

Chapter 1

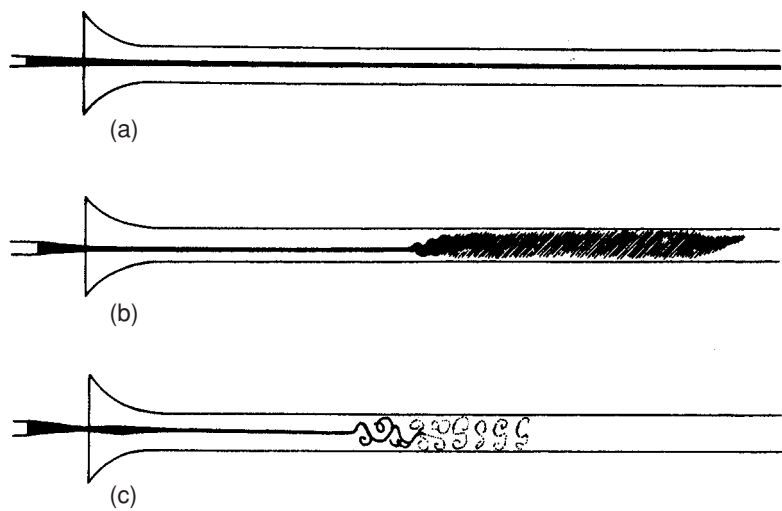
Heat, buoyancy, instability and turbulence

1.1 Introduction

The fluids, air and water, of the atmosphere and ocean<sup>1</sup> that cover the solid surface of the Earth, are almost everywhere in a state referred to as ‘turbulent’. At its simplest level, turbulence involves the sort of eddying motions visible in clouds and smoke plumes, and that are felt in the gustiness of the wind or are seen in the movement of patches of foam on the surface of the sea. Some of the manifestations of turbulence in the ocean are illustrated in figures in this and subsequent chapters. Turbulence is very effective in the transfer of momentum and heat in the ocean. It disperses, stresses and strains the particles and living organisms within the ocean, and it stirs, spreads and dilutes the chemicals that are dissolved in the seawater or released into the ocean from natural and anthropogenic sources.

Knowledge of ocean turbulence and its effects is crucial in understanding how the ocean works and in the construction of models to predict how the ocean will change or how its interactions with the atmosphere will be altered in the future. Although estimates of the rate of dissipation of the energy of the tides through turbulence in shallow seas were made as early as 1919, direct observations of turbulence in the ocean date back only to the measurements of near-bed turbulent stress made in the 1950s and to studies of the spectra of small-scale motions in the upper ocean in the early 1960s. In spite of ingenious developments in techniques for measuring turbulent motions, the ocean is still grossly under-sampled and, in comparison with the atmosphere, there are

1 By the ‘ocean’ is meant, here and later, the sum of the major oceans and their connected seas, including the continental shelf seas and those seas, such as the Mediterranean, Black Sea and Baltic, connected by straits to the larger ocean basins.



**Figure 1.1.** Reynolds’ sketches of the appearance of a ribbon of dye in liquid flowing from the left through a narrow tube of circular cross-section. (a) Laminar flow at sub-critical Reynolds number. (b) Transition to turbulent flow at super-critical Reynolds number. (c) Transition to turbulent flow seen when the dye is illuminated by a spark. (From Reynolds, 1883.)

few sets of data against which to test models of the ocean that include representation of its turbulent nature. Much is still to be discovered and quantified.

This chapter describes some of the ideas that underlie the understanding of the part played by turbulence in the ocean. Much of this background is derived from studies of turbulence and heat transfer in laboratory experiments.

The scientific study of turbulence began late in the nineteenth century. In 1883 a paper by Osborne Reynolds was published describing how a smooth flow of water down circular tubes with diameters,  $d$ , ranging from about 0.6 to 2.5 cm, breaks down when the speed of the flow,  $U$ , becomes sufficiently large. In his laboratory experiment, Reynolds introduced a thin line of dye into the water entering the tube to make the flow visible (Fig. 1.1). He describes his observations as follows.

