From Eudoxus to Einstein

A History of Mathematical Astronomy

Since man first looked towards the heavens, a great deal of effort has been put into trying to predict and explain the motions of the Sun, Moon, and planets. Developments in man's understanding have been closely linked to progress in the mathematical sciences. Whole new areas of mathematics, such as trigonometry, were developed to aid astronomical calculations, and on numerous occasions throughout history breakthroughs in astronomy have been possible only because of progress in mathematics. This book describes the theories of planetary motion that have been developed through the ages, beginning with the homocentric spheres of Eudoxus and ending with Einstein's general theory of relativity. It emphasizes the interaction between progress in astronomy and in mathematics, showing how the two have been linked inextricably since Babylonian times. This valuable text is accessible to a wide audience, from amateur astronomers to professional historians of astronomy.

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Preface

'It is extremely hard these days to write mathematical books, especially astronomical ones.' Thus begins Kepler's *New Astronomy*, published in 1609. While I would not attempt to claim that the problems I have faced are comparable with those Kepler had to tackle, he was after all working at the cutting edge of both the mathematics and astronomy of his day; the peculiar difficulties that face authors of books containing mathematics, but the major thrusts of which lie elsewhere, do not appear to have changed much over the passage of time. A balancing act is required: how does one include sufficient technical detail accurately to describe the procedures involved and not end up losing sight of the main focus? In the context of a historical survey there is another question: how faithful should one be to the form in which the mathematics was originally written? Here, historians and mathematicians writing about history tend to take different approaches. As a member of the latter category I have taken the view in this book that it is more important to understand what it is that was accomplished than precisely how it was achieved.

The mathematical details that are given hopefully serve to provide a more comprehensive description of the development of astronomical theories than is usually found in general histories of astronomy. Sometimes the mathematics is described using the methods available at the time, but on other occasions modern mathematical language has been used to make the discussion easier to follow for the modern reader. I have tried to ensure that anachronisms are labelled clearly as such and that the resulting mixture of old and new is both informative and not misleading.

There is an enormous number of books written about the history of astronomy, a vast subject spanning 4000 years of human history. The story of man's gradual appreciation of the nature of the heavens and the development of techniques for predicting the future positions of celestial bodies is fascinating. It can be appreciated on many levels, and histories of astronomy abound which are designed to appeal to the general public as well as to expert astronomers. Having a continuous history from 2000 BC to the present day, it is inevitable that astronomical thought has interacted with progress in many other branches of science (e.g. mathematics and physics) as well as areas such as philosophy and theology. Most astronomy today is concerned with objects which lie outside our Solar System but, until the nineteenth century, astronomy was the study of the motion of the Sun, Moon and planets measured against the background of 'fixed' stars. It is my aim in this book to describe the theories of planetary motion that have been developed, beginning with the homocentric spheres of Eudoxus and ending with Einstein's general theory of relativity, with particular regard to the interaction between progress in astronomy and in mathematics.

Since Babylonian times, astronomy and mathematics have been linked inextricably. The needs of astronomy have provided the impetus for research into many areas of mathematics, and whole new branches of mathematics (e. g. trigonometry) were developed to aid astronomical calculations. Conversely, on numerous occasions throughout history breakthroughs in astronomy have been possible only because of progress in mathematics. This two-way process pervades science:

> Mathematics is an indispensable medium by which and within which science expresses, formulates, continues, and communicates itself. And just as the language of true literacy not only specifies and expresses thoughts and processes of thinking but also creates them in turn, so does mathematics not only specify, clarify and make rigorously workable concepts and laws of science which perhaps, partially at least, could be put forward within it; but at certain crucial instances it is an indispensable constituent of their creation and emergence as well.
>
> (Bochner (1966), p. 256.)

However, most books on the history of astronomy provide only a cursory treatment of the underlying mathematics on the assumption that such topics would put people off, and in most books on the history of mathematics the interaction with astronomical thought is discussed only briefly. For example, in the preface to *The Cambridge Concise History of Astronomy* (1998), Michael Hoskin, describing the post-Newtonian celestial mechanicians, writes: 'But while their conclusions were of the keenest interest to astronomers, they were not themselves astronomers but mathematicians working in the service of astronomy, and so we can disregard the details of their calculations with a clear conscience.' All this is entirely understandable: the history of astronomy *can* be told in a non-mathematical way, and there is far more to the history of mathematics than just those parts which are relevant to astronomy.

