Multiple Scattering of Light by Particles

This volume provides a thorough and up-to-date treatment of multiple scattering of light and other electromagnetic radiation in media composed of randomly and sparsely positioned particles. For the first time in monographic literature, the radiative transfer theory (RTT) is systematically and consistently presented as a branch of classical macrosopic electromagnetics. The book traces the fundamental link between the RTT and the effect of coherent backscattering (CB) and explains their place in the context of a comprehensive hierarchy of electromagnetic scattering problems. Dedicated sections present a thorough discussion of the physical meaning and range of applicability of the radiative transfer equation (RTE) and compare the self-consistent microphysical and the traditional phenomenological approaches to radiative transfer. The work describes advanced techniques for solving the RTE and gives examples of physically based applications of the RTT and CB in noninvasive particle characterization and remote sensing. This thorough and self-contained book will be valuable for science professionals, engineers, and graduate students working in a wide range of disciplines including optics, electromagnetics, remote sensing, atmospheric radiation, astrophysics, and biomedicine.

MICHAEL I. MISHCHENKO is a senior scientist at the NASA Goddard Institute for Space Studies in New York City. After gaining a Ph.D. in physics in 1987, he has been principal investigator on several NASA and DoD projects and has served as topical editor and editorial board member of several leading scientific journals. Dr. Mishchenko is a recipient of the Henry G. Houghton Award of the American Meteorological Society and is an elected Fellow of the American Geophysical Union, the Optical Society of America, and the Institute of Physics. His research interests include electromagnetic scattering, radiative transfer, and remote sensing.

LARRY D. TRAVIS is presently Associate Chief of the NASA Goddard Institute for Space Studies. He gained a Ph.D. in astronomy at Pennsylvania State University in 1971. Dr. Travis has acted as principal investigator on several NASA projects and was awarded a NASA Exceptional Scientific Achievement Medal. His research interests include the theoretical interpretation of remote sensing measurements of polarization, planetary atmospheres, atmospheric dynamics, and radiative transfer.

ANDREW A. LACIS is a senior scientist at the NASA Goddard Institute for Space Studies, and teaches radiative transfer at Columbia University. He gained a Ph.D. in physics at the University of Iowa in 1970 and has acted as principal investigator on numerous NASA and DoE projects. His research interests include radiative transfer in planetary atmospheres, the absorption of solar radiation by the Earth's atmosphere, and climate modeling.

Among the numerous scientific publications by these authors is the monograph on *Scattering, Absorption, and Emission of Light by Small Particles* published by Cambridge University Press in 2002.

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Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis
Frontmatter
More information

Multiple Scattering of Light by Particles

Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko
Larry D. Travis
Andrew A. Lacis
NASA Goddard Institute for Space Studies, New York

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Preface

Since the seminal papers by Lommel (1887), Chwolson (1889), and Schuster (1905), the radiative transfer equation (RTE) has been widely used in diverse areas of science and engineering to describe multiple scattering of light and other electromagnetic radiation in media composed of randomly and sparsely distributed particles. Analytical studies of the RTE have formed a separate branch of mathematical physics. However, despite the importance and the widespread use of the radiative transfer theory (RTT), its physical basis had not been established firmly until quite recently.

Indeed, the traditional "phenomenological" way to introduce the RTE has been to invoke an eclectic combination of principles borrowed from classical radiometry (i.e., intuitively appealing arguments of energy balance and the simple heuristic concepts of light rays and ray pencils), classical electromagnetics (electromagnetic scattering, Stokes parameters, and phase and extinction matrices), and even quantum electrodynamics ("photons"). Furthermore, the phenomenological approach has always relied on an illusive concept of an "elementary (or differential) volume element" of the discrete scattering medium. To sew together these motley concepts, one needs a set of postulates that appear to be plausible at first sight but turn out to be artificial upon close examination.

This inconsistent approach to radiative transfer is quite deceptive since it implies that in order to derive the RTE for media composed of elastically scattering particles one needs postulates other than those already contained in classical electromagnetics. The phenomenological "derivation" becomes especially questionable when one attempts to include the effects of polarization described by the so-called vector RTE and/or to take into account the effects of particle nonsphericity and orientation. Furthermore, it does not allow one to determine the range of applicability of the RTE and trace the fundamental link between the RTT and the effect of coherent backscattering.

During the past few decades, there has been significant progress in studies of the statistical wave content of the RTT. This research has resulted in a much improved understanding of the basic assumptions leading to the RTE and has indeed demonstrated it to be a corollary of the Maxwell equations. Hence, the main goal of this monograph is to consistently present the RTT as a branch of classical electromagnetics as applied to discrete random media and to clarify the relationship between radiative transfer and coherent backscattering.