When the velocities were sufficiently low, the streak of colour extended in a beautiful straight line through the tube,

but later he makes the following comments.

As the velocity was increased by small stages, at some point in the tube, always at a considerable distance from . . . the intake, the colour band would all at once mix up with the surrounding water, and fill the tube with a mass of coloured water. On viewing the tube by light of an electric spark, the mass of colour resolved itself into a mass of more or less distinct curls, showing eddies.

The experiment shows that the ‘laminar flow’, the smooth flow through the tube at low flow speeds, undergoes a transition to a random eddying ‘turbulent’ motion at

higher speeds when a non-dimensional parameter,  $Ud/\nu$ , exceeded a value of about  $1.3 \times 10^4$ . Here  $\nu$  is the kinematic viscosity, which for water has a value of about  $10^{-6} \text{ m}^2 \text{ s}^{-1}$ . The parameter is now known as the Reynolds number, usually denoted by  $Re$ .<sup>2</sup> The eddies illustrated in Fig. 1.1c are of a size comparable to  $d$ .

In geophysical contexts, the values of the speed,  $U$ , and length,  $d$ , are usually taken to be those characterizing the flow, for example the changes in mean speed of the local flow and the vertical distance over which such changes occur. The precise value of  $Re$  at which turbulence sets in depends on the geometry of the flow and the nature of disturbances to which it is exposed, but flows in which  $Re$  exceeds a critical value of about  $10^4$ , common in the ocean, are generally turbulent unless constrained by effects not represented in Reynolds' experiment.

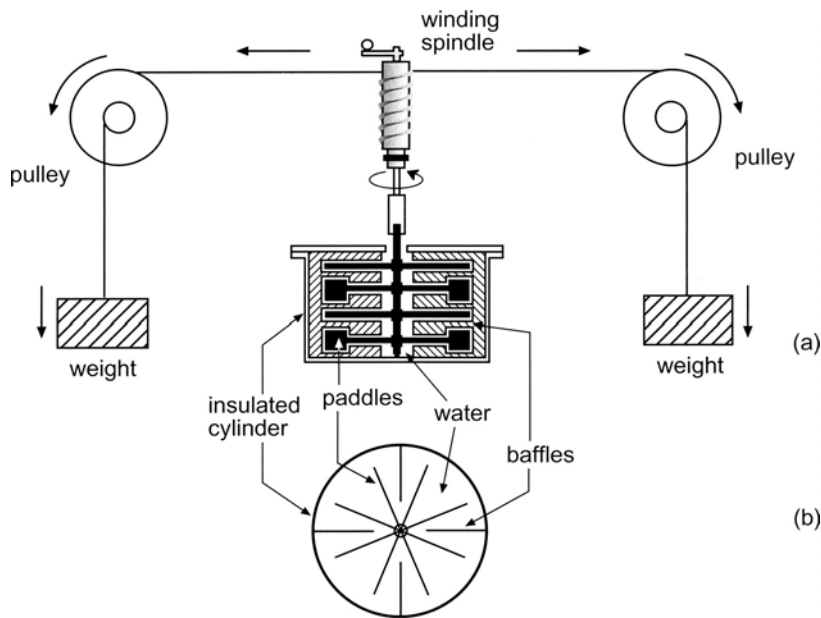
No really precise, robust, unambiguous or clear definition of what is meant by turbulence has, however, been devised. Turbulence is generally accepted to be an energetic, rotational and eddying state of motion that transpires to produce dispersion of material and to transfer momentum, heat and solutes at rates far higher than those of molecular processes alone. Perhaps its most important property, and one that is generally used to characterize it, is that by generating relatively large gradients of velocity at small scales, typically 1 mm to 1 cm, turbulence promotes conditions in which viscous dissipation transfers its kinetic energy into heat. Reynolds' experiment provides a first example of a transition, in this case a very rapid transition, from a steady flow to turbulence. Not all transitions from laminar to turbulent motions are so rapid. Several distinct stages have been identified within transitions found to occur in the ocean that involve buoyancy forces, and in these cases a Reynolds number is not necessarily the principal parameter that determines the onset of turbulence: examples are described in Chapters 3 and 4.

