

## Principles of Glacier Mechanics

Second Edition

This book provides students and practicing glaciologists with the tools they need to understand modern glaciology. Relatively simple concepts are introduced first, followed by mathematically more sophisticated chapters. A knowledge of basic calculus is assumed, but important equations describing physical processes are developed from elementary principles. Emphasis is placed on connections between modern research in glaciology and the origin of features of glacial landscapes. Student exercises are included. This new edition builds on the successful first edition: it has been completely updated, and important new sections and whole chapters have been added. *Principles of Glacier Mechanics* is designed to be used as a primary textbook in upper division and graduate courses in glaciology, and can be used as either a primary or supplementary text in courses in glacial geology. Practicing glacial geologists and glaciologists will also find it useful as a reference book.

ROGER LEB. HOOKE is Research Professor in the Department of Earth Sciences and the Climate Change Institute, University of Maine. He has been involved in glaciological research for over 30 years, focusing on processes relevant to the origin of glacial landforms. In addition to the first edition of *Principles of Glacier Mechanics*, he has published over 80 refereed research papers in journals such as the *Geological Society of America Bulletin*, *Geology*, the *Journal of Glaciology*, *Quaternary Research*, and the *Journal of Geology*.

Cambridge University Press  
0521836093 - Principles of Glacier Mechanics, Second Edition  
Roger LeB. Hooke  
Frontmatter  
[More information](#)

---

# Principles of Glacier Mechanics

## Second Edition

**Roger LeB. Hooke**  
Research Professor  
Department of Earth Sciences  
and Climate Change Institute  
University of Maine, Orono



Cambridge University Press  
0521836093 - Principles of Glacier Mechanics, Second Edition  
Roger LeB. Hooke  
Frontmatter  
[More information](#)

CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo  
Cambridge University Press  
The Edinburgh Building, Cambridge, CB2 2RU, UK

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9780521836093](http://www.cambridge.org/9780521836093)

© R. LeB. Hooke 2005

This book is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2005

Printed in the United Kingdom at the University Press, Cambridge

*A catalog record for this book is available from the British Library*

*Library of Congress Cataloging in Publication data*

Hooke, Roger LeB.

Principles of glacier mechanics / Roger LeB. Hooke. – 2nd ed.

p. cm.

Includes bibliographical references and index.

ISBN 0 521 83609 3 (hardback : alk. paper) ISBN 0 521 54416 5 (paperback : alk. paper)

1. Glaciers. 2. Glacial landforms. 3. Ice mechanics. I. Title.

GB2403.2.H66 2005

551.31 – dc22 2004054611

ISBN-13 978-0-521-83609-3 hardback

ISBN-10 0-521-83609-3 hardback

---

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this book, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

---

It is with a deep sense of gratitude that I dedicate this book to those who, at various times through the formative stages of my life, guided me into the most exciting and rewarding career I can imagine: the study of our Earth.

To my parents, who opened many doors for me;  
to my older brother, Richard, who led me through a door leading to  
the wilderness;  
to John Muir who opened my eyes to the spirituality in wilderness;  
to my wife, Ann, who introduced me to Geology;  
to John P. Miller who focused my attention on processes at the Earth's  
surface; and  
to Robert P. Sharp who taught me that basic physical principles could  
be used to understand these processes.

## Contents

Preface to the first edition	<i>page</i> xi
Preface to the second edition	xiii
Physical constants relevant to ice	xiv
Derived SI units and conversion factors	xvii
<b>1 Why study glaciers?</b>	<b>1</b>
<b>2 Some basic concepts</b>	<b>5</b>
A note on units and coordinate axes	5
Glacier size, shape, and temperature	6
The condition of incompressibility	9
Stresses, strains, and strain rates	10
<b>3 Mass balance</b>	<b>17</b>
The transformation of snow to ice	18
Snow stratigraphy	20
Mass balance principles	23
Climatic causes of mass balance fluctuations	26
The budget gradient	29
Other modes of ice loss from valley glaciers	31
Mass balance of polar ice sheets	34
Effect of atmospheric circulation patterns on mass balance	37
Global mass balance	40
Summary	41
<b>4 Flow and fracture of a crystalline material</b>	<b>43</b>
Crystal structure of ice	43
Dislocations	44
Rate-limiting processes	48
Internal stresses	53
Recrystallization	54
Deformation mechanism maps	63
	vii

