Quantum mechanics is our most successful physical theory. However, it raises conceptual issues that have perplexed physicists and philosophers of science for decades. This book develops a new approach, based on the proposal that quantum theory is not a complete, final theory, but is in fact an emergent phenomenon arising from a deeper level of dynamics. The dynamics at this deeper level is taken to be an extension of classical dynamics to non-commuting matrix variables, with cyclic permutation inside a trace used as the basic calculational tool. With plausible assumptions, quantum theory is shown to emerge as the statistical thermodynamics of this underlying theory, with the canonical commutation–anticommutation relations derived from a generalized equipartition theorem. Brownian motion corrections to this thermodynamics are argued to lead to state vector reduction and to the probabilistic interpretation of quantum theory, making contact with recent phenomenological proposals for stochastic modifications to Schrödinger dynamics.

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QUANTUM MECHANICS AS AN EMERGENT PHENOMENON

The Statistical Mechanics of Matrix Models as the Precursor of Quantum Field Theory

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Quantum mechanics as an emergent phenomenon: the statistical dynamics of global unitary invariant matrix models as the precursor of quantum field theory

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To Sarah Brett-Smith, with love and admiration
Contents

Acknowledgements x

Introduction and overview 1
1 The quantum measurement problem 2
2 Reinterpretations of quantum mechanical foundations 6
3 Motivations for believing that quantum mechanics is incomplete 9
4 An overview of this book 13
5 Brief historical remarks on trace dynamics 18

1 Trace dynamics: the classical Lagrangian and Hamiltonian dynamics of matrix models 21
1.1 Bosonic and fermionic matrices and the cyclic trace identities 21
1.2 Derivative of a trace with respect to an operator 24
1.3 Lagrangian and Hamiltonian dynamics of matrix models 27
1.4 The generalized Poisson bracket, its properties, and applications 29
1.5 Trace dynamics contrasted with unitary Heisenberg picture dynamics 32

2 Additional generic conserved quantities 39
2.1 The trace “fermion number” N 39
2.2 The conserved operator Ĉ 42
2.3 Conserved quantities for continuum spacetime theories 52
2.4 An illustrative example: a Dirac fermion coupled to a scalar Klein–Gordon field 58
2.5 Symmetries of conserved quantities under \( p_F \leftrightarrow q_F \) 62

3 Trace dynamics models with global supersymmetry 64
3.1 The Wess–Zumino model 64
3.2 The supersymmetric Yang–Mills model 67
3.3 The matrix model for M theory 70
3.4 Superspace considerations and remarks 72

4 Statistical mechanics of matrix models 75
4.1 The Liouville theorem 76
4.2 The canonical ensemble 81
4.3 The microcanonical ensemble 88
4.4 Gauge fixing in the partition function 93
4.5 Reduction of the Hilbert space modulo $i_{\text{eff}}$ 100
4.6 Global unitary fixing 106

5 The emergence of quantum field dynamics 117
5.1 The general Ward identity 119
5.2 Variation of the source terms 124
5.3 Approximations/assumptions leading to the emergence of quantum theory 128
5.4 Restrictions on the underlying theory implied by further Ward identities 139
5.5 Derivation of the Schrödinger equation 147
5.6 Evasion of the Kochen–Specker theorem and Bell inequality arguments 151

6 Brownian motion corrections to Schrödinger dynamics and the emergence of the probability interpretation 156
6.1 Scenarios leading to the localization and the energy-driven stochastic Schrödinger equations 157
6.2 Proof of reduction with Born rule probabilities 170
6.3 Phenomenology of stochastic reduction – reduction rate formulas 174
6.4 Phenomenology of energy-driven reduction 175
6.5 Phenomenology of reduction by continuous spontaneous localization 185

7 Discussion and outlook 190
Appendices 193

Appendix A: Modifications in real and quaternionic Hilbert space 194
Appendix B: Algebraic proof of the Jacobi identity for the generalized Poisson bracket 194
Appendix C: Symplectic structures in trace dynamics 198
Appendix D: Gamma matrix identities for supersymmetric trace dynamics models 201
Appendix E: Trace dynamics models with operator gauge invariance 204
Contents

Appendix F: Properties of Wightman functions needed for reconstruction of local quantum field theory 206
Appendix G: BRST invariance transformation for global unitary fixing 208

References 212
Index 220
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