1.2 Heat and temperature

Determination of the processes and rates of transfer of energy between its kinetic and potential forms in the ocean, and the dissipation of energy caused by turbulence and its transfer to heat, are central to the science of oceanography.

Heat is a form of energy contained at a molecular level within a body of fluid, and temperature is a measure of heat content. The relation between heat and temperature can be expressed in terms of a change,  $\Delta H$ , in the heat per unit mass (measured in  $\text{J kg}^{-1}$ ) and corresponding change,  $\Delta T$ , in temperature (measured in degrees kelvin, K, or in  $^{\circ}\text{C}$ ) by  $\Delta H = \rho c_p \Delta T$ , where  $\rho$  is the fluid density (in  $\text{kg m}^{-3}$ ) and  $c_p$  is the specific heat at constant pressure which, for seawater, has a value now known to be about  $3.99 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$ .<sup>3</sup> The experimental determination of the value of  $c_p$

2 Later experiments have shown that the critical  $Re$  depends on the level of the background disturbances to the flow, values consequently ranging from about  $1 \times 10^3$  for relatively substantial disturbances to about  $4.5 \times 10^4$  in very carefully controlled, low disturbance, pipe flows.  
3 The specific heat varies with temperature, salinity and pressure. For more precise values of  $c_p$  and for values of  $\rho$  see Gill (1982, Section A3.4 and Table A3.7).



**Figure 1.2.** A sketch of Joule’s original apparatus for determining the ‘mechanical equivalent of heat’ (or  $c_p$ ), showing (a) a side view and (b) a plan view of the cylinder. The cylinder, containing a little over 6 kg of water, is 0.2 m in diameter and about the same in height. (An additional experiment was made using mercury in an apparatus of about half this size.) The weights are each about 13.6 kg and descended 1.6 m in about 26 s driving the paddles within the cylinder via pulleys and the winding spindle. Heat transmission from the water up the brass paddle shaft was reduced by inserting a piece of boxwood. Experiments were made in a ‘spacious cellar’ in which the temperature was fairly uniform, and were repeated 23 times to provide reliable mean estimates of  $c_p$ . (After Joule, 1850.)

provides a nice illustration of the role of fluid motion, in particular of turbulent motion, in the transfer of energy.

The unit in which energy is measured is named after J. P. Joule.<sup>4</sup> Joule’s most celebrated experiment is that to estimate of  $c_p$  in the apparatus sketched in Fig. 1.2. In essence, the falling weights drive paddles, which churn water in the cylinder, leading to its heating. In the experiment the weights descended through a distance of 1.6 m at a speed of about  $6.1 \text{ cm s}^{-1}$ . They were repeatedly lifted, and over a period of about 35 min in which the weights descended some 20 times, the temperature of the water in the cylinder increased by about 0.31 K. The temperature change was carefully

<sup>4</sup> James Prescott Joule (1818–1889) received private lessons in chemistry in his home city of Manchester from John Dalton (1766–1844), by inclination a meteorologist (see, for example, Oliver and Oliver, 2003) and now best known as the discoverer of the law of partial pressures of gases. As a young man, James observed the aurora borealis and sounded the depth of Lake Windermere in northwest England with his elder brother, Benjamin. The Joule family owned and managed a brewery but to what extent James was actively engaged in its running is unclear; Osborne Reynolds (see Section 1.1), a friend and biographer, asserts that James had little to do with the brewery, although he did make experiments within its premises as part of an extensive study of the relationship between different forms of energy. Cardwell’s (1989) biography of J. P. Joule provides informative details of his early years and of his contacts with other scientists of the time, including Michael Faraday and G. G. Stokes.

