Chapter 1
Why study glaciers?

Before delving into the mathematical intricacies with which much of this book is concerned, one might well ask why we are pursuing this topic – glacier mechanics? For many who would like to understand how glaciers move, how they sculpt the landscape, how they respond to climatic change, mathematics does not come easily. I assure you that all of us have to think carefully about the meaning of the expressions that seem so simple to write out but so difficult to understand. Only then do they become part of our vocabulary, and only then can we make use of the added precision which mathematical analysis, properly formulated, is able to bring. Is it worth the effort? That depends upon your objectives; on why you chose to study glaciers.

There are many reasons, of course. Some are personal, some academic, and some socially significant. To me, the personal reasons are among the most important: glaciers occur in spectacular areas, often remote, that have not been scarred by human activities. Through glaciology, I have had the opportunity to live in these areas; to drift silently in a kayak on an ice-dammed lake in front of our camp as sunset gradually merges with sunrise on an August evening; to marvel at the northern lights while out on a short ski tour before bedtime on a December night; and to reflect on the meaning of life and of our place in nature. Maybe some of you will share these needs, and will choose to study glaciers for this reason. I have found that many glaciologists do share them, and this leads to a comradeship which is rewarding in itself.

Academic reasons for studying glaciers are perhaps difficult to separate from socially significant ones. However, in three academic disciplines, the application of glaciology to immediate social problems is at least one step removed from the initial research. The first of these is glacial geology. Glaciers once covered 30% of the land area of the Earth, and left deposits of diverse shape and composition. How were these deposits formed, and what can they tell us about the glaciers that made them? The second discipline is structural geology; glacier ice is a metamorphic rock that can be observed in the process of deformation at temperatures close to the melting point. From the study of this deformation, both in the laboratory and in the field, much can be learned about the origin of metamorphic structures in other crystalline rocks that were deformed deep within the Earth. The final discipline is paleoclimatology. Glaciers record climatic fluctuations in two ways: the deposits left during successive advances and retreats provide a coarse record of climatic change which, with careful study, a little luck, and a good deal of skill, can be placed in correct chronological order and dated. A more detailed record is contained in ice cores from polar glaciers such as the Antarctic and Greenland ice sheets. Isotopic and chemical variations in these cores reflect present atmospheric circulation patterns and past changes in the temperature and composition of the atmosphere. Changes during the past several centuries to several millennia can be rather precisely dated by using core stratigraphy. Changes further back in time are dated with less certainty using flow models.

Relatively recent changes in climate and in concentrations of certain anthropogenic substances in the atmosphere are attracting increasing attention as humans struggle with problems of maintaining a healthy living environment in the face of overpopulation and the resulting demands on natural resources. Studies of ice cores and other dated ice samples provide a baseline from which to measure these anthropogenic changes. For example, levels of lead in the Greenland ice sheet increased about four-fold when Greeks and Romans began extracting silver from lead sulphides (Hong *et al.*, 1994). Then, after dropping slightly in the first millennium AD, they increased to ~80 times natural levels during the industrial revolution and to ~200 times natural levels when lead additives became common in gasoline (Murozumi *et al*., 1969). These studies are largely responsible for the fact that lead is no longer used in gasoline. Similarly, measurements of CO_2 and CH_4 in ice cores have documented levels of these greenhouse gases in pre-industrial times.

Other applications of glaciology are not hard to find. An increasing number of people in northern and mountainous lands live so close to glaciers that their lives would be severely altered by ice advances comparable in magnitude to the retreats that have taken place during the past century in many parts of the world. Tales of glacier advances gobbling up farms and farm buildings and of ice falls smashing barns and houses are common from the seventeenth and eighteenth centuries, a period of ice advance as the world entered the Little Ice Age. Records tell of buildings being crushed into small pieces and mixed with “soil, grit, and great rocks” (Grove, 1988, p. 72). The Mer de Glace in France presented a particular problem during this time period, and several times during the seventeenth century exorcists were sent out to deal with the “spirits” responsible for its advance. They appeared to have been successful, as the glaciers there were then near their Little Ice Age maxima and beginning to retreat.

Other people live in proximity to streams draining lakes dammed by glaciers. Some of the biggest floods known from the geologic record resulted from failure of such ice dams, and smaller floods of the same origin have devastated communities in the Alps and Himalayas.