At the other extreme, there are books that investigate episodes in the history of mathematical astronomy in great detail – O. Pedersen's *A Survey of the*

Almagest or N. M. Swerdlow and O. Neugebauer's *Mathematical Astronomy in Copernicus's De revolutionibus*, for example – but a huge amount of technical astronomical knowledge is required before these works become accessible.

This state of affairs can be frustrating for mathematicians, or people interested in mathematics, who want to explore the history of astronomy but who are not already well-versed in technical astronomy. For example, it is difficult to find out about the mathematical problems that Kepler struggled with in his *New Astronomy* without access to technical journals, and yet a proper appreciation of Kepler's contributions to astronomy surely requires a knowledge of the mathematical difficulties which he had to overcome. It is my aim in this book to provide a history of theoretical astronomy that to some extent fills this gap. The book is not simply a study of the interactions between mathematical and astronomical developments – its main aim is to tell the story of how we have come to understand the motions of the Sun, Moon and planets – but it does so recognizing that the mathematics that challenged astronomers throughout preceding millennia forms an integral part of the tale.

The first mathematical model of the Universe was that of Eudoxus in the fourth century BC, and this provides (more or less) the starting point. Eudoxus' model represented the motions of the celestial bodies in a qualitative way, but was not satisfactory when it came to quantitative prediction. Over the centuries, through the work of men such as Ptolemy, Ibn al-Shāṭir, Copernicus, Tycho Brahe, Kepler, and Newton, models of the heavens came to reproduce the results of observations with greater and greater accuracy. During the eighteenth and nineteenth centuries, astronomers believed that the fundamental principles on which Newtonian dynamics rested provided the final word on celestial mechanics (a phrase coined by Laplace) though mathematical progress would be required to enable people to extract more information from Newton's theory of universal gravitation.

Discoveries of new celestial bodies led to new challenges, and the sophistication of the mathematical theories designed to meet them increased. The discovery of Neptune in 1846 brought the power of theoretical astronomy to the attention of a wide audience and cemented its position as the ultimate exact science. And yet, within 70 years, its omnipotence had been challenged successfully – twice. First, new techniques for analysing problems in mechanics led Poincaré to the conclusion that there was a theoretical limit to the predictive power of the differential equations of celestial mechanics; here were the beginnings of 'chaos theory'. Second, and finally as far as this book is concerned, Einstein showed in 1915 that the whole of Newtonian mechanics is an approximation valid only at speeds much less than that of light. The general theory of relativity provided the explanation for the one planetary phenomenon that had refused to succumb to the power of Newtonian theory: the anomalous advance of the perihelion of Mercury. Since 1915, theory and observation can be said to be in agreement within the limits of observational accuracy. This provides a suitable endpoint.

The book is intended for anyone who is interested in the history of astronomy and who is not afraid of mathematics. Large parts of the book can be read by someone with only a rudimentary mathematical knowledge, but there are parts where the level of mathematics required is that taught at undergraduate level. No prior knowledge of astronomy is required, however, and so the book will be suitable particularly for undergraduates reading a mathematics-based degree programme, and who are taking a course in the history of either astronomy or mathematics. It was through teaching a course on the history of mathematics at Loughborough University that I first saw the need for, and decided to write, this book.

For those wishing to explore specific topics in more detail, extensive references are provided via footnotes. These include both works of scholarship and popular accounts. For reasons of practical necessity I have restricted myself to sources written in English – which of course, rules out the original works of most of the people discussed in the book – but every effort has been made not to propagate errors and misconceptions from the secondary literature. Some will remain, and some errors will have been committed that I cannot blame anyone for but myself. For these I apologize in advance.

I would like to express my gratitude to colleagues and friends for their encouragement and, in particular, to Peter Shiu for reading a preliminary draft and making numerous helpful suggestions. Finally, I would like to thank Joanna for her patience.