GENERAL EQUILIBRIUM THEORY
Second Edition

General Equilibrium Theory: An Introduction presents the mathematical economic theory of price determination and resource allocation from elementary to advanced levels, suitable for advanced undergraduates and graduate students of economics. This Arrow-Debreu model (known for two of its most prominent founders, both Nobel Laureates) is the basis of modern price theory and of a wide range of applications. The text starts with elementary models: Robinson Crusoe, the Edgeworth Box, and a two-commodity two-household two-firm model. It gives a brief introduction to the mathematics used in the field (continuity, convexity, separation theorems, Brouwer fixed-point theorem, point-to-set mappings, and Shapley-Folkman theorem). It then presents the mathematical general equilibrium model in progressively more general settings, including point-valued, set-valued, and nonconvex set-valued demand and supply. Existence of general equilibrium, fundamental theorems of welfare economics, core convergence, and futures markets with time and uncertainty are treated fully. This new edition updates the discussion throughout and expands the number and variety of exercises. It offers a revised and extended treatment of core convergence, including the case of nonconvex preferences, and introduces the investigation of approximate equilibrium with U-shaped cost curves and nonconvex preferences.

Ross M. Starr is Professor of Economics at the University of California, San Diego, where he has taught since 1980. He has also served on the faculties of Yale University, the London School of Economics, and the University of California, Davis, and he held a Guggenheim Fellowship at the University of California, Berkeley. Professor Starr's research focuses on general equilibrium theory, mathematical economics, and monetary theory. He is the editor of *General Equilibrium Models of Monetary Economies* (1989) and coeditor of the three-volume *Essays in Honor of Kenneth Arrow* (Cambridge University Press, 1986). His articles have appeared in journals such as *Econometrica; Economic Theory; Journal of Economic Theory; Journal of Money, Credit, and Banking; Quarterly Journal of Economics*; and *Review of Economic Studies*. Cambridge University Press published the first edition of *General Equilibrium Theory: An Introduction* in 1997.

GENERAL EQUILIBRIUM THEORY

An Introduction

Second Edition

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For Susan

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Introduction to the second edition

The foundations of modern economic general equilibrium theory are contained in a surprisingly short list of references. For primary sources, it is sufficient to master Arrow and Debreu (1954), Arrow (1951), Arrow (1953), and Debreu and Scarf (1963). An even shorter list is comprehensive; Debreu (1959) and Debreu and Scarf (1963) cover the topic admirably. Why should anyone write (or read!) a secondary source, a textbook? Because, unfortunately, this body of material is extremely difficult for most students to read and comprehend. Professor Hahn described Debreu's (1959) book as "very short, but it may well take as long to read as many works three times as long. This is not due to faulty exposition but to the demands rigorous analysis makes on the reader. It is to be hoped that no one will be put off by this, for the . . . return . . . is very high indeed" (Hahn [1961]). Unfortunately, in teaching economic theory we find that many capable students are indeed put off by the mathematical abstraction of the above works. What theorists regard as elegantly terse expression, students may find inaccessible formality. The focus of this textbook is to overcome this barrier and to make this body of work accessible to a wider audience of advanced undergraduate and graduate students in economics.

This book presents the theory of general economic equilibrium incrementally, from elementary to more sophisticated treatments. Part A (Chapters 1 through 5) presents an elementary introduction. Chapters 2 and 3 present a nontechnical introduction to the Robinson Crusoe and Edgeworth box models of general equilibrium and Pareto efficiency using differential calculus. Chapter 4 goes over the $2 \times 2 \times 2$ (two commodities, two households, two factors) model using differential calculus, including the marginal equivalence results typical of the classical welfare economics. Chapter 5 briefly presents an introduction to the use of the Brouwer Fixed-Point Theorem to prove the existence of general equilibrium.

Part B (Chapters 6 through 9) introduces the mathematics used throughout the rest of the book: analysis and convexity in $\mathbf{R}^N$, separation theorems, the

Shapley-Folkman Theorem, and the Brouwer Fixed-Point Theorem (including a combinatorial proof of the Brouwer Theorem on the simplex). Although it is not a substitute for a course in real analysis, Part B does provide a useful summary and presents the mathematical issues important to economic theory that are sometimes omitted from a real analysis course.

