

Contents

<i>Preface</i>	<i>page xi</i>
1 Preliminaries, notations and conventions	1
1.1 Elements of topology	1
1.2 Measure theory	3
1.3 Functions of bounded variation. Riemann–Stieltjes integral	17
1.4 Sequences of independent random variables	23
1.5 Convex functions. Hölder and Minkowski inequalities	29
1.6 The Cauchy equation	33
2 Basic notions in functional analysis	37
2.1 Linear spaces	37
2.2 Banach spaces	44
2.3 The space of bounded linear operators	63
3 Conditional expectation	80
3.1 Projections in Hilbert spaces	80
3.2 Definition and existence of conditional expectation	87
3.3 Properties and examples	91
3.4 The Radon–Nikodym Theorem	101
3.5 Examples of discrete martingales	103
3.6 Convergence of self-adjoint operators	106
3.7 ... and of martingales	112
4 Brownian motion and Hilbert spaces	121
4.1 Gaussian families & the definition of Brownian motion	123
4.2 Complete orthonormal sequences in a Hilbert space	127

Cambridge University Press

0521831660 - Functional Analysis for Probability and Stochastic Processes: An Introduction

A. Bobrowski

Table of Contents

[More information](#)

viii

Contents

4.3	Construction and basic properties of Brownian motion	133
4.4	Stochastic integrals	139
5	Dual spaces and convergence of probability measures	147
5.1	The Hahn–Banach Theorem	148
5.2	Form of linear functionals in specific Banach spaces	154
5.3	The dual of an operator	162
5.4	Weak and weak* topologies	166
5.5	The Central Limit Theorem	175
5.6	Weak convergence in metric spaces	178
5.7	Compactness everywhere	184
5.8	Notes on other modes of convergence	198
6	The Gelfand transform and its applications	201
6.1	Banach algebras	201
6.2	The Gelfand transform	206
6.3	Examples of Gelfand transform	208
6.4	Examples of explicit calculations of Gelfand transform	217
6.5	Dense subalgebras of $C(S)$	222
6.6	Inverting the abstract Fourier transform	224
6.7	The Factorization Theorem	231
7	Semigroups of operators and Lévy processes	234
7.1	The Banach–Steinhaus Theorem	234
7.2	Calculus of Banach space valued functions	238
7.3	Closed operators	240
7.4	Semigroups of operators	246
7.5	Brownian motion and Poisson process semigroups	265
7.6	More convolution semigroups	270
7.7	The telegraph process semigroup	280
7.8	Convolution semigroups of measures on semigroups	286
8	Markov processes and semigroups of operators	294
8.1	Semigroups of operators related to Markov processes	294
8.2	The Hille–Yosida Theorem	309
8.3	Generators of stochastic processes	327
8.4	Approximation theorems	340
9	Appendixes	363
9.1	Bibliographical notes	363

Cambridge University Press

0521831660 - Functional Analysis for Probability and Stochastic Processes: An Introduction

A. Bobrowski

Table of Contents

[More information](#)

	<i>Contents</i>	ix
9.2	Solutions and hints to exercises	366
9.3	Some commonly used notations	383
	<i>References</i>	385
	<i>Index</i>	390