## viii Contents

	A flow law for glacier ice	66
	Fracture	70
	Summary	74
<b>5</b>	<b>The velocity field in a glacier</b>	<b>76</b>
	Measurement of velocity	77
	Balance velocity	78
	Shear stress distribution	79
	Horizontal velocity at depth in an ice sheet	81
	Horizontal velocity in a valley glacier	83
	Mean horizontal velocity and ice flux	87
	Vertical velocity	88
	Submergence and emergence velocities	91
	Flow field	92
	Transverse profiles of surface elevation on a valley glacier	94
	Radar stratigraphy	96
	Effect of drifting snow on the velocity field	98
	Ice streams	105
	Summary	110
<b>6</b>	<b>Temperature distribution in polar ice sheets</b>	<b>112</b>
	Energy balance in an ice sheet	112
	Dependence of $K$ on temperature	117
	The steady-state temperature profile at the center of an ice sheet	117
	Temperature profiles in the ablation zone	127
	Temperature profiles near the surface of an ice sheet	127
	Temperature distributions far from a divide	131
	Englacial and basal temperatures along a flow line calculated using the Column model	135
	Basal temperatures in Antarctica – comparison of solutions using the Column model and a numerical model	138
	Geomorphic implications	142
	Summary	144
<b>7</b>	<b>The coupling between a glacier and its bed</b>	<b>147</b>
	Sliding	148
	Deformation of subglacial till	168
	Stability of ice streams	190

	Effect of a frozen bed	193
	Summary	194
<b>8</b>	<b>Water flow in and under glaciers: geomorphic implications</b>	<b>197</b>
	The upper part of the englacial hydraulic system	197
	Equipotential surfaces in a glacier	201
	Melt rates in conduits	205
	Water pressures in subglacial conduits on hard beds	208
	Types of subglacial drainage system	215
	Surges	230
	Subglacial drainage paths and the formation of eskers	232
	Tunnel valleys	241
	Water pressure and glacier quarrying	244
	Origin of cirques and overdeepenings	248
	Summary	250
<b>9</b>	<b>Stress and deformation</b>	<b>252</b>
	Stress	252
	Momentum balance	261
	Deformation	262
	Condition that principal axes of stress and strain rate coincide	267
	Summary	269
<b>10</b>	<b>Stress and velocity distribution in an idealized glacier</b>	<b>271</b>
	Solutions for stresses and velocities in plane strain	271
	Comparison with real glaciers	286
	Summary	287
<b>11</b>	<b>Numerical modeling</b>	<b>288</b>
	Goals of modeling	289
	Numerical integration	289
	Finite-difference models	291
	Finite-element models	298
	Initial conditions and forcing	299
	Validation	301

## x Contents

	Intercomparison of models	301
	Sensitivity testing and tuning	302
	Coupling thermal and mechanical models	303
	Examples	304
	Summary	313
<b>12</b>	<b>Applications of stress and deformation principles to classical problems</b>	<b>315</b>
	Collapse of a cylindrical hole	315
	Calculating basal shear stresses using a force balance	326
	Creep of floating ice shelves	333
	Analysis of borehole-deformation data	338
	Summary	348
<b>13</b>	<b>Finite strain and the origin of foliation</b>	<b>349</b>
	The strain ellipse	349
	Simple and pure shear	351
	Parameters describing cumulative deformation	352
	Calculating cumulative strain	353
	Components of foliation	356
	Summary	364
<b>14</b>	<b>Response of glaciers to changes in mass balance</b>	<b>365</b>
	Positive feedback processes	366
	Response of a temperate glacier	367
	Elementary kinematic wave theory	368
	Analysis of the effect of a small change in mass balance using a perturbation approach	371
	Effect of diffusion	375
	The problem at the terminus	376
	Further study of the response time	376
	Numerical modeling of glacier responses	381
	Comparison with observation	383
	Summary	390
	Appendix Problems	391
	References	399
	Index	421