Somewhat further from human living environments, one finds glaciers astride economically valuable deposits, or discharging icebergs into the shipping lanes through which such deposits are moved. What complications would be encountered, for example, if mining engineers were to make an open pit mine through the edge of the Greenland Ice Sheet to tap an iron deposit? What is the possibility that the present rapid retreat of Columbia Glacier in Alaska will increase perhaps ten-fold, perhaps one hundred-fold, the flux of icebergs into shipping lanes leading to the port of Valdez, at the southern end of the trans-Alaska pipeline? Were shipping to be halted there for an extended period so that the oil flow through the pipeline had to be stopped, oil would congeal in the pipe, making what one glaciologist referred to as the world’s longest candle.

Glacier ice itself is an economically valuable deposit; glaciers contain 60% of the world’s fresh water, and peoples in arid lands have seriously studied the possibility of towing icebergs from Antarctica to serve as a source of water. People in mountainous countries use the water not only for drinking, but also as a source of hydroelectric power. By tunneling through the rock under a glacier and thence up to the ice–rock interface, they trap water at a higher elevation than would be possible otherwise, and thus increase the energy yield. Glaciologists provided advice on where to find streams beneath the glaciers.

With the threat of global warming hanging over the world, the large volume of water locked up in glaciers and ice sheets represents a potential hazard for human activities in coastal areas. Collapse of the West Antarctic Ice Sheet could lead to a worldwide rise in sea level of 7 m in, perhaps, less than a century. Were this to be followed by melting of the East Antarctic Ice Sheet, sea level could rise an additional 50 m or so. Concern over these prospects has stimulated a great deal of research in the past two decades.

Lastly, we should mention a proposal to dispose of radioactive waste by letting it melt its way to the base of the Antarctic Ice Sheet. How long would such waste remain isolated from the biologic environment? How would the heat released affect the flow of the ice sheet? Might it cause a surge, with thousands of cubic kilometers of ice dumped into the oceans over a period of a few decades? This would raise sea level several tens of meters, with, again, interesting consequences! To accommodate these concerns, later versions of the proposal called for suspending the waste canisters on wires anchored at the glacier surface. The whole project was later abandoned, however, but not on glaciological grounds. Rather, there seemed to be no risk-free way to transport the waste to the Antarctic.

A good quantitative understanding of the physics of glaciers is essential for rigorous treatment of a number of these problems of academic interest, as well as for accurate analysis of various engineering and environmental problems of concern to humans. The fundamental principles upon which this understanding is based are those of physics and, to a lesser extent, chemistry. Application of these principles to glacier dynamics is initially straightforward but, as with many problems, the better we seek to understand the behavior of glaciers the more involved, and in many respects the more interesting, the applications become.

So we have answered our first question; we study glaciers for the same reasons that we study many other features of the natural landscape, but also for a special reason which I will try to impart to you, wordlessly, if you will stand with me looking over a glacier covered with a thick blanket of fresh powder snow to distant peaks, bathed in alpine glow, breathless from a quick climb up a steep slope after a day of work, but with skis ready for the telemark run back to camp. “Mäktig,” my companion said – powerful.

Chapter 2
Some basic concepts

In this chapter, we will introduce a few basic concepts that will be used frequently throughout this book. First, we review some commonly used classifications of glaciers by shape and thermal characteristics. Then we consider the mathematical formulation of the concept of conservation of mass and, associated with it, the condition of incompressibility. This will appear again in Chapters 6 and 9. Finally, we discuss stress and strain rate, and lay the foundation for understanding the most commonly used flow laws for ice. Although a complete consideration of these latter concepts is deferred to Chapter 9, a modest understanding of them is essential for a fuller appreciation of some fundamental concepts presented in Chapters 4–8.

A note on units and coordinate axes

SI (Système International) units are used in this book. The basic units of length, mass, and time are the meter (m), kilogram (kg), and second (s) (MKS). Temperatures are measured in Kelvins (K) or in the derived unit, degrees Celsius (°C). Some other derived units and useful conversion factors are given on p. xvii.

In comparison with the earlier glaciological literature, one of the most significant changes introduced by use of SI units is that from bars to pascals as the principal unit of stress. The bar (= 0.1 MPa ≈ 1 atmosphere) was a convenient unit because stresses in glaciers are typically ∼1 bar.