measured, the accuracy attained being about 3 mK. (Temperature is now routinely measured at sea to an accuracy of 1 mK, and often in particular studies, e.g. in boundary layers where the temperature is relatively uniform, with a resolution of 0.1 mK or better.) Great care was taken to minimize heat loss during the period of the experiment, and a wooden screen was erected to avoid effects of radiant heat from the observer. Joule calculated the potential energy lost by the weights in descending and, by subtracting their kinetic energy at the end of their descent and accounting for a small heat loss from the cylinder during the experiment, was able to relate the energy imparted to the water per unit volume through the paddles, i.e. its change in heat energy, to its rise in temperature. The results were communicated to the Royal Society by no less august a person than Michael Faraday, and published in 1850.

This experiment was later refined to obtain greater accuracy. But, as it is, it contains a major subtlety that involves the motion of the water within the cylinder. As Fig. 1.2 shows, there are baffles fixed to the inside of the cylinder. They are important for two reasons. The first is that without them a circulatory flow would be set up which, having kinetic energy, would need to be accounted for in the energy balance. (Alternatively, Joule could have waited until the circulation died out before measuring the temperature, but that would have required a means to ensure there was no substantial residual motion and would have taken time, during which heat would have been lost from the cylinder to the air.) The second reason is perhaps more important. The rotating paddles drive fluid past the stationary fixed baffles and promote small-scale eddying and enhanced shearing motions, characteristic of turbulence. These greatly increase the rate at which molecular viscosity dissipates the kinetic energy imparted to the water, transferring mechanical energy into heat much more rapidly than can a mean circulation gradually spun down through viscous drag at the cylinder walls.

It is by increasing the mean square shear that turbulence enhances the effect of viscosity in the transfer from mechanical energy to heat, at a rate explained in Section 1.7.8. In the ocean and in lakes, the heating generated by turbulence is usually miniscule in comparison with other sources of heat<sup>5</sup> and has little effect on the dynamics but, as will be shown later, the energy lost by turbulent motions is substantial and provides a measure of ‘mixing’. The transfer of kinetic energy by turbulence into heat in the ocean is a one-way ‘irreversible’ process, meaning that the former state cannot be recovered except by doing work on the fluid system. Turbulent dissipation represents a loss in energy that, to maintain a quasi-steady state in the ocean, must be replenished from some external source such as the atmosphere or through the tide-generating gravitational forces of the Moon and Sun.

1.3 Density

The density of fluids has important effects on their dynamics and, in particular, on the onset and nature of turbulence. Forces are derived from the variations of density in fluids

5 An exception may be in the very high dissipation region of surf zones described in Section 11.2.1.

lying within gravitational fields. Differences in weight lead to pressure differences that, if unopposed, drive motion. These forces are termed buoyancy forces and, often derived from temperature or salinity variations or driven by heat fluxes at the boundaries of fluids, are the cause of a wide range of turbulence-related phenomena (Turner, 1973; Simpson, 1997). These include several that involve instability, a topic introduced in the next section and described with examples from the oceans and lakes in later chapters.

