was a picture that needed to be drawn more sharply in order to fit with the mathematical devices he had created. The *Begriffsschrift* notion of the content of a simple atomic sentence $S\alpha$ combined two distinct semantic strands: the part corresponding to the singular term was the reference of the expression and the part corresponding to the predicate was the sense of the expression. Keeping his eye firmly focused on the function/argument structure, Frege was able to win through (although twelve years later) to his sense/reference distinction: this helped enormously to clarify the important connections between the various types of expressions set in place by the compositionality principles. But confusion remained, most clearly in the application of the distinction to predicate expressions, and, relatedly, in the way in which the function/argument structure was to apply at the level of sense. We will examine an important example of the former error, namely, his enormously influential treatment of *existence:* although the problem of accounting for the informativeness of existence statements is on a par with the problem of accounting for the informativeness of identity statements, Frege ignored the parallel and persisted in denying that existence was a property of objects. Frege (1892*c*) drew his sense/reference distinction to explain the informativeness of descriptions without, unlike Russell after him, also providing a logical mechanism for them. Russell accounted for the sense of a description via the inferential connections of the underlying predicate construction; but Frege regarded descriptions as individual constants, and it remains an open problem how his notion of sense engages with these predicate constructions. Russell's famous account of definite descriptions provides a powerful foil for probing Frege's semantic theory. Russellian views will wend their way through our discussion of Frege's semantics, leading us to an example of the second sort of problem mentioned above, namely, Frege's analysis of indirect contexts. It is widely believed that Frege's semantics of indirect contexts leads to an infinite hierarchy of semantic primitives, a problem actually set in motion by Russell's (1905) criticism of Frege's distinction. We will examine both

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oratio obliqua and *oratio recta* contexts and show that neither leads to the absurdity charged. The critical distinction, as Dummett saw, is between customary sense and indirect sense; the differences in the levels of indirect sense pose no theoretical challenge to a rule-governed semantic story.

We will, in this book, be tracing some of the philosophical implications of what we take to be Frege's central innovation in philosophy of language, namely, the function/argument analysis. We do not pretend that this book is a comprehensive treatment of Frege's philosophy. We have little to offer on his important contributions to the foundations of mathematics. Even in our discussion of Frege's philosophy of language, there will be omissions: in particular, Frege's treatment of demonstratives – indeed, any in-depth analysis of Frege's notion of sense. These introduce a level of difficulty that we are not prepared to address. Our landscape is already sufficiently fraught with philosophical minefields, for we will be tackling some of the fundamental issues that exercised philosophers in the twentieth century, and we are pleased to have been able to advance as far as we have on them. Our goal here is, quite modestly, to illuminate Frege's central insight, which we take to be the function/argument analysis, at the level of reference, and to pursue this insight into the most difficult terrain of indirect contexts, hoping thereby to help clarify philosophical issues Frege grappled with.

On our reading, the sense/reference theory marked a sharp rejection of the view Frege had held earlier in *Begriffsschrift*, and which was later a standard of Russell and the early Wittgenstein, namely, the view that has come to be known as *direct reference*. Wittgenstein (1922) expressed the doctrine so:

> 3.203 A name means an object. The object is its meaning.

Although, as we just mentioned, Frege (1879) also upheld this principle, Frege (1892*c*) categorically rejected it. Frege (1892*c*) abandoned direct reference entirely, by contrast with Russell (1905), who, faced with the same puzzle, preserved direct reference for "genuine" proper names. The disagreement between the two is evident in the series of letters they exchanged.[2] In recent years, direct reference has once again become the focal point of philosophical controversy. Russellians accept the principle, while Fregeans reject it.

Within the context of the controversy, it is clearly inadvisable to translate Frege's *Bedeutung* into English as *meaning*. For on that suggested translation, Wittgenstein's words capture exactly the thought Frege (1892*c*)

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sought to uphold, and the disagreement between the two disappears.[3] A number of Frege scholars, including those who have worked so hard to make his views available to the English-speaking world, have replaced earlier choices, like the classical Black and Geach (1952) rendering as *reference*, in favor of *meaning*. But the virtues of this replacement are quite theoretical and have yet to reveal themselves. Whatever they might be, they are thoroughly outweighed by the confusion and discomfort engendered in a philosophically literate English-language reader for whom the issue of the *meaning* of a proper name, not its *Bedeutung*, is salient. Black and Geach's (1952) original choice of *reference* for *Bedeutung*, and secondarily, expressions like *designation* and *denotation*, are most comfortable. These preserve the truth value of the German original, and, in addition, provide us with a means of stating Frege's view with reasonable clarity in English. Because Black and Geach (1952) is no longer readily available, we will use Beaney's (1997) translation as the primary source for our citations. (All quotations of Frege's writings are drawn from the translations identified in the Bibliography.) Beaney (1997: 44) admits that "[i]f forced to choose, I myself would use 'reference' . . . ," but in the text he decided to leave the noun *'Bedeutung'* untranslated.

We will see in Chapter 1 that Frege's project was primarily technical. His *Logicist* program, as it has come to be called, involved (a) formalizing a logic sufficient to represent arithmetical reasoning, (b) providing definitions for arithmetical constants and operations, in purely logical terms, and (c) representing the definitionally expanded truths of arithmetic as truths of logic. Portions of this project were enormously successful, but others turned out to be disastrous. Russell located a contradiction in Frege's unrestricted comprehension schema for sets and communicated it to Frege just as the second volume of *Grundgesetze* was in press. Frege never found a solution to the problem and came to believe his program was in ruins. The Logicist program was dealt another severe setback years later when Gödel showed that not all the truths of arithmetic were provable. In any event, work on the foundations of mathematics and the philosophy of mathematics soon outstripped Frege's achievements, even his relevance. Frege's philosophy of language, however, remains intensely vital today. Not since medieval times has the connection between logic and language been so close.

Earlier versions of parts of this book have, over time, been published as separate essays. Portions of Chapters 2 and 8 are from "Frege and the Grammar of Truth," which appeared in *Grammar in Early Twentieth-Century*

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Philosophy, ed. Richard Gaskin (Routledge, London, 2001), pp. 28–53. Portions of Chapters 3 and 4 are from "Frege's *Begriffsschrift* Theory of Identity," *Journal of the History of Philosophy* **20** (1982), 279–99. Portions of Chapter 5 are from "Frege on Predication," *Midwest Studies in Philosophy* **6** (1981), 69–82. Portions of Chapter 9 are from "Frege's Treatment of Indirect Reference," in *Frege: Importance and Legacy*, ed. Matthias Schirn (Walter de Gruyter, Berlin, 1996), pp. 410–37. With respect to the first and last of these articles, however, we caution the reader that the position we adopt here is significantly different from the one we defended in those essays.

Our debt to the work of Michael Dummett should be evident throughout. Almost single-handedly, he brought Frege's philosophy into mainstream consciousness. And although we disagree with W. V. O. Quine on many of these pages, our debt to his work is evident as well. Our original interest in Frege was piqued by the way in which Quine applied technical devices to philosophical problems. Finally, we are very grateful to F. Fritsche, who helped correct earlier drafts of the two appendixes.

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