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Thomas Erneux, Pierre Glorieux
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LASER DYNAMICS

Bridging the gap between laser physics and applied mathematics, this book offers a new perspective on laser dynamics. Combining fresh treatments of classic problems with up-to-date research, asymptotic techniques appropriate for nonlinear dynamical systems are shown to offer a powerful alternative to numerical simulations. The combined analytical and experimental descriptions of dynamical instabilities provide a clear derivation of physical formulae and an evaluation of their significance.

Starting with the observation of different time scales of an operating laser, the book develops approximation techniques to systematically explore their effects. Laser dynamical regimes are introduced at different levels of complexity, from standard turn-on experiments to stiff, chaotic, spontaneous, or driven pulsations. Particular attention is given to quantitative comparisons between experiments and theory. The book broadens the range of analytical tools available to laser physicists and provides applied mathematicians with problems of practical interest, and is invaluable for both graduate students and researchers.

THOMAS ERNEUX is a Professor at the Université Libre de Bruxelles. His current interests concentrate on studying specific laser dynamical phenomena and the applications of delay differential equations in all areas of science and engineering.

PIERRE GLORIEUX is a Professor at the Laboratoire de Physique des Lasers, Atomes et Molécules, Université des Sciences et Technologies de Lille, and a Senior Member of the Institut Universitaire de France. Recently, he has studied spatio-temporal dynamics in liquid crystals and photorefractive oscillators.

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THOMAS ERNEUX
Université Libre de Bruxelles
PIERRE GLORIEUX
Université des Sciences et Technologies de Lille

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To Anne and Mijo
for their love and support

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Preface

Many of the physicists studying lasers in laboratories have been confronted by the appearance of *erratic intensity fluctuations* in the laser beam. This type of behavior was already evident *in the early days of the laser (1960s)* when it was found that the intensity of the light generated by the ruby laser displayed irregular spiking. Russian theoreticians showed that equations describing an active medium coupled to an electromagnetic field could display such pulsations. Laser physicists K. Shimoda and C.L. Tang tried to relate these outputs to saturable absorption and mode competition, respectively. But the discrepancy in the values for the instability frequencies, the fact that simple rate equations only predicted damped oscillations, and the development of stable lasers shifted interest towards new topics. About the same time, spontaneous instabilities were found to play key roles in fluid mechanics, chemistry, and the life sciences. Except for some isolated pioneers like L.W. Casperson, laser physicists only understood *in the early 1980s* that the pulsating outputs were not the result of environmental fluctuations but rather originated from the interaction between the radiation field and matter. On June 18–21, 1985, the University of Rochester organized the first International Meeting on "Instabilities and Dynamics of Lasers and Nonlinear Optical Systems" [1]. Two special issues of the *Journal of the Optical Society of America* later appeared [2, 3]. But it took until *the early 1990s* before the idea became widely accepted among physicists that lasers exhibit the same type of bifurcations as oscillating mechanical, chemical, and biological systems [4–7]. The possible laser outputs were then systematically explored by multi-disciplinary groups. Nonlinear laser dynamics became a hot topic of research following similar adventures in the physical and life sciences [8–12].

Early investigations have concentrated on gas and solid state lasers. *Semiconductor lasers* came in the 1990s thanks to an enormous effort in fundamental and applied research. They are the lasers used in most of our current applications. Systematic experimental and theoretical studies of their possible instabilities in

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a variety of set-ups have been undertaken during the last 20 years and significant progress has been made, to the point where we know how to exploit, avoid, or control them [13, 14].

Laser dynamical instabilities are of interest for a growing number of scientists and engineers, not only laser physicists, but also chemists, biologists, and others in a variety of obviously and not so obviously related fields. Placing pulsating lasers in the framework of *dynamical systems* means that many of the observed instabilities can be investigated using simple classical equations based on material properties rather than design. Twenty years ago, a book largely devoted to laser intensity oscillations using this approach would have been inconceivable without taking account of the quantum mechanical properties of the laser or its cavity design. The visually compelling phenomena observed with laser devices and their potential applications make laser dynamics a subject about which colleagues and graduate students with different experiences seek to become better informed. Special sessions entitled Laser Dynamics now appear at conferences, introductory courses are offered at universities, and research groups have concentrated their main activities on laser stability problems.

The primary objective of this book is to introduce a series of *simple laser dynamical problems* that are the building blocks of our current research in the field. These include a description of the relaxation oscillations of the laser, strongly pulsating outputs following a quick change of a parameter or resulting from a saturable absorber, phase-locking phenomena for a laser subject to an injected signal, resonance phenomena in modulated lasers, and oscillatory instabilities caused by a delayed optical feedback. Topics like the diagnostics of chaotic outputs, ultra-fast optics and mode-locked lasers, or the propagation of spatial solitons in fibers are too broad to be covered in this book.

As is largely the case for engineers and applied scientists, a theoretical model is often considered as a numerical model. The difficulty with this approach is that computation limits insight because of an inability to pose questions properly. We cannot ignore the possibilities offered by our computers but we also need to think about the main objectives of our research. To this end, *asymptotic approaches* [15–17] based on the natural values of the parameters smoothly complement simulations by emphasizing particular properties of our laser. From an applied mathematical point of view the laser rate equations offer challenging (singular) limits requiring the adaptation of known techniques to our laser equations. In this book, we introduce some of these techniques, helping the physicist to highlight the generic character of a specific phenomenon or to compare different lasers through their relevant effective parameters. Key to this approach is the hierarchy of different time scales as they appear in the experimental set-ups and observations. The book explores different laser systems whose descriptions require tools of

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increasing complexity. In each chapter, both theoretical and experimental points of view are confronted with the goal of finding the underlying physical mechanisms responsible for a specific dynamical output.

