THE PRIMORDIAL DENSITY PERTURBATION
Cosmology, Inflation and the Origin of Structure

The origin and evolution of the primordial perturbation is the key to understanding structure formation in the earliest stages of the Universe. It carries clues to the types of physical phenomena active in that extreme high-density environment. Through its evolution, generating first the observed cosmic microwave background anisotropies and later the distribution of galaxies and dark matter in the Universe, it probes the properties and dynamics of the present Universe.

This graduate-level textbook gives a thorough account of theoretical cosmology and perturbations in the early Universe, describing their observational consequences and showing how to relate such observations to primordial physical processes, particularly cosmological inflation. With ambitious observational programmes complementing ever-increasing sophistication in theoretical modelling, cosmological studies will remain at the cutting edge of astrophysical studies for the foreseeable future.

DAVID H. LYTH is Professor of Particle Astrophysics in the Physics Department of Lancaster University.

ANDREW R. LIDDLE is Professor of Astrophysics in the Department of Physics and Astronomy at the University of Sussex. They have a long-established research collaboration and have jointly developed some of the key concepts in studies of cosmological perturbations, particularly in relation to the inflationary cosmology. They previously co-authored the Cambridge University Press textbook *Cosmological Inflation and Large-Scale Structure* in 2000.

THE PRIMORDIAL DENSITY PERTURBATION

Cosmology, Inflation and the Origin of Structure

DAVID H. LYTH
University of Lancaster

ANDREW R. LIDDLE
University of Sussex

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521828499

First published 2009

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data

ISBN 978-0-521-82849-9 hardback

To Margaret, John, and Duncan

and to Ed, John, and Rocky

Contents

Cambridge University Press
978-0-521-82849-9 - The Primordial Density Perturbation: Cosmology, Inflation and the Origin of Structure
David H. Lyth and Andrew R. Liddle
Frontmatter
More information

Cambridge University Press
978-0-521-82849-9 - The Primordial Density Perturbation: Cosmology, Inflation and the Origin of Structure
David H. Lyth and Andrew R. Liddle
Frontmatter
More information

Cambridge University Press
978-0-521-82849-9 - The Primordial Density Perturbation: Cosmology, Inflation and the Origin of Structure
David H. Lyth and Andrew R. Liddle
Frontmatter
More information

Frequently used symbols, and their place of definition.

Symbol	*Page*	*Definition*
$a_{\ell m}$	154	Observed multipole of Θ
$\hat{a}_{\mathbf{k}}$ $(\hat{a}^\dagger_{\mathbf{k}})$	254	Annihilation (creation) operator
B	55	Baryon number
B_ℓ	155	A CMB polarization multipole
C_ℓ	155	Spectrum $\langle \lvert a_{\ell m}\rvert^2\rangle$ of CMB anisotropy
ds^2	7	Spacetime interval
E	12	Energy
E_ℓ	182	A CMB polarization multipole
$f_{\rm a}$	282	The axion parameter
f $(f_{\rm a})$	19	Distribution function (of species ‘a’)
$f_{\rm NL}$	100	Reduced bispectrum of ζ
g $(g_{\rm a})$	19	Number of spin states (of species ‘a’)
g_{ij}	34	Spatial metric tensor
$g_{\mu\nu}$	27	Metric tensor (generic coordinates)
h_{ij}	38, 199	Gravitational wave amplitude
H	42	Hubble parameter $\dot{a}/a$
H_0	60	Present Hubble parameter
H	216, 217	Hamiltonian
$\hat{H}$	247	Hamiltonian operator
I	79	Unit matrix or unit operator
j^μ	16	Any conserved current
k, $\mathbf{k}$	87	Wavenumber, wave-vector (physical or comoving)
k, $\mathbf{k}$	254	Momentum of a single particle (physical or comoving)
L	55	Lepton number
L	214	Lagrangian
L	87	Comoving box size for Fourier expansion
$\mathcal{L}$	218	Lagrangian density
m	12	Mass of a particle
M	71	Cosmological mass within a comoving sphere
n $(n_{\rm a})$	20	Number density (of species ‘a’)
n	99	Spectral index of ζ

Cambridge University Press
978-0-521-82849-9 - The Primordial Density Perturbation: Cosmology, Inflation and the Origin of Structure
David H. Lyth and Andrew R. Liddle
Frontmatter
More information

Symbol	*Page*	*Definition*
N	81	e-folds between any two epochs
N	81	N such that $\zeta = \delta N$
N	313	e-folds of observable inflation
$\mathbf{p}$	12	Momentum
p^μ	12	4-momentum
$P\ (P_{\rm a})$	17	Pressure (of species ‘a’)
P_g	89	Spectrum of a perturbation g
$\mathcal{P}_g$	89	Spectrum of a perturbation g (alternative definition)
Q	55	Electric charge
Q	266	Renormalization scale
r	199	Tensor fraction $\mathcal{P}_h/\mathcal{P}_\zeta$
R	42	Cosmological comoving radius
s	53	Entropy density
S	214	Action
S_i	190	An isocurvature perturbation
t	7, 39	Time, cosmic time
T	24	Temperature (usually of the early Universe or CMB)
T	125, 159	A transfer function
$T^{\mu\nu}$	16	Energy–momentum tensor
u^μ	12	4-velocity
$\mathbf{v}\ (\mathbf{v}_{\rm a})$	106, 120	Fluid velocity perturbation (of species ‘a’)
$V\ (V_{\rm a})$	107, 120	Fluid velocity scalar (of species ‘a’)
V	219	Scalar field potential
W	290	Superpotential
$x\ (x^\mu)$	7	Minkowski spacetime coordinates $(\mu = 0, 1, 2, 3)$
$x\ (x^\mu)$	26	Generic spacetime coordinates $(\mu = 0, 1, 2, 3)$
x^i	34	Generic spatial coordinates $(i = 1, 2, 3)$
$\mathbf{x}$	7	Cartesian coordinates (x^1, x^2, x^3)
$\mathbf{x}$	41	Comoving Cartesian coordinates (x^1, x^2, x^3)
$w\ (w_{\rm a})$	122	Ratio P/ρ (for species ‘a’), equation of state
z	52	Redshift

