From Classical to Quantum Mechanics

This book provides a pedagogical introduction to the formalism, foundations and applications of quantum mechanics. Part I covers the basic material that is necessary to an understanding of the transition from classical to wave mechanics. Topics include classical dynamics, with emphasis on canonical transformations and the Hamilton–Jacobi equation; the Cauchy problem for the wave equation, the Helmholtz equation and eikonal approximation; and introductions to spin, perturbation theory and scattering theory. The Weyl quantization is presented in Part II, along with the postulates of quantum mechanics. The Weyl programme provides a geometric framework for a rigorous formulation of canonical quantization, as well as powerful tools for the analysis of problems of current interest in quantum physics. In the chapters devoted to harmonic oscillators and angular momentum operators, the emphasis is on algebraic and group-theoretical methods. Quantum entanglement, hidden-variable theories and the Bell inequalities are also discussed. Part III is devoted to topics such as statistical mechanics and black-body radiation, Lagrangian and phase-space formulations of quantum mechanics, and the Dirac equation.

This book is intended for use as a textbook for beginning graduate and advanced undergraduate courses. It is self-contained and includes problems to advance the reader's understanding.

Giampiero Esposito received his PhD from the University of Cambridge in 1991 and has been INFN Research Fellow at Naples University since November 1993. His research is devoted to gravitational physics and quantum theory. His main contributions are to the boundary conditions in quantum field theory and quantum gravity via functional integrals.

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An Introduction to the Formalism, Foundations and Applications

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Preface

The present manuscript represents an attempt to write a modern monograph on quantum mechanics that can be useful both to expert readers, i.e. graduate students, lecturers, research workers, and to educated readers who need to be introduced to quantum theory and its foundations. For this purpose, part I covers the basic material which is necessary to understand the transition from classical to wave mechanics: the key experiments in the development of wave mechanics; classical dynamics with emphasis on canonical transformations and the Hamilton–Jacobi equation; the Cauchy problem for the wave equation, the Helmholtz equation and the eikonal approximation; physical arguments leading to the Schrödinger equation and the basic properties of the wave function; quantum dynamics in one-dimensional problems and the Schrödinger equation in a central potential; introduction to spin and perturbation theory; and scattering theory. We have tried to describe in detail how one arrives at some ideas or some mathematical results, and what has been gained by introducing a certain concept.

Indeed, the choice of a first chapter devoted to the experimental foundations of quantum theory, despite being physics-oriented, selects a set of readers who already know the basic properties of classical mechanics and classical electrodynamics. Thus, undergraduate students should study chapter 1 more than once. Moreover, the choice of topics in chapter 1 serves as a motivation, in our opinion, for studying the material described in chapters 2 and 3, so that the transition to wave mechanics is as smooth and 'natural' as possible. A broad range of topics are presented in chapter 7, devoted to perturbation theory. Within this framework, after some elementary examples, we have described the nature of perturbative series, with a brief outline of the various cases of physical interest: regular perturbation theory, asymptotic perturbation theory and summability methods, spectral concentration and singular perturbations. Chapter

8 starts along the advanced lines of the end of chapter 7, and describes a lot of important material concerning scattering from potentials.

Advanced readers can begin from chapter 9, but we still recommend that they first study part I, which contains material useful in later investigations. The Weyl quantization is presented in chapter 9, jointly with the postulates of the currently accepted form of quantum mechanics. The Weyl programme provides not only a geometric framework for a rigorous formulation of canonical quantization, but also powerful tools for the analysis of problems of current interest in quantum mechanics. We have therefore tried to present such a topic, which is still omitted in many textbooks, in a self-contained form. In the chapters devoted to harmonic oscillators and angular momentum operators the emphasis is on algebraic and group-theoretical methods. The same can be said about chapter 12, devoted to algebraic methods for the analysis of Schrödinger operators. The formalism of the density matrix is developed in detail in chapter 13, which also studies some very important topics such as quantum entanglement, hidden-variable theories and Bell inequalities; how to transfer the polarization state of a photon to another photon thanks to the projection postulate, the production of statistical mixtures and phase in quantum mechanics.

Part III is devoted to a number of selected topics that reflect the authors' taste and are aimed at advanced research workers: statistical mechanics and black-body radiation; Lagrangian and phase-space formulations of quantum mechanics; the no-interaction theorem and the need for a quantum theory of fields.

The chapters are completed by a number of useful problems, although the main purpose of the book remains the presentation of a conceptual framework for a better understanding of quantum mechanics. Other important topics have not been included and might, by themselves, be the object of a separate monograph, e.g. supersymmetric quantum mechanics, quaternionic quantum mechanics and deformation quantization. But we are aware that the present version already covers much more material than the one that can be presented in a two-semester course. The material in chapters 9–16 can be used by students reading for a master or Ph.D. degree.

Our monograph contains much material which, although not new by itself, is presented in a way that makes the presentation rather original with respect to currently available textbooks, e.g. part I is devoted to and built around wave mechanics only; Hamiltonian methods and the Hamilton–Jacobi equation in chapter 2; introduction of the symbol of differential operators and eikonal approximation for the scalar wave equation in chapter 3; a systematic use of the symbol in the presentation of the Schrödinger equation in chapter 4; the Pauli equation with time-dependent magnetic fields in chapter 6; the richness of examples in chapters 7 and 8; Weyl quantization in chapter 9; algebraic methods for eigenvalue problems in chapter 12; the Wigner theorem and geometrical phases in chapter 13; and a geometrical proof of the no-interaction theorem in chapter 16.

So far we have defended, concisely, our reasons for writing yet another book on quantum mechanics. The last word is now with the readers.

Acknowledgments

Our dear friend Eugene Saletan has kindly agreed to act as merciless reviewer of a first draft. His dedicated efforts to assess our work have led to several improvements, for which we are much indebted to him. Comments by Giuseppe Bimonte, Volodya Man'ko, Giuseppe Morandi, Saverio Pascazio, Flora Pempinelli and Patrizia Vitale have also been helpful.

Rosario Peluso has produced a substantial effort to realize the figures we needed. The result shows all his skills with computer graphics and his deep love for fundamental physics. Charo Ivan Del Genio, Gabriele Gionti, Pietro Santorelli and Annamaria Canciello have drawn the last set of figures, with patience and dedication. Several students, in particular Alessandro Zampini and Dario Corsi, have discussed with us so many parts of the manuscript that its present version would have been unlikely without their constant feedback, while Andrea Rubano wrote notes which proved very useful in revising the last version.

Our Italian sources have not been cited locally, to avoid making unhelpful suggestions for readers who cannot understand textbooks written in Italian. Here, however, we can say that we relied in part on the work in Caldirola *et al.* (1982), Dell'Antonio (1996), Onofri and Destri (1996), Sartori (1998), Picasso (2000) and Stroffolini (2001).

We are also grateful to the many other students of the University of Naples who, in attending our lectures and asking many questions, made us feel it was appropriate to collect our lecture notes and rewrite them in the form of the present monograph.

Our Editor Tamsin van Essen at Cambridge University Press has provided invaluable scientific advice, while Suresh Kumar has been assisting us with TeX well beyond the call of duty.