In most discussions herein we use a rectangular coordinate system with the *x*-axis horizontal or subhorizontal and in the direction of flow, the y-axis horizontal and transverse, and the z-axis normal to the other two and thus vertical or slightly inclined to the vertical. Some derivations are easier to approach with the z-axis directed upward, while in others it is simpler to have the z-axis directed downward.

Glacier size, shape, and temperature

As humans, one way in which we try to organize knowledge and enhance communication is by classifying objects into neat compartments, each with it own label. The natural world persistently upsets these schemes by presenting us with particular items that fit neither in one such pigeonhole nor the next, but rather have characteristics of both, for continua are the rule rather than the exception. This is as true of glaciers as it is of other natural systems.

One way of classifying glaciers is by shape. Herein, we will be concerned with only two basic shapes. Glaciers that are long and comparatively narrow, and that flow in basically one direction, down a valley, are called *valley glaciers*. When a valley glacier reaches the coast and interacts with the sea, it is called a *tidewater glacier*. (I suppose this name is appropriate even in circumstances in which the tides are negligible, although with luck no one will ever find a valley glacier encroaching on such a tideless marine environment.) Valley glaciers that are very short, occupying perhaps only a small basin in the mountains, are called *cirque glaciers*. In contrast to these forms are glaciers that spread out in all directions from a central dome. These are called either *ice caps*, or, if they are large enough, *ice sheets*.

There is, of course, a continuum between valley glaciers and ice caps or ice sheets. For example, Jostedalsbreen in Norway and some ice caps on islands in the Canadian arctic feed *outlet glaciers,* which are basically valley glaciers flowing outward from an ice cap or ice sheet. However, the end members, valley glaciers and ice sheets, typically differ in other significant ways (see, for example, Figure 3.1). Thus, a classification focusing on these two end members is useful.

Glaciers are also classified by their thermal characteristics, although once again a continuum exists between the end members. We normally think of water as freezing at 0 °C, but may overlook the fact that once all the water in a space is frozen, the temperature of the resulting ice can be lowered below 0 °C as long as heat can be removed from it. Thus, the temperature of ice in glaciers in especially cold climates can be well below 0 °C. We call such glaciers *polar glaciers*. More specifically, polar glaciers are glaciers in which the temperature is below the melting temperature of ice everywhere, except possibly at the bed. Because the presence of meltwater at the base of a polar glacier has dramatic

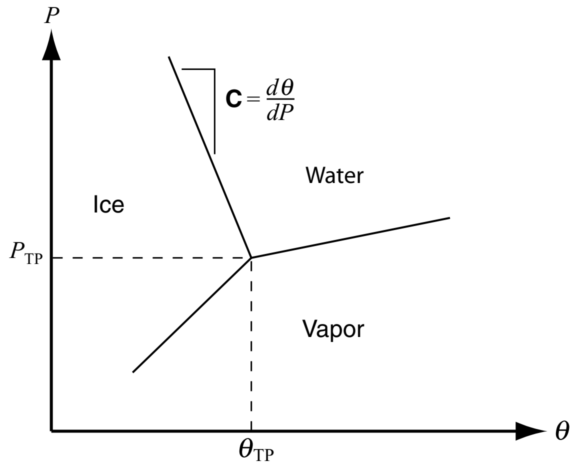

Figure 2.1. Schematic phase diagram for H_2O near the triple point, TP. At the triple point, liquid, solid, and vapor phases are in equilibrium. As long as all three phases are present, neither the pressure nor the temperature can depart from their triple-point values.

consequences for both glacier kinematics and landform development, it will be convenient to refer to such glaciers as *Type II* polar glaciers and to those that are frozen to their beds as *Type I* polar glaciers. In Chapter 6, we will investigate the temperature distribution in such glaciers in some detail.

Glaciers that are not polar are either *polythermal* or *temperate*. Polythermal glaciers, which are sometimes called *subpolar* glaciers, contain large volumes of ice that are cold, but also large volumes that are at the melting temperature. Most commonly, the cold ice is present as a surface layer, tens of meters in thickness, on the lower part of the glacier (the ablation area).