The book is *organized into three parts*, namely, I Basic tools, II Driven laser systems, and III Particular laser systems. The first part aims to address how the laser physicist studies simple dynamical outputs by using rate equations and the mathematical tools used for their exploration. There is an extensive discussion of time scales and their relevance in slow-time dynamics. There is confusion in the literature and we hope to clarify some of the questions arising in choosing time scales. Another objective of Part I is to introduce the basic bifurcation transitions that appear in a variety of laser set-ups. To this end, we examine explicit examples and introduce methods in the most friendly way. After many years of teaching the subject of laser dynamics, we have found that this is the best way to introduce bifurcation theory to the physicist. Part II is devoted to specific laser systems that are driven either by a modulated signal or by a slowly varying control parameter. The literature of periodically forced lasers is abundant because modulated lasers are important in telecom applications, but also at a more fundamental level because strongly forced lasers lead to chaotic outputs. This part will not review all that has been realized on driven lasers but rather will emphasize the variety of synchronization mechanisms from weak to strongly modulated. Slow passage problems are a key topic in applied mathematics because they appear in a variety of problems with applications in physics, chemistry, and biology. Surprisingly, many experiments on basic slow passage problems have been realized with lasers or optically bistable devices. Part III is devoted to specific laser set-ups that became important on their own and were motivated by specific applications. Of particular interest is the fact that they each introduce a new dynamical phenomenon, such as the onset of spiking pulses, multimode antiphase dynamics, or instabilities caused by a delayed optical feedback.

The book contains more than enough for *two one-semester courses* and some flexibility is possible in selecting topics. Part I collects simple concepts in both laser physics and nonlinear dynamics such as stability, bifurcation, and multiple time scales that must be understood before exploring Parts II and III. The first two chapters of Part II are linked while the third one on slow passage effects is rather independent. The five chapters of Part III consider specific laser systems and can be read separately. Some of these chapters cover classical areas (such as the laser with a saturable absorber) that are introduced in almost every course. Other chapters concentrate on less known areas (such as the far-infrared laser) which we critically revisit, benefiting from the current capacities of our computers or from new asymptotic investigations. To cover the whole book, the student will need a background in linear algebra and ordinary differential equations. However,

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the details of the calculations are given each time a new technique is introduced so that the reader less oriented towards theory may follow each step. Similarly, experimental details are introduced in the simplest way and avoid technical descriptions of set-ups. To limit the size of the book, we have combined solved problems and additional material usually relegated to appendices in an associated website, http://www.ulb.ac.be/sciences/ont. This site includes detailed answers to exercises in the book, links to other useful sites, and illustrations of specific mathematical techniques such as the method of matched asymptotic expansions (MAE).

We are very much indebted to many colleagues for help during the years while this book was being written. Over the past 30 years, we greatly profited from collaboration or discussion with our friends at the Laboratoire de Physique des Lasers, Atomes et Molécules and in the department of Optique Nonlinéaire Théorique who shared our enthusiasm comparing experimental and theoretical data. This book pays tribute to the memory of Gilbert Grynberg, Lorenzo Narducci, Yakov I. Khanin, and Fréderic Stoekel who have particularly contributed to many aspects of laser dynamics. Finally, we acknowledge the Belgian National Science Foundation and the Pole Attraction Pole program of the Belgian government for the support we received during the preparation of this book.

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Abbreviations

AOM	Acousto-Optic Modulator
BD	Bifurcation Diagram
CCD	Charge Coupled Device (in CCD camera)
cw	continuous-wave
cw and *ccw*	clockwise and counterclockwise
DBR	Distributed Bragg Reflector
DDE	Delayed Differential Equations
DFB	Distributed FeedBack
DROPO	Doubly Resonant Optical Parametric Oscillator
DOPO	Degenerate Optical Parametric Oscillator
ECM	External Cavity Mode
EDT	Exponentially Decaying Terms
EOM	ElectroOptic Modulator
FIR	Far InfraRed (for FIR laser)
FWM	Four-Wave Mixing
KTP	($KTiOPO_4$) Potassium Titanium Oxide Phosphate
LaROFI	Laser Relaxation Oscillation Frequency Imaging
LFF	Low Frequency Fluctuations
LIS	Laser with an Injected Signal
LK	Lang and Kobayashi (for Lang and Kobayashi equations)
LSA	Laser with a Saturable Absorber
MAE	Matched Asymptotic Expansions
MB	Mode Beating
MZ	Mach–Zehnder (for Mach–Zehnder interferometer)
OEO	OptoElectronic Oscillator
OFB	Optical FeedBack

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OPA	Optical Parametric Amplification
OPO	Optical Parametric Oscillator
PIN	Positive-Intrinsic-Negative (for PIN photodiode)
PQS	Passive Q-Switching
RE	Regular Emission
RO	Relaxation Oscillations
RF	Radio Frequency
SE	Spontaneous Emission
SHG	Second Harmonic Generation
SL	Semiconductor Laser
SN	Saddle-Node (for the saddle-node bifurcation)
SRE	Standard Rate Equations
SROPO	Singly Resonant Optical Parametric Oscillator
TROPO	Triply Resonant Optical Parametric Oscillator
VCSEL	Vertical Cavity Surface Emitting Laser
YAG	$(Y_3Al_{15}O_{12})$Yttrium Aluminum Garnet

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