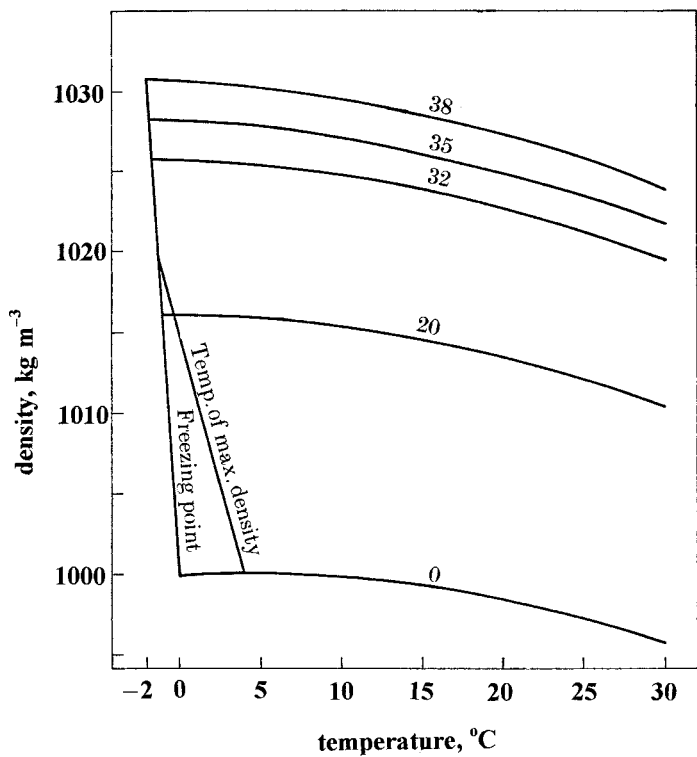
The majority of liquids expand when heated and contract when cooled. Their density consequently increases as their temperature decreases. Fresh water between its freezing point at 0 °C and a temperature of about 4 °C is a well-known exception, becoming denser as temperature increases. The salts dissolved in seawater and measured as ‘salinity’,<sup>6</sup> modify this behaviour. The freezing point of seawater differs from that of fresh water, and decreases as salinity increases. The density of seawater with salinity greater than about 24.7 psu (and most of the ocean has a higher salinity, typically about 35 psu) behaves like that of most liquids, increasing as temperature decreases until freezing occurs (at a temperature of −1.92 °C for a seawater with a salinity of 35 psu). As a consequence, the dynamical properties of the ocean and freshwater lakes may differ at low temperatures.

Temperature, salinity and depth (or pressure) are now measured routinely to the full depth of the ocean basins by using lowered Conductivity–Temperature–Depth (CTD) probes (Lawson and Larson, 2001), salinity being derived from the conductivity with a temperature correction. (The mean depth of the oceans is about 3795 m, and the depth of abyssal plains is typically 5000 m.) The density of seawater is determined from the temperature, salinity and pressure by using an expression known as the equation of state. This is illustrated in Fig. 1.3, which shows several of the factors mentioned in the preceding paragraph, the density maximum of fresh water at about 4 °C, the decrease in freezing point as salinity increases, and the monotonic increase in density as temperature decreases when the salinity exceeds 24.7 psu. Increased salinity increases density because of the increase in the mass of salts, and may be a very substantial component in some circumstances. For small variations in temperature,  $T$ , and salinity,  $S$ , from reference values where the density is  $\rho_0$ , the equation of state for density,  $\rho$ , may be approximated by

$$\rho = \rho_0(1 - \alpha T + \beta S). \tag{1.1}$$

The coefficients,  $\alpha$  and  $\beta$ , relate to the expansion of seawater and are specified at the reference values of  $T$  and  $S$ , but depend on depth. Seawater near the sea surface with a salinity of 35 psu has a thermal expansion coefficient,  $\alpha$ , the increase in volume per unit volume per kelvin, of  $5.26 \times 10^{-5} \text{ K}^{-1}$  at 0 °C,  $7.81 \times 10^{-5} \text{ K}^{-1}$  at 2 °C,  $1.67 \times 10^{-4}$  at 10 °C and  $2.97 \times 10^{-4}$  at 25 °C. The value of  $\alpha$  increases with depth, at 1000 m being equal to  $1.84 \times 10^{-4} \text{ K}^{-1}$  in water at 10 °C and 35 psu. The coefficient for the

6 The component ions contributing to salinity in the ocean are in nearly constant ratio. The main ions (and proportions) are chlorine 55.0%, sodium 30.6%, sulphate 7.7%, magnesium 3.7%, calcium 1.2%, and potassium 1.1%. The unit of salinity is the psu, approximately equal to 1000 times the mass of dissolved salts per unit mass of seawater (Unesco, 1983).



