

Cambridge University Press

978-0-521-83324-0 - From Classical to Quantum Mechanics: An Introduction to the Formalism, Foundations and Applications

Giampiero Esposito, Giuseppe Marmo and George Sudarshan

Frontmatter

[More information](#)

From Classical to Quantum Mechanics

This book provides a pedagogical introduction to the formalism, foundations and applications of quantum mechanics. Part I covers the basic material that is necessary to an understanding of the transition from classical to wave mechanics. Topics include classical dynamics, with emphasis on canonical transformations and the Hamilton–Jacobi equation; the Cauchy problem for the wave equation, the Helmholtz equation and eikonal approximation; and introductions to spin, perturbation theory and scattering theory. The Weyl quantization is presented in Part II, along with the postulates of quantum mechanics. The Weyl programme provides a geometric framework for a rigorous formulation of canonical quantization, as well as powerful tools for the analysis of problems of current interest in quantum physics. In the chapters devoted to harmonic oscillators and angular momentum operators, the emphasis is on algebraic and group-theoretical methods. Quantum entanglement, hidden-variable theories and the Bell inequalities are also discussed. Part III is devoted to topics such as statistical mechanics and black-body radiation, Lagrangian and phase-space formulations of quantum mechanics, and the Dirac equation.

This book is intended for use as a textbook for beginning graduate and advanced undergraduate courses. It is self-contained and includes problems to advance the reader's understanding.

GIAMPIERO ESPOSITO received his PhD from the University of Cambridge in 1991 and has been INFN Research Fellow at Naples University since November 1993. His research is devoted to gravitational physics and quantum theory. His main contributions are to the boundary conditions in quantum field theory and quantum gravity via functional integrals.

GIUSEPPE MARMO has been Professor of Theoretical Physics at Naples University since 1986, where he is teaching the first undergraduate course in quantum mechanics. His research interests are in the geometry of classical and quantum dynamical systems, deformation quantization, algebraic structures in physics, and constrained and integrable systems.

GEORGE SUDARSHAN has been Professor of Physics at the Department of Physics of the University of Texas at Austin since 1969. His research has revolutionized the understanding of classical and quantum dynamics. He has been nominated for the Nobel Prize six times and has received many awards, including the Bose Medal in 1977.

Cambridge University Press
978-0-521-83324-0 - From Classical to Quantum Mechanics: An Introduction to the Formalism, Foundations
and Applications
Giampiero Esposito, Giuseppe Marmo and George Sudarshan
Frontmatter
[More information](#)

Cambridge University Press

978-0-521-83324-0 - From Classical to Quantum Mechanics: An Introduction to the Formalism, Foundations and Applications

Giampiero Esposito, Giuseppe Marmo and George Sudarshan

Frontmatter

[More information](#)

FROM CLASSICAL TO QUANTUM MECHANICS

An Introduction to the Formalism, Foundations
and Applications

GIAMPIERO ESPOSITO, GIUSEPPE MARMO

INFN, Sezione di Napoli and
Dipartimento di Scienze Fisiche,
Università Federico II di Napoli

GEORGE SUDARSHAN

Department of Physics,
University of Texas, Austin

Cambridge University Press
978-0-521-83324-0 - From Classical to Quantum Mechanics: An Introduction to the Formalism, Foundations
and Applications
Giampiero Esposito, Giuseppe Marmo and George Sudarshan
Frontmatter
[More information](#)

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521833240

© G. Esposito, G. Marmo and E. C. G. Sudarshan 2004

This publication is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without the written
permission of Cambridge University Press.

First published 2004

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication Data

Esposito, Giampiero.
From classical to quantum mechanics / Giampiero Esposito, Giuseppe Marmo,
George Sudarshan.
p. cm.
Includes bibliographical references and index.
ISBN 0 521 83324 8
1. Quantum theory. I. Marmo, Giuseppe. II. Sudarshan, George, 1931– III. Title.
QC174. I2.E95 2004 530.12–dc21 2003053212

ISBN 978-0-521-83324-0 Hardback
ISBN 978-0-521-14362-2 Paperback

Cambridge University Press has no responsibility for the persistence or
accuracy of URLs for external or third-party internet websites referred to in
this publication, and does not guarantee that any content on such websites is,
or will remain, accurate or appropriate. Information regarding prices, travel
timetables, and other factual information given in this work is correct at
the time of first printing but Cambridge University Press does not guarantee
the accuracy of such information thereafter.