Like all scientific theories, the theory of general economic equilibrium is a family of "if–then" statements: "If the world looks like this family of assumptions, then here's what the outcome will be." The unifying view of firms and households throughout microeconomic theory is to characterize their behavior as maximization of a criterion function (profit or utility) subject to constraint.

A technical issue that persistently arises is the possibility that those maxima may not exist if constraint sets are unbounded (a budget constraint where some prices are nil or a technology constraint where outputs are limited only by available inputs). When the price of a desirable good is zero, there may be no well-defined value for the demand function at those prices, since the quantity demanded will be arbitrarily large. Nevertheless, it is important that we be able to deal with free goods (zero prices). The classic means of dealing with this issue (Arrow and Debreu [1954]) is to recognize that attainable outputs of the economy are bounded. It is then possible to impose *the modeler's* bounds on individual firms' supplies and households' demands (bounds slightly larger than the bounds naturally arising from the limited production possibilities of the real economy). The economy with modeler-bounded individual opportunities has well-defined maxima for firms and households.

This approach to solving the problem of ill-defined maxima appears completely wrongheaded! The concept of decentralized market allocation using the price system is that *prices* (not the economic modeler) should communicate scarcity and resource constraints to firms and households. Here is the strategy of proof:

> Find a general equilibrium in the model of the economy where firms and households are subject to the *modeler's* bounds.
>
> In equilibrium, the bounds are not binding constraints. The bounds can be deleted and the equilibrium prices of the bounded economy are equilibrium prices of the original economy described without the modeler's bound on individual firm and household behavior.

Part C (Chapters 11 through 14) presents the special case where technology really is bounded. Here, the bounds are not exogenously imposed by the modeler but are supposed to represent the underlying technology. Chapter 11 introduces most of the theory of the firm used throughout the book. Chapter 12 introduces most of the theory of the household (consumer), including derivation of a continuous utility function from the household preference ordering. Chapters 13 and 14 develop Walras's Law and the existence of general equilibrium.

Part D (Chapters 15 to 18) generalizes the results of Part C to the case of unbounded technology. We prove in Chapter 15 that the set of attainable outputs is bounded, using the assumptions of convexity, irreversibility, and no free lunch (no output without input). In Chapters 15 and 16, the modeler's bound on the opportunity sets of firms and households is introduced as a bound tight enough that maximizing behavior is well defined but loose enough that all attainable outputs are (strictly) included in their opportunity sets. Chapter 17 restates Walras's Law in this setting. Chapter 18 presents a proof of the existence of general equilibrium in the artificially bounded economy created in Chapters 15 and 16. That economy is an example of the bounded model of Part C, so the existence of general equilibrium in the artificially bounded economy is merely an application of the existence theorem of Chapter 14 (using the mathematician's trick of reducing the current problem to one previously solved). But an equilibrium is necessarily attainable; the constraint that firm and household behavior lie in the bounded set is not binding in equilibrium. The artificial constraint of modeler-bounded opportunity sets can be removed, and the prices and allocations constitute a general equilibrium for the unconstrained economy. That is the existence of general equilibrium result of Chapter 18.

Chapter 19 (Part E) presents the classic First and Second Fundamental Theorems of Welfare Economics, which describe the relationship of general equilibrium to efficient allocation. Chapter 20 presents the reinterpretation of the model in terms of allocation over time and uncertainty using futures and contingent commodities.

Part F (Chapters 21 and 22) presents the theory of the core of a market economy, the modern counterpart to the Edgeworth box. This includes, in Chapter 22, proof of the classic result that in a large economy individual economic agents have no significant bargaining power, so that a competitive price-taking allocation is sustainable (core convergence). The treatment in Chapter 22 includes the proof of core convergence, using both Debreu-Scarf–style replication of the economy and the Anderson-style treatment using the Shapley-Folkman Theorem.