**Figure 1.3.** The equation of state: the relation between density and temperature for fluids of different salinity. The salinity for each curve is shown, ranging from 0 to 38 psu. The freezing point and temperature of maximum density, both decreasing as salinity increases, are also shown.

contribution of salinity to density,  $\beta$ , is  $0.82 \text{ psu}^{-1}$  at  $0^\circ\text{C}$  and about  $0.79 \text{ psu}^{-1}$  at  $10^\circ\text{C}$ . An increase in temperature of  $\Delta T = 0.1 \text{ K}$  in an upper layer of thickness,  $h = 1000 \text{ m}$  at a temperature of  $10^\circ\text{C}$  would result in a ‘steric’ rise in sea level of  $\alpha h \Delta T$ , or about  $1.76 \times 10^{-4} \times 0.1 \times 1000 \text{ m} = 1.76 \text{ cm}$ . Changes in sea level are a very important consequence of climate change.<sup>7</sup>

The density of seawater is often expressed as  $\sigma_T$  (sigma-T), the difference in density from  $1 \times 10^3 \text{ kg m}^{-3}$  but with units conventionally omitted (and with qualifications explained by Gill, 1982, Appendix 2). The density of water in the upper 100 m of the sea is typically about  $1.028 \times 10^3 \text{ kg m}^{-3}$  or  $\sigma_T = 28$ . Surfaces of constant density in the ocean are known as isopycnal surfaces (or just isopycnals). Much of the attention given to ocean mixing at small scales relates to the transfers that occur across such surfaces.

<sup>7</sup> Calculation of sea-level change in response to climatic variations is complicated by the contributions of fresh water entering the oceans from the continents as a result, for example, of the melting of glaciers, causing a ‘eustatic’ rise in sea level. Change in the salinity of the sea resulting from the melting of ice must also be taken into account (Munk, 2003).



Pressure-induced adiabatic heating (changes in temperature that occurs without any exchange of heat with surroundings) caused by the compression of seawater leads to a gradually increasing temperature with increasing depth in an otherwise uniform layer. The ‘adiabatic lapse rate’,  $\Gamma_a = g\alpha\vartheta/c_p$ , where  $\vartheta$  is the temperature in kelvin, increases with depth and temperature in the surface waters, being about  $1.16 \times 10^{-4} \text{ K m}^{-1}$  at the surface in a temperature of  $10^\circ\text{C}$  and about  $1.31 \times 10^{-4} \text{ K m}^{-1}$  at 4000 m depth in a temperature of  $4^\circ\text{C}$ , both values being at a salinity of 35 psu.<sup>8</sup> Such gradients are sometimes apparent in relatively uniform regions of the ocean, for example in the deep convectively mixed layers described in Section 4.5.2. Care must be taken to account for adiabatic changes of temperature particularly in conditions where water is transported over large vertical distances or in comparing the density of water at widely separated depths, with the use of potential temperatures and densities, where appropriate. (The potential temperature is that which a fluid parcel would have if it were moved adiabatically to a given pressure reference level. Potential density is defined similarly; see section 3.7 in Gill, 1982.)