Cambridge University Press
978-0-521-83324-0 - From Classical to Quantum Mechanics: An Introduction to the Formalism, Foundations
and Applications
Giampiero Esposito, Giuseppe Marmo and George Sudarshan
Frontmatter
[More information](#)

For Michela, Patrizia, Bhamathi, and Margherita, Giuseppina, Nidia

Cambridge University Press
978-0-521-83324-0 - From Classical to Quantum Mechanics: An Introduction to the Formalism, Foundations
and Applications
Giampiero Esposito, Giuseppe Marmo and George Sudarshan
Frontmatter
[More information](#)

Contents

<i>Preface</i>	<i>page</i> xiii
<i>Acknowledgments</i>	xvi
Part I From classical to wave mechanics	1
1 Experimental foundations of quantum theory	3
1.1 The need for a quantum theory	3
1.2 Our path towards quantum theory	6
1.3 Photoelectric effect	7
1.4 Compton effect	11
1.5 Interference experiments	17
1.6 Atomic spectra and the Bohr hypotheses	22
1.7 The experiment of Franck and Hertz	26
1.8 Wave-like behaviour and the Bragg experiment	27
1.9 The experiment of Davisson and Germer	33
1.10 Position and velocity of an electron	37
1.11 Problems	41
<i>Appendix 1.A</i> The phase 1-form	41
2 Classical dynamics	43
2.1 Poisson brackets	44
2.2 Symplectic geometry	45
2.3 Generating functions of canonical transformations	49
2.4 Hamilton and Hamilton–Jacobi equations	59
2.5 The Hamilton principal function	61
2.6 The characteristic function	64
2.7 Hamilton equations associated with metric tensors	66
2.8 Introduction to geometrical optics	68
2.9 Problems	73
<i>Appendix 2.A</i> Vector fields	74

viii	<i>Contents</i>	
	<i>Appendix 2.B</i> Lie algebras and basic group theory	76
	<i>Appendix 2.C</i> Some basic geometrical operations	80
	<i>Appendix 2.D</i> Space–time	83
	<i>Appendix 2.E</i> From Newton to Euler–Lagrange	83
3	Wave equations	86
3.1	The wave equation	86
3.2	Cauchy problem for the wave equation	88
3.3	Fundamental solutions	90
3.4	Symmetries of wave equations	91
3.5	Wave packets	92
3.6	Fourier analysis and dispersion relations	92
3.7	Geometrical optics from the wave equation	99
3.8	Phase and group velocity	100
3.9	The Helmholtz equation	104
3.10	Eikonal approximation for the scalar wave equation	105
3.11	Problems	114
4	Wave mechanics	115
4.1	From classical to wave mechanics	115
4.2	Uncertainty relations for position and momentum	128
4.3	Transformation properties of wave functions	131
4.4	Green kernel of the Schrödinger equation	136
4.5	Example of isometric non-unitary operator	142
4.6	Boundary conditions	144
4.7	Harmonic oscillator	151
4.8	JWKB solutions of the Schrödinger equation	155
4.9	From wave mechanics to Bohr–Sommerfeld	162
4.10	Problems	167
	<i>Appendix 4.A</i> Glossary of functional analysis	167
	<i>Appendix 4.B</i> JWKB approximation	172
	<i>Appendix 4.C</i> Asymptotic expansions	174
5	Applications of wave mechanics	176
5.1	Reflection and transmission	176
5.2	Step-like potential; tunnelling effect	180
5.3	Linear potential	186
5.4	The Schrödinger equation in a central potential	191
5.5	Hydrogen atom	196
5.6	Introduction to angular momentum	201
5.7	Homomorphism between $SU(2)$ and $SO(3)$	211
5.8	Energy bands with periodic potentials	217
5.9	Problems	220