## Preface to the first edition

One might well ask why one should write a book about so specialized a subject as glacier mechanics when there are already other good books on this subject written by eminent glaciologists. This book is an outgrowth of a course that I teach to students who, in many cases, do not have any background in continuum mechanics. Consequently, it was necessary to start at a level considerably less advanced than that at which other similar books begin, and to develop the theoretical principles one step at a time. Thus, unlike other books on the subject and the general scientific literature, in which space is at a premium, the steps leading from one equation to another are, in most cases, easily seen. In addition, qualitative interpretations of the equations are often provided to clarify the physics behind the mathematics. Capable students with a solid background in basic physics and in differential and integral calculus, and with some modest exposure to differential equations, will have little difficulty understanding the concepts and derivations presented.

My goal in writing this book was not to produce a comprehensive treatise on glacier mechanics, but rather to develop the basic foundation upon which the modern literature on this subject rests. Thus, many topics are not covered, or are treated in less detail than some readers might wish. However, students who have a full appreciation for the concepts in this book will have the background they need to understand most of the current literature.

Beginning students in glaciology will find that this book will save them many long hours of searching through the background literature to clarify basic concepts. Glacial geologists and geomorphologists will also find much of value, including applications of glacier physics to the origin of some glacial landforms. Structural geologists and others with interest in stress and deformation will likewise discover that glaciers are, in fact, monomineralic rock masses that are deforming at the Earth's surface where they can be observed in detail. The book is, thus, appropriate for upper division and graduate level courses in glaciology, and as a supplementary text for courses in glacial geology and in structural geology.

xii Preface to the first edition

In the preliminary pages, readers will find a compilation of physical constants relevant to ice, and a list of SI units and conversion factors. A series of problems keyed to individual chapters is also included.

The encouragement I have received in this undertaking from many present and former students, as well as from other glaciologists, has been a major stimulus in bringing it to completion. I trust the final product is worthy of their confidence. The book has benefited from the critical comments of R. W. Baker at the University of Wisconsin, River Falls; C. R. Bentley at the University of Wisconsin, Madison; G. K. C. Clarke at the University of British Columbia; E. M. Grace, and B. Hanson at the University of Delaware; N. R. Iverson at the University of Minnesota; T. Jóhannesson at the Icelandic Meteorological Office; M. Kuhn at the University of Innsbruck, Austria; M. F. Meier at the University of Colorado; J. F. Nye at the University of Bristol, England; C. F. Raymond at the University of Washington; R. L. Shreve at the University of California, Los Angeles; J. Weertman at Northwestern University, and especially I. Whillans at Ohio State University.

June 25, 1996

## Preface to the second edition

When I wrote the preface to the first edition of this book seven years ago, nothing was further from my mind than a second edition. The first edition was well received, however, and on numerous occasions colleagues have lamented the fact that it was no longer available. When Cambridge University Press agreed that a new edition was desirable, little did I realize what I had gotten into.

When I told Matt Lloyd (my editor at Cambridge) that my goal was to have the text ready by a certain time, he graciously gave me a target date that was nearly double that time. I told him that his time schedule was fine, but that I did not want to be held too strictly to it. As it happens, I had an unrealistic view of the volume of new material that needed to be sifted through, absorbed, and translated into language appropriate for the upper-division undergraduate and graduate-level students for whom this book is written. As with the first edition, my goal is not to provide an encyclopedia of research in glaciology, as other books do that well, but rather to give students the basic background they will need to understand the modern literature. At the same time, the book has proven to be a useful reference for professionals who don't keep all of the equations and conversion factors stored for instant recall. I myself use it for that purpose frequently.