In simplest terms, a temperate glacier is one that is at the melting temperature throughout. However, the melting temperature, θ_m, is not easily defined. As the temperature of an ice mass is increased towards the melting point, veins of water form along the lines where three ice crystals meet (Figure 8.1). At the wall of such a vein:

$$\theta_m = \theta_{TP} - \mathbf{C}P - \frac{\theta_{mK}\gamma_{SL}}{L\rho_i r_p} - \zeta\frac{s}{W} \tag{2.1}$$

(Raymond and Harrison, 1975; Lliboutry, 1976). Here, θ_{TP} is the triple point temperature, 0.0098 °C (Figure 2.1); $\mathbf{C}$ is the depression of the melting point with increased pressure, P (Figure 2.1); θ_{mK} is the melting point temperature in Kelvin, 273.15 K; γ_{SL} is the liquid–solid surface energy, 0.034 J m^{-2}; L is the latent heat of fusion, 3.34×10^5 J kg^{-1}; ρ_i is the density of ice; r_p is the radius of curvature of liquid–solid interfaces; s is the solute content of the ice in mols kg^{-1}, W is the fractional water content of the ice by weight (kg kg^{-1}), and ζ is the depression of the melting point resulting from solutes in the ice, 1.86 °C kg mol^{-1}. The third term on the right in Equation (2.1) represents a change in melting temperature in the immediate vicinity of veins. **C** is the Clausius–Clapeyron slope, given by:

$$\mathbf{C} = \frac{d\theta}{dP} = \left(\frac{1}{\rho_i} - \frac{1}{\rho_w}\right)\frac{\theta_{TPK}}{L} \qquad (2.2)$$

Here, ρ_w is the density of water and θ_{TPK} is the triple point temperature in Kelvins. **C** is 0.0742 K MPa^{-1} in pure water, but rises to 0.098 K MPa^{-1} in air-saturated water. As glacier ice normally contains air bubbles, the water is likely to contain air even if it is not saturated with air. Thus, under most circumstances it is probably appropriate to use a value higher than 0.0742 K MPa^{-1} (Lliboutry, 1976).

Clearly, the melting temperature varies on many length scales in a glacier (Equation (2.1)). On the smallest scales, it varies within the veins that occur along crystal boundaries. On a slightly larger scale, it varies from the interiors of crystals to the boundaries because solutes become concentrated on the boundaries during crystal growth. On the largest scale, it varies with depth owing to the change in hydrostatic pressure.

As a result of these variations, small amounts of liquid are apparently present on grain boundaries at temperatures as low as about −10 °C, and the amount of liquid increases as the temperature increases. This led Harrison (1972) to propose a more rigorous definition of a temperate glacier. He suggested that a glacier be considered temperate if its heat capacity is greater than twice the heat capacity of pure ice. In other words, this is when the temperature and liquid content of the ice are such that only half of any energy put into the ice is used to warm the ice (and existing liquid), while the other half is used to melt ice in places where the local melting temperature is depressed.

Harrison's definition, while offering the benefit of rigor, is not easily applied in the field. However, as we shall see in Chapter 4, relatively small variations in the liquid content of ice can have a major influence on its viscosity and crystal structure, among other things. Thus, this discussion serves to emphasize that the class of glaciers that we loosely refer to as temperate may include ice masses with a range of physical properties that are as wide as, or wider than, those of glaciers that we refer to as polar.

Ice caps and ice sheets are commonly polar, while valley glaciers are more often temperate. However, there is nothing in the respective classification schemes that requires this. In fact, many valley glaciers in high Arctic areas and in Antarctica are at least polythermal, and some are undoubtedly polar. However, none of the major ice caps or ice sheets that exist today is temperate.

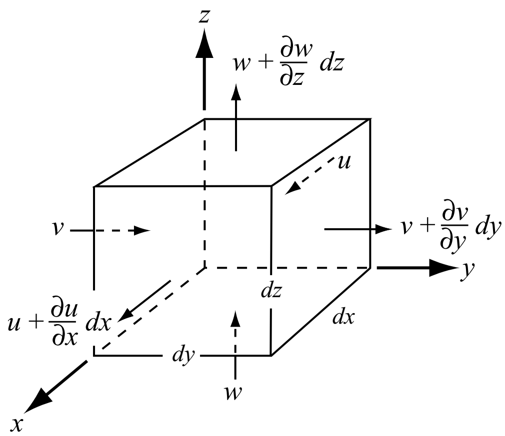

Figure 2.2. Derivation of the condition of incompressibility.