<i>Contents</i>		ix
	<i>Appendix 5.A</i> Stationary phase method	221
	<i>Appendix 5.B</i> Bessel functions	223
6	Introduction to spin	226
6.1	Stern–Gerlach experiment and electron spin	226
6.2	Wave functions with spin	230
6.3	The Pauli equation	233
6.4	Solutions of the Pauli equation	235
6.5	Landau levels	239
6.6	Problems	241
	<i>Appendix 6.A</i> Lagrangian of a charged particle	242
	<i>Appendix 6.B</i> Charged particle in a monopole field	242
7	Perturbation theory	244
7.1	Approximate methods for stationary states	244
7.2	Very close levels	250
7.3	Anharmonic oscillator	252
7.4	Occurrence of degeneracy	255
7.5	Stark effect	259
7.6	Zeeman effect	263
7.7	Variational method	266
7.8	Time-dependent formalism	269
7.9	Limiting cases of time-dependent theory	274
7.10	The nature of perturbative series	280
7.11	More about singular perturbations	284
7.12	Problems	293
	<i>Appendix 7.A</i> Convergence in the strong resolvent sense	295
8	Scattering theory	297
8.1	Aims and problems of scattering theory	297
8.2	Integral equation for scattering problems	302
8.3	The Born series and potentials of the Rollnik class	305
8.4	Partial wave expansion	307
8.5	The Levinson theorem	310
8.6	Scattering from singular potentials	314
8.7	Resonances	317
8.8	Separable potential model	320
8.9	Bound states in the completeness relationship	323
8.10	Excitable potential model	324
8.11	Unitarity of the Möller operator	327
8.12	Quantum decay and survival amplitude	328
8.13	Problems	335

x	<i>Contents</i>	
	Part II Weyl quantization and algebraic methods	337
9	Weyl quantization	339
9.1	The commutator in wave mechanics	339
9.2	Abstract version of the commutator	340
9.3	Canonical operators and the Wintner theorem	341
9.4	Canonical quantization of commutation relations	343
9.5	Weyl quantization and Weyl systems	345
9.6	The Schrödinger picture	347
9.7	From Weyl systems to commutation relations	348
9.8	Heisenberg representation for temporal evolution	350
9.9	Generalized uncertainty relations	351
9.10	Unitary operators and symplectic linear maps	357
9.11	On the meaning of Weyl quantization	363
9.12	The basic postulates of quantum theory	365
9.13	Problems	372
10	Harmonic oscillators and quantum optics	375
10.1	Algebraic formalism for harmonic oscillators	375
10.2	A thorough understanding of Landau levels	383
10.3	Coherent states	386
10.4	Weyl systems for coherent states	390
10.5	Two-photon coherent states	393
10.6	Problems	395
11	Angular momentum operators	398
11.1	Angular momentum: general formalism	398
11.2	Two-dimensional harmonic oscillator	406
11.3	Rotations of angular momentum operators	409
11.4	Clebsch–Gordan coefficients and the Regge map	412
11.5	Postulates of quantum mechanics with spin	416
11.6	Spin and Weyl systems	419
11.7	Monopole harmonics	420
11.8	Problems	426
12	Algebraic methods for eigenvalue problems	429
12.1	Quasi-exactly solvable operators	429
12.2	Transformation operators for the hydrogen atom	432
12.3	Darboux maps: general framework	435
12.4	$SU(1,1)$ structures in a central potential	438
12.5	The Runge–Lenz vector	441
12.6	Problems	443