I am indebted to many who have encouraged me in this undertaking, and especially to those who have generously given their time to review new sections or entire chapters, who have resurrected archived computer files to provide images or data files from which new figures were produced, or who have made new calculations especially for this volume. The following have assisted me in this effort: Richard Alley, Bob Bindschadler, Ginny Catania, Chris Clark, Lee Clayton, Paul Cutler, Gordon Hamilton, Brian Hanson, Bruce Hooke, Peter Hudleston, Kolumbian Hutter, Philippe Huybrechts, Neal Iverson, Peter Jansson, Susan Kaspari, Katie Leonard, Paul Mayewski, Shawn Marshall, Howard Mooers, Nadine Nereson, Felix Ng, Charlie Raymond, Vandy Spikes, Slawek Tulaczyk, and Joe Walder.

## Physical constants relevant to ice

Symbol	Parameter	Value
$g$	Acceleration of gravity	$9.81 \text{ m s}^{-2}$
$\rho$	Density of bubble-free ice	$916 \text{ kg m}^{-3}$
$\rho$	Density of water at $0^\circ\text{C}$	$999.84 \text{ kg m}^{-3}$
$\theta_m$	Melting point at atmospheric pressure	$0.0^\circ\text{C}$ $273.15 \text{ K}$
$C$	Heat capacity (For temperatures above $\sim -0.5^\circ\text{C}$ the effective heat capacity increases owing to the presence of a liquid phase, the amount of which depends upon the concentration of chemical impurities.)	$2093 \text{ J kg}^{-1} \text{ K}^{-1}$
$C_w$	Heat capacity of air-free water at constant pressure	$256.9 \text{ J kg}^{-1} \text{ K}^{-1}$
$C$	Depression of the melting point due to pressure Pure ice and air-free water Pure ice and air-saturated water (Harrison, 1972)	$0.074 \text{ K MPa}^{-1}$ $0.098 \text{ K MPa}^{-1}$
$\zeta$	Depression of the melting point due to solutes	$1.86^\circ\text{C kg mol}^{-1}$
$L$	Heat of fusion	$3.34 \times 10^5 \text{ J kg}^{-1}$
$K$	Thermal conductivity at $-1^\circ\text{C}$ K varies with temperature, thus: $K = 7.10 \times 10^7 - 1.96 \times 10^5 \theta = 3.63 \times 10^3 \theta^2$ where $\theta$ is in degrees Celsius (a negative number) (Ratcliffe, 1962)	$7.1 \times 10^7 \text{ J m}^{-1} \text{ a}^{-1} \text{ K}^{-1}$

Symbol	Parameter	Value
$\kappa$	Thermal diffusivity at $-1\text{ }^{\circ}\text{C}$ (Below $-0.5\text{ }^{\circ}\text{C}$ , $\kappa$ varies with temperature owing to the variation in $K$ (see above). Above $-0.5\text{ }^{\circ}\text{C}$ , $\kappa$ decreases owing to the increase in effective $C$ (see above). Paterson (1971) estimates that at $-0.1\text{ }^{\circ}\text{C}$ $\kappa$ is half its value for pure ice, and at $-0.01\text{ }^{\circ}\text{C}$ it is 1% of the value for pure ice. These estimates assume a salinity of $10^{-6}$ .)	$37.2\text{ m}^2\text{ a}^{-1}$
$R$	Gas constant	$8.31\text{ J mol}^{-1}\text{ K}^{-1}$ $462\text{ J kg}^{-1}\text{ K}^{-1}$
$Q$	Activation energy for creep below $-10\text{ }^{\circ}\text{C}$ (Barnes <i>et al.</i> , 1971) (Above $-10\text{ }^{\circ}\text{C}$ , $Q$ is presumably the same but the $\dot{\epsilon}$ vs $1/\theta$ curve steepens, probably owing to the presence of a liquid phase.)	$78.8\text{ kJ mol}^{-1}$
$\nu$	Poisson's ratio for polycrystalline ice (Gold, 1958) (The ratio of the transverse strain (contraction) to the axial strain (extension) of a bar in a uniaxial tensile test.)	0.31*
$\mu$	Shear modulus (at $-5\text{ }^{\circ}\text{C}$ ) (Gold, 1958) (The ratio of shear stress to elastic shear strain in a test in simple shear.)	$3.8 \times 10^3\text{ MPa}^*$
$K$	Bulk modulus (at $-5\text{ }^{\circ}\text{C}$ ) (Gold, 1958) (The ratio of applied pressure to fractional change in volume.)	$8.7 \times 10^3\text{ MPa}^*$
$E$	Young's modulus (Gold, 1958) (The ratio of axial stress to elastic axial strain in a test in uniaxial tension. $E = 2\mu(1 + \nu)$ .)	$8.3 \times 10^3\text{ MPa}^*$
$a$	Coefficient of linear thermal expansion of ordinary water at $0\text{ }^{\circ}\text{C}$ (Kell, 1967) ice at $\sim -10\text{ }^{\circ}\text{C}$	$-22.3 \times 10^{-6}\text{ K}^{-1}$ $51.6 \times 10^{-6}\text{ K}^{-1}$
$b$	Burgers vector	$4.5 \times 10^{-10}\text{ m}$