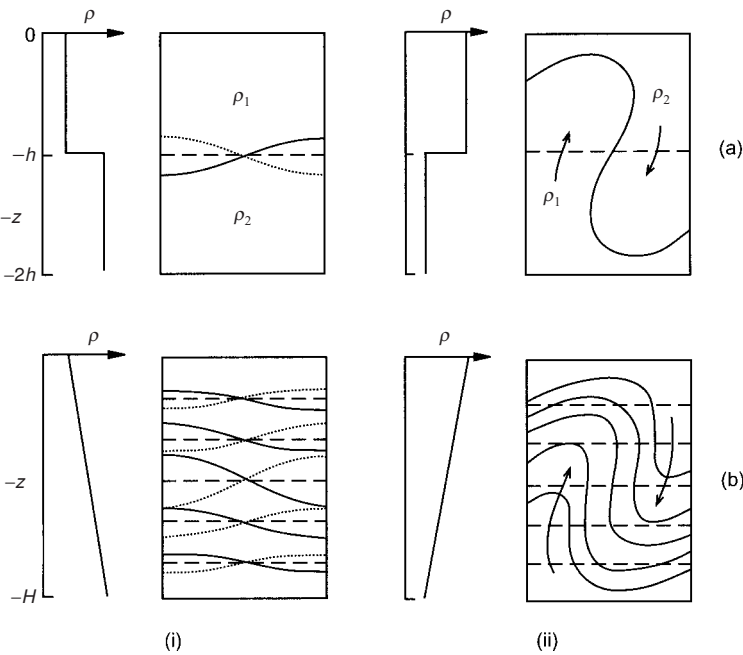
Although compressibility plays an important role in the atmosphere, it is generally small in the ocean. The first floats capable of remaining at one level and of following the slow drift of deep ocean currents were those devised in the 1950s and now called after their inventor, John Swallow (1923–1994). The main body of a Swallow float consists of a sealed aluminium tube. This depends for its stability in the water column on being *less* compressible than seawater. If displaced downwards from a level at which it has the same density as the sea, it will compress less than the seawater and so be less dense and buoyant, rising back towards its original position. Similarly, if moved upwards, its reduction in volume is less than that of the seawater, and so it is denser, and descends. The float is consequently ‘stable’ (a topic discussed further in the next section), tending to follow a highly damped oscillation about its mean level if perturbed from it. Tracked acoustically from a ship<sup>9</sup>, Swallow floats provided the first evidence of the variable nature of deep ocean flows in the early 1960s. With greatly improved means of tracking and design, floats now provide a means to measure mean circulation and dispersion at large scales which is described in Chapter 13.

1.4     **Instability and oscillations resulting from buoyancy forces**

The density of the ocean generally increases with depth, and this is an important factor in turbulence and mixing. The effect of density is illustrated by the two idealized distributions of density shown in Fig. 1.4. In both, water is supposed incompressible and inviscid. In the first, part (a), two layers each of height  $h$  but of different densities

8 Values of  $\alpha$  and  $c_p$  from which other values of  $\Gamma_a$  can be derived are listed by Gill (1982, Table A3.1).  
9 Acoustic tracking was no simple matter. There was no Global Positioning System (GPS) when the floats were first used, and precise position fixing was made relative to topographic features identified in careful echo-sounding surveys.





**Figure 1.4.** Stable (i, on left) and unstable (ii, right) stratification, for (a, top) two layers of different density and (b, bottom) a linear density variation with depth. The horizontal undisturbed interface in (a) and undisturbed isopycnals in (b) are shown as dashed lines. Periodic disturbances, internal waves, are shown in (i) by full lines and dotted lines, whilst growing disturbances in (ii) are illustrated as full lines.

(e.g. different salinities or temperatures) are separated by a thin interface. In (i), to the left, the upper layer is of density  $\rho_1$  and the lower is of greater density,  $\rho_2$ . In (ii), to the right, the two layers have been reversed so that now the water of greater density,  $\rho_2$ , is on top. As anyone who has turned a beaker of water upside down will know, in this situation the upper layer will begin to descend into the lower. But why? Case (ii) has greater potential energy, PE, than (i). The weight of the layer of density  $\rho_2$  is  $g\rho_2h$  per unit area and its centre of gravity is a distance  $h$  higher in (ii) than in (i), so that its potential energy per unit area is  $g\rho_2h^2$  greater. Similarly, the PE of the layer of density  $\rho_1$  is  $g\rho_1h^2$  less in (ii) than in (i). By subtraction, the PE per unit area of (ii) is therefore  $g(\rho_2 - \rho_1)h^2$  greater than (i).<sup>10</sup> An adjustment that converts the situation illustrated in (ii) to that in (i) releases energy, whereas energy must be provided to convert (i) into (ii).