	<i>Contents</i>	xi
13	From density matrix to geometrical phases	445
13.1	The density matrix	446
13.2	Applications of the density matrix	450
13.3	Quantum entanglement	453
13.4	Hidden variables and the Bell inequalities	455
13.5	Entangled pairs of photons	459
13.6	Production of statistical mixtures	461
13.7	Pancharatnam and Berry phases	464
13.8	The Wigner theorem and symmetries	468
13.9	A modern perspective on the Wigner theorem	472
13.10	Problems	476
	Part III Selected topics	477
14	From classical to quantum statistical mechanics	479
14.1	Aims and main assumptions	480
14.2	Canonical ensemble	481
14.3	Microcanonical ensemble	482
14.4	Partition function	483
14.5	Equipartition of energy	485
14.6	Specific heats of gases and solids	486
14.7	Black-body radiation	487
14.8	Quantum models of specific heats	502
14.9	Identical particles in quantum mechanics	504
14.10	Bose–Einstein and Fermi–Dirac gases	516
14.11	Statistical derivation of the Planck formula	519
14.12	Problems	522
	<i>Appendix 14.A</i> Towards the Planck formula	522
15	Lagrangian and phase-space formulations	526
15.1	The Schwinger formulation of quantum dynamics	526
15.2	Propagator and probability amplitude	529
15.3	Lagrangian formulation of quantum mechanics	533
15.4	Green kernel for quadratic Lagrangians	536
15.5	Quantum mechanics in phase space	541
15.6	Problems	548
	<i>Appendix 15.A</i> The Trotter product formula	548
16	Dirac equation and no-interaction theorem	550
16.1	The Dirac equation	550
16.2	Particles in mutual interaction	554
16.3	Relativistic interacting particles. Manifest covariance	555
16.4	The no-interaction theorem in classical mechanics	556
16.5	Relativistic quantum particles	563

Cambridge University Press
978-0-521-83324-0 - From Classical to Quantum Mechanics: An Introduction to the Formalism, Foundations
and Applications
Giampiero Esposito, Giuseppe Marmo and George Sudarshan
Frontmatter
[More information](#)

xii	<i>Contents</i>	
16.6	From particles to fields	564
16.7	The Kirchhoff principle, antiparticles and QFT	565
	<i>References</i>	571
	<i>Index</i>	588

Cambridge University Press

978-0-521-83324-0 - From Classical to Quantum Mechanics: An Introduction to the Formalism, Foundations and Applications

Giampiero Esposito, Giuseppe Marmo and George Sudarshan

Frontmatter

[More information](#)

Preface

The present manuscript represents an attempt to write a modern monograph on quantum mechanics that can be useful both to expert readers, i.e. graduate students, lecturers, research workers, and to educated readers who need to be introduced to quantum theory and its foundations. For this purpose, part I covers the basic material which is necessary to understand the transition from classical to wave mechanics: the key experiments in the development of wave mechanics; classical dynamics with emphasis on canonical transformations and the Hamilton–Jacobi equation; the Cauchy problem for the wave equation, the Helmholtz equation and the eikonal approximation; physical arguments leading to the Schrödinger equation and the basic properties of the wave function; quantum dynamics in one-dimensional problems and the Schrödinger equation in a central potential; introduction to spin and perturbation theory; and scattering theory. We have tried to describe in detail how one arrives at some ideas or some mathematical results, and what has been gained by introducing a certain concept.

Indeed, the choice of a first chapter devoted to the experimental foundations of quantum theory, despite being physics-oriented, selects a set of readers who already know the basic properties of classical mechanics and classical electrodynamics. Thus, undergraduate students should study chapter 1 more than once. Moreover, the choice of topics in chapter 1 serves as a motivation, in our opinion, for studying the material described in chapters 2 and 3, so that the transition to wave mechanics is as smooth and ‘natural’ as possible. A broad range of topics are presented in chapter 7, devoted to perturbation theory. Within this framework, after some elementary examples, we have described the nature of perturbative series, with a brief outline of the various cases of physical interest: regular perturbation theory, asymptotic perturbation theory and summability methods, spectral concentration and singular perturbations. Chapter

Cambridge University Press

978-0-521-83324-0 - From Classical to Quantum Mechanics: An Introduction to the Formalism, Foundations and Applications

Giampiero Esposito, Giuseppe Marmo and George Sudarshan

Frontmatter

[More information](#)

xiv

Preface

8 starts along the advanced lines of the end of chapter 7, and describes a lot of important material concerning scattering from potentials.