Another motivation for writing this book was the recognition of the scarcity of comprehensive monographs describing the fundamentals of polarized radiative transfer and its applications in a way intelligible to graduate students and non-expert scientists.[1] This factor has significantly impeded the development and wide dissemination of physically-based remote sensing and particle characterization techniques. Hence, the additional purpose of this volume is to present a broad and coherent outline of the subject and to make the technical material accessible to a larger audience than those specializing in this research area. Consistent with this purpose, our presentation assumes minimal prior knowledge of the subject matter and the relevant theoretical approaches. We expect, therefore, that the book will be useful to science professionals, engineers, and graduate students working in a broad range of disciplines: optics, electromagnetics, atmospheric radiation and remote sensing, radar meteorology, oceanography, climate research, astrophysics, optical engineering and technology, particle characterization, and biomedical optics.

This volume is a natural continuation of our recent monograph on *Scattering, Absorption, and Emission of Light by Small Particles* (Mishchenko *et al*., 2002; hereinafter referred to as MTL[2]) in that it consistently uses the same general methodology and notation system while applying them to multiple scattering by random particle ensembles. However, the present book contains all the necessary background material and is self-contained.

As in MTL, we usually denote vectors using the Times bold font and matrices using the Arial bold font. Unit vectors are denoted by a caret, whereas dyads and dyadics are denoted by the symbol ↔. The Times italic font is reserved for scalar quantities, important exceptions being the square root of minus one, the differential sign, and the base of natural logarithms, which are denoted by Times roman characters i, d, and e, respectively. Another exception is the relative refractive index, which is denoted by a sloping sans serif m. For the reader's convenience, a glossary listing the symbols used, their meaning and dimension, and the section where they first appear is provided at the end of the book (Appendix I). Appendix H contains a list of abbreviations.

[1] The recent book by Hovenier *et al*. (2004) is a notable exception.
[2] By agreement with Cambridge University Press, MTL is now publicly available in the .pdf format at http://www.giss.nasa.gov/~crmim/books.html.

We did not try to compile a comprehensive and detailed reference list. Instead, preference was given to seminal publications as well as to relevant books and reviews where further references can be found.

We mention several relevant computer programs made publicly available on-line. These programs have been thoroughly tested and are expected to generate reliable results provided that they are implemented as instructed. It is not inconceivable, however, that some of these programs contain errors and/or are not platform-independent. Also, it is possible that users could specify input parameter values that are outside the intended range for which accurate results can be expected. For these reasons the authors of this book and the publisher disclaim all liability for any damage that may result from the use of the programs. Although the authors and the publisher have used their best endeavors to ensure that the URLs for external Internet sites referred to in this book are correct and active at the time of this book going to press, they cannot guarantee that a site will remain live or that its content is or will remain appropriate.

Michael I. Mishchenko
Larry D. Travis
Andrew A. Lacis

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Dedication and acknowledgments

The phenomenological theory of radiative transfer in discrete random media had been widely used for many decades in numerous research and engineering disciplines despite the fact that its physical origin had not been established. This uncomfortable situation has finally changed, and the RTT has become a legitimate branch of classical electromagnetics. It was very exciting for us to be able to write this entire monograph on radiative transfer without ever having to leave the firm grounds of electromagnetic theory. We, therefore, appreciatively dedicate this book to James Clerk Maxwell, whose monumental contribution to physics can be compared only to that of Sir Isaac Newton and whose equations of electromagnetism have been voted by scientists to be the greatest equations ever (Crease, 2004).

Several prominent scientists have made important contributions to the evolving subject of multiple wave scattering by small particles and microphysical justification of the RTT. Our own research has been most influenced by the publications of Yuri Barabanenkov, Anatoli Borovoi, Akira Ishimaru, Leung Tsang, Victor Twersky, and Hendrik van de Hulst to whom we express sincere appreciation.

We are deeply indebted to Joop Hovenier, Michael Kahnert, and Cornelis van der Mee for numerous discussions, continued encouragement, and valuable comments on a preliminary version of this book that resulted in a much improved manuscript. We also gratefully acknowledge illuminating discussions with Yuri Barabanenkov, Anatoli Borovoi, Oleg Bugaenko, Brian Cairns, Barbara Carlson, Zhanna Dlugach, Helmut Domke, James Hansen, Vsevolod Ivanov, Nikolai Khlebtsov, Kuo-Nan Liou, Daniel Mackowski, Bart van Tiggelen, Victor Tishkovets, Gorden Videen, Edgard Yanovitskij, and many other colleagues.

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Our special thanks go to Nadia Zakharova who provided invaluable assistance during the preparation of the manuscript and the proofreading stage. She also contributed many numerical results and almost all computer graphics.