Cambridge University Press
978-0-521-82849-9 - The Primordial Density Perturbation: Cosmology, Inflation and the Origin of Structure
David H. Lyth and Andrew R. Liddle
Frontmatter
More information

Symbol	*Page*	*Definition*
δ ($\delta_{\rm a}$)	76	Density contrast $\delta\rho/\rho$ (of species 'a')
ϵ	316	Slow-roll flatness parameter $\frac{1}{2}M_{\rm Pl}^2(V'/V)^2$
$\epsilon_{\rm H}$	317	Slow-roll parameter $-\dot{H}/H^2$
ζ	79	Primordial curvature perturbation
η	315	Slow-roll flatness parameter $M_{\rm Pl}^2V''/V$
η	41	Conformal time $d\eta = dt/a$
$\eta_{\rm ls}$	66	Conformal time at photon decoupling
η_0	44	Present conformal time
$\eta_{\mu\nu}$	7	Metric tensor (Minkowski coordinates)
Θ	153	Photon brightness function dT/T
Θ_ℓ	158	Multipole of a Fourier component of Θ
Λ	46	Cosmological constant
μ ($\mu_{\rm a}$)	24	Chemical potential (of species 'a')
Π ($\Pi_{\rm a}$)	120	Anisotropic stress scalar (of species 'a')
Π_{ij}	120	Dimensionless anisotropic stress
ρ ($\rho_{\rm a}$)	16	Energy density (of species 'a')
$\sigma_{\rm T}$	130	Thomson scattering cross-section
Σ_{ij}	17	Anisotropic stress
τ	65	Optical depth
ϕ (ϕ_n)	219	Scalar field (the nth scalar field)
Φ	106	Newtonian peculiar gravitational potential
Φ	121	One of the relativistic gravitational potentials
φ	383	Conformal inflaton field perturbation $a\delta\phi$
Ψ	121	One of the relativistic gravitational potentials
Ω ($\Omega_{\rm a}$)	47, 61	Ratio $\rho/\rho_{\rm crit}$ (for species 'a')

Preface

The beginning of the twenty-first century stands a good chance of being identified in history as the time when humankind first came to grips with the Universe. In a rush of observational progress, the charge led by the 1992 discovery of cosmic microwave background anisotropies by the Cosmic Background Explorer (COBE) satellite, the elements required to build accurate cosmological models were assembled. Complementing this, development of theoretical methods allowed accurate predictions to be made to confront those observations.

The landmark was the 2003 announcement of precision cosmic microwave measurements from the team operating the Wilkinson Microwave Anisotropy Probe (WMAP). Ironically, this success lay in a kind of failure — a failure to uncover anything new and unexpected. In the words of astrophysicist John Bahcall at NASA's announcement press conference, "the biggest surprise is that there are no surprises". Instead, then, the power of the observations became fully focussed on determining the properties of the cosmological model, and for the first time many of its components were determined to a satisfying degree of accuracy: the percent level for quantities such as the geometry of the Universe, its age, and the density of the baryonic material, and the ten percent level for many other aspects.

In 2000, we published a graduate-level textbook, *Cosmological Inflation and Large-Scale Structure*, written during the late 1990s and which described many of the ideas underpinning the modern cosmology. We had been considering a second edition which would bring everything up to date. It became apparent, however, that the subject had already developed too far for a simple update to be possible. The emergence of the standard cosmological model had left by the wayside many concepts which in the late 1990s had still had possible relevance for our Universe. Meanwhile, the community's attention had moved on to the new frontiers, such as cosmic non-gaussianity, where the next discoveries illuminating our Universe may lie. New ideas on the origin of inflationary perturbations had emerged, as had more elegant and streamlined ways of deriving established results.

Cambridge University Press
978-0-521-82849-9 - The Primordial Density Perturbation: Cosmology, Inflation and the Origin of Structure
David H. Lyth and Andrew R. Liddle
Frontmatter
More information

So, instead, a new book! Some of the subject coverage is the same as the previous one, but the focus is more narrowly fixed on the now-standard cosmology, and as such goes deeper. The emphasis is shifted somewhat in a theoretical direction, and looks forward to the new frontiers of cosmology. After all, setting in place a viable and well-specified cosmological model is only the start, suggesting that we are learning *how* the Universe works. But what we really want to know is *why*!

We have learned much from research collaborators, and from many others acknowledged in our published papers. In addition, we thank Anthony Challinor, Eung Jin Chun, Paolo Creminelli, Wayne Hu, Kazunori Kohri, Eiichiro Komatsu, George Lazarides, Antony Lewis, Andrei Linde, Marta Losada, John McDonald, Ian Moss, Joe Polchinski, Misao Sasaki, Quasar Shafi, Alex Vilenkin, David Wands, Steven Weinberg and Martin White for input relating to matters dealt with in this book that we didn't have the opportunity to acknowledge in published papers.

Of course, we have done our best to ensure that the contents of this book are accurate; however, experience tells us that some errors will have slipped through. Please let us know of any you spot. There will be an up-to-date record of known errors, plus other updates, accessible at the book's World Wide Web Home Page at

http://astronomy.sussex.ac.uk/∼andrewl/pdp.html