The situation in (ii), with denser water overlying less-dense water, is ‘statically unstable’, a perturbation gaining in energy and growing. In this example, the instability involves convection (as sketched), the release of potential energy through the

<sup>10</sup> The acceleration due to gravity appears in product with the density difference. The acceleration due to gravity times the fractional density difference, denoted as  $g' = 2g(\rho_2 - \rho_1)/(\rho_1 + \rho_2)$ , appears frequently in connection with the effects of buoyancy and is often referred to as the ‘reduced gravitational acceleration’ or simply as ‘reduced gravity’.

descent and growth of one or more plumes of the denser water and ascent of similar plumes of the lighter water, the potential energy of the initial state being converted into kinetic energy, KE, of the developing convective motion. The instability is called ‘Rayleigh–Taylor instability’. Lord Rayleigh (1883) studied the stability of fluids at rest with density increasing upwards, and G. I. Taylor (1950)<sup>11</sup> recognized the essential ingredient of the instability, that the acceleration producing instability must be directed from the lighter fluid towards the denser, usefully generalizing Rayleigh’s results to cases in which the vertical acceleration varies, as in wave motions.<sup>12</sup>

Equally, any change that converts (i) into (ii) requires additional energy to be found, and such changes, without suitable sources of energy, are inhibited. In (i), the water column is ‘statically stable’ or stably stratified in density. Small disturbances, providing a rise in the centre of gravity or modest supply of KE, result in interfacial waves at the boundary between the two layers (as shown in the figure). A large energy input, tending towards the development of state (ii), may however result in instability. An example is provided in Fig. 5.5a.

Generally in the ocean the density changes gradually with depth and not discontinuously. Figure 1.4b shows a fluid of height,  $H$ , with lines representing isopycnal surfaces and a density that varies linearly in depth. (Here, and later,  $z$  is taken as the upward vertical coordinate.)<sup>13</sup> In (i), the density,  $\rho$ , decreases upwards at a rate,  $d\rho/dz = -q\rho_0$ , where  $q$  is a positive constant with dimensions  $(\text{length})^{-1}$ , and in (ii) it increases at the same rate. The PE in (ii) is greater than that in (i) by an amount  $gq\rho_0 H^3/6$ , and, as before, the illustrated situation in (ii) is unstable to disturbances that convert it into (i). Indeed any mass-conserving adjustment of the density distribution (or ‘profile’) in (ii) will lead to a state of lower PE, so that any disturbance, however small, will lead to a release of PE.<sup>14</sup> A condition of ‘static instability’ occurs whenever denser fluid lies above less dense.

In this case there is a simple alternative argument that indicates the dynamically unstable nature of (ii) and the stability of (i). Imagine that a small volume,  $V$ , of water of density  $\rho_0$  in (i) is moved upwards by a small distance into less-dense water. By Archimedes’ Principle, there is an up-thrust or buoyancy force acting on the displaced volume equal to the weight of water it displaces. Since the displaced water is of lower

11 Sir Geoffrey Ingram Taylor (1886–1975), known to friends and colleagues as ‘GI’, was a brilliant theoretician and experimentalist whose research provided the basis of understanding many aspects of fluid mechanics, particularly turbulent flow. His work is characterized by the requirement to find experimental verification of predictions, and by the elegant simplicity of explanations of phenomena in terms of the *essential* physical processes – and no more. Batchelor (1996) gives an entertaining and comprehensive account of Taylor’s life and achievements. Turner (1997) provides a shorter personal description of some aspects of Taylor’s research, particularly during his later years.

12 Taylor’s results mean that a downward acceleration exceeding  $g$  is necessary at the sea surface before the air–water interface will become unstable as a consequence of the induced acceleration. A similar downward acceleration of a layer of dense fluid overlying a less-dense layer would promote stability.

13 Depth is usually measured from the mean level of the sea surface. In some figures, the vertical axis is given as pressure in pascals (Pa) or decibars (db). In seawater, 1 MPa  $\approx$  97 m and 1 db  $\approx$  0.97 m, but the conversion factor depends on the density of seawater.

14 Exchange of any two thin layers of equal thickness in the water column in (ii) will result in the denser being lowered and the lighter raised, a situation which case (a) leads to decreased PE.