

INTRODUCTION TO QUANTUM OPTICS

From Light Quanta to Quantum Teleportation

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Translated from German by
IGOR JEX



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1

Introduction

And the Lord saw that the light was good.
Genesis 1:4

Most probably all people, even though they belong to different cultures, would agree on the extraordinary role that light – the gift of the Sun-god – plays in nature and in their own existence. Optical impressions mediated by light enable us to form our views of the surrounding world and to adapt to it. The warming power of the sun’s rays is a phenomenon experienced in ancient times and still appreciated today. We now know that the sun’s radiation is the energy source for the life cycles on Earth. Indeed, it is photosynthesis in plants, a complicated chemical reaction mediated by chlorophyll, that forms the basis for organic life. In photosynthesis carbon dioxide and water are transformed into carbohydrates and oxygen with the help of light. Our main energy resources, coal, oil and gas, are basically nothing other than stored solar energy.

Finally, we should not forget how strongly seeing things influences our concepts of and the ways in which we pursue science. We can only speculate whether the current state of science could have been achieved without sight, without our ability to comprehend complicated equations, or to recognize structures at one glance and illustrate them graphically, and record them in written form.

The most amazing properties, some of which are completely alien to our common experiences with solid bodies, can be ascribed to light: it is weightless; it is able to traverse enormous distances of space with incredible speed (Descartes thought that light spreads out instantaneously); without being visible itself, it creates, in our minds, via our eyes, a world of colors and forms, thus “reflecting” the outside world. Due to these facts it comes as no surprise that optical effects confronted our knowledge-seeking mind with more difficult problems than those of moving material objects. Over several hundred years a bitter war was fought between two parties. One group, relying on Newton’s authority, postulated

the existence of elementary constituents of light. The other, inspired by the ideas of Huygens, fought for light as a wave phenomenon. It seemed that the question was ultimately settled in favor of the wave alternative by Maxwell's theory, which conceived light as a special form of the electromagnetic phenomena. All optical phenomena could be related without great difficulty and to a high degree of accuracy to special solutions of the basic equations of classical electrodynamics, the Maxwell equations.

However, not more than 40 years passed and light phenomena revealed another surprise. The first originated in studies of black-body radiation (radiation emitted from a cavity with walls held at a constant temperature). The measured spectral properties of this radiation could not be theoretically understood. The discrepancy led Max Planck to a theory which brought about a painful break with classical physics. Planck solved the problem by introducing as an *ad hoc* hypothesis the quantization of energy of oscillators interacting with the radiation field.

On the other hand, special features of the photoelectric effect (or photoeffect) led Einstein to the insight that they are most easily explained by the "light quantum hypothesis". Based on an ingenious thermodynamic argument Einstein created the concept of a light field formed from energy quanta $h\nu$ localized in space (h is Planck's constant and ν is the light frequency).

The newly created model was fully confirmed in all its quantitative predictions by studies of the photoeffect that followed, but there was also no doubt that many optical phenomena like interference and diffraction can be explained only as wave phenomena. The old question, Is light formed from particles or waves?, was revived on a new, higher level. Even though painful for many physicists, the question could not be resolved one way or the other. Scientists had to accept the idea that light quanta, or photons as they were later called, are objects more complicated than a particle or a wave. The photon resembles a Janus head: depending on the experimental conditions it behaves either like a particle or as a wave. We will face this particle-wave dualism several times in the following chapters when we analyze different experiments in our quest to elucidate the essence of the photon. Before this, let us take a short stroll through the history of optics.