The condition of incompressibility

Let us next examine the consequences of the requirement that mass be conserved in a glacier. In Figure 2.2 a control volume of size $dx{\cdot}dy{\cdot}dz$ is shown. The velocities into the volume in the x-, y-, and z-directions are u, v, and w respectively. The velocity out in the x-direction is:

$$u + \frac{\partial u}{\partial x} dx$$

Here $\partial u/\partial x$ is the velocity gradient through the volume, which, when multiplied by the length of the volume, dx, gives the change in velocity through the volume in the x-direction. The mass fluxes into and out of the volume in the x-direction are:

$$\underset{\frac{\mathrm{kg}}{\mathrm{m}^3}}{\rho}\ \underset{\frac{\mathrm{m}}{\mathrm{a}}}{u}\ \underset{\mathrm{m}}{dy}\ \underset{\mathrm{m}}{dz} = \frac{\mathrm{kg}}{\mathrm{a}} \qquad \text{and} \qquad \left(\rho u + \frac{\partial \rho u}{\partial x} dx\right) dy\, dz$$

Here, ρ is the density of ice. (The dimensions of the various parameters are shown beneath the left-hand term to clarify the physics. This is a procedure that we will use frequently in this book, and that the reader is likely to find useful, as errors in equations can often be detected in this way.) Similar relations may be written for the mass fluxes into and out of the volume in the y- and z-directions. Summing these fluxes, we find that the change in mass with time, $\partial m/\partial t$, in the control volume is:

$$\frac{\partial m}{\partial t} = \rho u\, dy\, dz - \left(\rho u + \frac{\partial \rho u}{\partial x} dx\right) dy\, dz + \rho v\, dx\, dz - \left(\rho v + \frac{\partial \rho v}{\partial y} dy\right) dx\, dz$$
$$+ \rho w\, dx\, dy - \left(\rho w + \frac{\partial \rho w}{\partial z} dz\right) dx\, dy$$

Note that each term on the right-hand side has the dimensions $\mathrm{M} \cdot \mathrm{T}^{-1}$, or, in the units which we will use most commonly herein, kg a^{-1}.

Simplifying by canceling like terms of opposite sign and dividing by $dx{\cdot}dy{\cdot}dz$ yields:

$$-\frac{1}{dx\,dy\,dz}\frac{\partial m}{\partial t}=\frac{\partial \rho u}{\partial x}+\frac{\partial \rho v}{\partial y}+\frac{\partial \rho w}{\partial z} \tag{2.3}$$

Ice is normally considered to be incompressible, which means that ρ is constant. This is not true near the surface of a glacier where snow and firn are undergoing compaction, but to a good approximation it is valid throughout the bulk of most ice masses. In this case, Equation (2.3) becomes:

$$-\frac{1}{\rho dx\,dy\,dz}\frac{\partial m}{\partial t}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} \tag{2.4}$$

The mass of ice in the control volume can change if the control volume is not full initially. When it is full of incompressible ice, however, $\partial m/\partial t = 0$, and Equation (2.4) becomes:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{2.5}$$

This is the condition of incompressibility; it describes the condition that mass and density are not changing.

Stresses, strains, and strain rates

A stress is a force per unit area, and has the dimensions N m^{-2}, or Pa. Stresses are vector quantities in that they have a magnitude and direction. Stresses that are directed normal to the surface on which they are acting are called normal stresses, while those that are parallel to the surface are shear stresses.

Notation

Referring to Figure 2.3, σ_{xz} is the shear stress in the z-direction on the plane normal to the x-axis. Thus, the first subscript in a pair identifies the plane on which the stress acts, and the second gives the direction of the stress.

The sign convention used in such situations is as follows. Let $\hat{\mathbf{n}}$ be the outwardly directed normal to a surface; $\hat{\mathbf{n}}$ is positive if it is directed in the positive direction and conversely. If a normal stress is in the positive direction and $\hat{\mathbf{n}}$ is also positive on this face, the normal stress is defined as positive, and conversely if one is positive and the other negative, the stress is negative. Thus in Figure 2.3, σ_{zz} is positive on both of the faces normal to the z-axis and σ_{xx} is negative on both of the faces normal to the x-axis. In other words, *tension is positive and compression is negative*.