Throughout Chapters 11 through 18, we use strict convexity of tastes and technology to ensure point-valued demands and supplies. That treatment excludes the set-valued supply-and-demand behavior that can arise from perfect substitutes in consumption or from linear production technologies. In Part G (Chapters 23 to 25), we generalize those results to the case of set-valued demands and supplies. Chapter 23 introduces the mathematics of correspondences: point-to-set mappings. Particularly important in this setting are the continuity concepts and the Kakutani Fixed-Point Theorem. Chapter 24 presents the economic model of firms, households, the market economy, and general equilibrium with (upper hemi-)continuous, convex, set-valued supply and demand. Chapter 25 introduces the approximate equilibrium results associated with bounded scale economies (U-shaped cost curves) and preferences for concentrated consumption. The U-shaped cost curve model is a staple of undegraduate economics, but the more advanced student of general equilibrium is often led to believe that the general equilibrium theory cannot treat this conventional case; Chapter 25 bridges that gap.

The careful reader will note that the preceding outline includes four developments of demand, supply, excess demand, and existence of general equilibrium. Repetition aids comprehension, but isn't that overdoing it? For advanced undergraduates in economics, typically the answer is "no." They generally benefit from seeing the ideas developed in a simple and then a more complex context. For advanced graduate students in economic theory, the answer is probably "yes." These students will want to avoid some repetition to achieve the most complete and general treatment of these classic issues.

How should the reader/student make use of this material without wasting time and attention?

A typical one-semester advanced undergraduate course in mathematical general equilibrium theory would include Chapters 1 through 14 and Chapter 19. A two-semester course would cover the whole book in order, with the possible omission of Part G. A several-week segment on general equilibrium in the graduate core microeconomic theory course would include Chapters 11 to 14 and 19 through 22. A one-semester graduate introduction to general equilibrium theory would include Chapters 10 through 25.

What portions of the book can be omitted without loss of continuity? Which parts are essential?

Part A introduces Robinson Crusoe and the Edgeworth box; it is intended to introduce the concepts of general equilibrium and Pareto efficiency in a simple tractable context. The well-prepared student can skip this material without loss of continuity.

Proofs are provided for most of the mathematical results in the pure mathematics chapters (6 through 9, and 23). The proofs are there because mathematical theory necessarily involves the understanding and development of mathematical results. Nevertheless, the student can – without loss of continuity – skip the proofs in these chapters; only an understanding of the definitions and results is essential. Conversely, the student unfamiliar with real analysis will want to supplement the material in Part B with a sound text in real analysis such as Bartle (1976), Bartle and Sherbert (1992), Bryant (1990), or Rudin (1976). Excellent treatments focusing on mathematics for economic theory include Carter (2001); Corbae, Stinchcombe, and Zeman (2009); and Ok (2007).

Chapters 11 and 12, which introduce the firm and the household, cannot easily be omitted.

Chapters 13 and 14 present Walras's Law and equilibrium in the market economy with bounded technology. The substance of these chapters is repeated in Chapters 17 and 18 in the setting of an economy with unbounded production technology. The student who loathes repetition may wish to skip Chapters 13 and 14 and go on to Chapters 15 through 18 (for the point-valued case) or to Chapters 23 and 24 (for the most general and difficult, set-valued, case).

The student who has completed Part C can, without loss of continuity, skip Part D (Chapters 15 to 18).

Welfare economics – the relationship of equilibrium to efficiency – is a cornerstone of microeconomic theory that recurs throughout the book. Most readers will want to complete Chapter 19. The notion of contingent commodities and Arrow insurance contracts is central to theoretical finance and to applications of the general equilibrium model in macroeconomics. Most readers will want to review Chapter 20.

Parts E, F, and G are virtually independent of one another. They can be read in any order or combination.

Notation

Vectors, coordinates. Most variables treated in this book are vectors in $\mathbf{R}^N$, real N-dimensional space. For $\boldsymbol{x} \in \mathbf{R}^N$, we will typically denote the coordinates of x by subscripts. Thus,

$$x = (x_1, x_2, x_3, \ldots, x_{N-1}, x_N).$$

We will generally designate ownership or affiliation by superscripts (with rare exceptions). Thus, x^i will be household i's consumption vector and y^j will be firm j's production vector.

Vector inequalities. For two N-dimensional vectors, $\boldsymbol{x}$ and $\boldsymbol{y} \in \mathbf{R}^N$, inequalities can be read in the following ways: $x \geq y$ means that for all $k = 1, 2, \ldots, N$, $x_k \geq y_k$; the weak inequality holds coordinatewise. The expression $x > y$ means $x_k \geq y_k$, $k = 1, 2, \ldots, N$, but $x \neq y$. $x \gg y$ means that for all $k = 1, 2, \ldots, N$, $x_k > y_k$; a strict inequality holds coordinatewise.