## xvi Physical constants relevant to ice

Symbol	Parameter	Value
$\gamma_{\text{SL}}$	Specific surface energy of liquid–solid interface (Ketcham and Hobbs, 1969)	$0.034 \text{ J m}^{-2}$
$\gamma_{\text{gb}}$	Specific surface energy of grain boundary	$0.065 \text{ J m}^{-2}$
$\beta$	Dihedral angle ( $\cos \beta = \gamma_{\text{gb}}/2\gamma_{\text{SL}}$ ) (Nye and Mae, 1972)	$2\beta = 32 \pm 3^\circ$
$P_{\text{TP}}$	Triple point pressure	600 Pa
$\theta_{\text{TP}}$	Triple point temperature	$+0.0098^\circ\text{C}$
$S_{\text{cr}}$	Crushing strength of ice formed from natural snow. The strength increases substantially with decreasing temperature. Hobbs (1974, p. 331) gives a graph from Butkovitch (1954) that can be approximated by: $S_{\text{cr}} = 1.8 - 0.266\theta - 0.0202\theta^2 - 7.72 \times 10^{-4}\theta^3 - 1.39 \times 10^{-5}\theta^4 - 9.37 \times 10^{-8}\theta^5$ where $\theta$ is the temperature in degrees Celsius (a negative number). There is considerable variability depending on the type of ice tested and its orientation.	1.8 MPa at $0^\circ\text{C}$
$K_{\text{Ic}}$	Fracture toughness (Rist <i>et al.</i> , 1999)	0.05–0.15 MPa m <sup>1/2</sup>

\* Values given are based on the work of Gold (1958) as reported by Hobbs (1974, pp. 255–258). Hobbs also reports other values based on the work of other (earlier) investigators.

## Derived SI units and conversion factors

$$1 \text{ N} = 1 \text{ kg m s}^{-2}$$

$$1 \text{ Pa} = 1 \text{ N m}^{-2} = 1 \text{ kg m}^{-1} \text{ s}^{-2}$$

$$1 \text{ J} = 1 \text{ N m} = 1 \text{ kg m}^2 \text{ s}^{-2}$$

$$1 \text{ W} = 1 \text{ J s}^{-1} = \text{N m s}^{-1}$$

$$1 \text{ bar} = 0.1 \text{ MN m}^{-2} = 0.1 \text{ MPa} = 0.9868 \text{ atm}$$

$$1 \text{ cal} = 4.18 \text{ J}$$

$$1 \text{ a} = 3.155\,69 \times 10^7 \text{ s}$$

$$0 \text{ }^\circ\text{C} = 273.15 \text{ K}$$

Force (mass · acceleration)

Stress

Work or energy

Power