Advanced readers can begin from chapter 9, but we still recommend that they first study part I, which contains material useful in later investigations. The Weyl quantization is presented in chapter 9, jointly with the postulates of the currently accepted form of quantum mechanics. The Weyl programme provides not only a geometric framework for a rigorous formulation of canonical quantization, but also powerful tools for the analysis of problems of current interest in quantum mechanics. We have therefore tried to present such a topic, which is still omitted in many textbooks, in a self-contained form. In the chapters devoted to harmonic oscillators and angular momentum operators the emphasis is on algebraic and group-theoretical methods. The same can be said about chapter 12, devoted to algebraic methods for the analysis of Schrödinger operators. The formalism of the density matrix is developed in detail in chapter 13, which also studies some very important topics such as quantum entanglement, hidden-variable theories and Bell inequalities; how to transfer the polarization state of a photon to another photon thanks to the projection postulate, the production of statistical mixtures and phase in quantum mechanics.

Part III is devoted to a number of selected topics that reflect the authors' taste and are aimed at advanced research workers: statistical mechanics and black-body radiation; Lagrangian and phase-space formulations of quantum mechanics; the no-interaction theorem and the need for a quantum theory of fields.

The chapters are completed by a number of useful problems, although the main purpose of the book remains the presentation of a conceptual framework for a better understanding of quantum mechanics. Other important topics have not been included and might, by themselves, be the object of a separate monograph, e.g. supersymmetric quantum mechanics, quaternionic quantum mechanics and deformation quantization. But we are aware that the present version already covers much more material than the one that can be presented in a two-semester course. The material in chapters 9–16 can be used by students reading for a master or Ph.D. degree.

Our monograph contains much material which, although not new by itself, is presented in a way that makes the presentation rather original with respect to currently available textbooks, e.g. part I is devoted to and built around wave mechanics only; Hamiltonian methods and the Hamilton–Jacobi equation in chapter 2; introduction of the symbol of differential operators and eikonal approximation for the scalar wave equation in chapter 3; a systematic use of the symbol in the presentation of the Schrödinger equation in chapter 4; the Pauli equation with time-dependent magnetic

Cambridge University Press

978-0-521-83324-0 - From Classical to Quantum Mechanics: An Introduction to the Formalism, Foundations and Applications

Giampiero Esposito, Giuseppe Marmo and George Sudarshan

Frontmatter

[More information](#)

Preface

xv

fields in chapter 6; the richness of examples in chapters 7 and 8; Weyl quantization in chapter 9; algebraic methods for eigenvalue problems in chapter 12; the Wigner theorem and geometrical phases in chapter 13; and a geometrical proof of the no-interaction theorem in chapter 16.

So far we have defended, concisely, our reasons for writing yet another book on quantum mechanics. The last word is now with the readers.

Cambridge University Press

978-0-521-83324-0 - From Classical to Quantum Mechanics: An Introduction to the Formalism, Foundations and Applications

Giampiero Esposito, Giuseppe Marmo and George Sudarshan

Frontmatter

[More information](#)

Acknowledgments

Our dear friend Eugene Saletan has kindly agreed to act as merciless reviewer of a first draft. His dedicated efforts to assess our work have led to several improvements, for which we are much indebted to him. Comments by Giuseppe Bimonte, Volodya Man'ko, Giuseppe Morandi, Saverio Pascazio, Flora Pempinelli and Patrizia Vitale have also been helpful.

Rosario Peluso has produced a substantial effort to realize the figures we needed. The result shows all his skills with computer graphics and his deep love for fundamental physics. Charo Ivan Del Genio, Gabriele Gionti, Pietro Santorelli and Annamaria Canciello have drawn the last set of figures, with patience and dedication. Several students, in particular Alessandro Zampini and Dario Corsi, have discussed with us so many parts of the manuscript that its present version would have been unlikely without their constant feedback, while Andrea Rubano wrote notes which proved very useful in revising the last version.

Our Italian sources have not been cited locally, to avoid making unhelpful suggestions for readers who cannot understand textbooks written in Italian. Here, however, we can say that we relied in part on the work in Caldirola *et al.* (1982), Dell'Antonio (1996), Onofri and Destri (1996), Sartori (1998), Picasso (2000) and Stroffolini (2001).

We are also grateful to the many other students of the University of Naples who, in attending our lectures and asking many questions, made us feel it was appropriate to collect our lecture notes and rewrite them in the form of the present monograph.

Our Editor Tamsin van Essen at Cambridge University Press has provided invaluable scientific advice, while Suresh Kumar has been assisting us with TeX well beyond the call of duty.