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Preface to the second edition

Like the first edition of this work, the second edition begins with a celebration. In 2005 at the University of California at Berkeley there was an enthusiastic conference celebrating the life and work of our late colleague Gerard Debreu. Professors, researchers, and students gathered literally from all over the world. For three days and nights, papers were presented, reminiscences shared, testimonials and tributes spoken. Gerard Debreu – half of the Arrow-Debreu team – had reshaped our field and created the specialty we loved. Prof. Hugo Sonnenschein remarked:

> The Arrow-Debreu model, as communicated in *Theory of Value* changed basic thinking, and it quickly became the standard model of price theory. It is the "benchmark" model . . . it was no longer "as it is" in Marshall, Hicks, and Samuelson; rather it became "as it is" in *Theory of Value*.

That's why the present volume appeared: to make *Theory of Value* more easily accessible to a wide audience, because the Arrow-Debreu model is the standard of the field. We who work the field should understand it well.

For the past decade, students and colleagues have remarked on the first edition of this book: appreciating, criticizing, suggesting revisions and corrections.

It is a pleasure to acknowledge two distinctive contributions. Colleagues at the University of Copenhagen have been extraordinarily helpful. Professor Peter Sørensen and the late Professor Birgit Grodal both went over the entire volume, making immensely useful suggestions and corrections. Professor Sørensen prepared a very detailed richly scholarly thoughtful corrigendum, emphasizing mathematical precision and elegance.

Birgit prepared a large and varied family of notes, covering precision, mathematical elegance, and taste in presentation. During four decades, Birgit was a vibrant presence and a frequent visitor in California. In this volume, her contributions to clear and precise expression are a living presence. It is hard to believe that a woman of such intensity is gone.

I – and this volume's readers – owe Peter and Birgit fulsome thanks.

Many friends, colleagues, and students have left their marks on the second edition. The students come from UC Berkeley, UC Santa Barbara, UC San Diego, and European University Institute. These contributors include an anonymous referree, Robert Anderson, Phillip Babcock, Michael Bacci, Blake Barton, Aislinn Bohren, Marika Cabral, Tolga Cenesizoglu, Karim Chalak, Christopher Chambers, Qun Del-Homme, Susana Ferreira Martinez, R. Garcia-Cobian Yonatan Harel, Khashayar Khorasani, Young Do Kim, David Kovo, Troy Kravitz, Bernhard Lamel, David Miller, George Monokroussos, William Nelson, Augusto Nieto Barthaburu, Thien T. Nguyen, Tatsuyoshi Okimoto, Lindsay Oldenski, Luis Pinto, Adam Sanjurjo, Greg Scott, Jason Shafrin, Joel Sobel, Steven Sumner, Leslie Wallace, and Jonathan Weare. Readers of this volume benefit from their contributions. They and a generation of undergraduate and graduate students have refined this book.

Remaining errors are, of course, my own.

Ross M. Starr
La Jolla, California
November 2009

Preface to the first edition

In June 1993, a remarkable birthday party took place at CORE (Center for Operations Research and Econometrics) of the Université Catholique de Louvain in Louvain-la-neuve, Belgium. The gathering celebrated the fortieth anniversary of one of the great achievements of modern economic theory: the mathematical theory of general economic equilibrium. For several days and nights, hundreds of professors, researchers, and students from around the world presented papers, discussions, and reminiscences of the specialty they had pursued for years. At the center of the celebration were the modern founders of the field: Professors Kenneth Arrow (Nobel laureate), Gerard Debreu (Nobel laureate), and Lionel McKenzie.

This book presents the cause of that celebration, the field of mathematical general equilibrium theory. The approach of the field is revolutionary: It fundamentally changes your way of thinking. Once you see things this way, it is hard to conceive of them otherwise.

This book reflects the experience of students at Yale University, University of California at Davis, University of California at San Diego, and the Economics Training Center of the People's University of China (Renda) in Beijing. They deserve my thanks for their patience, the stimulus they provided for this book, and their contributions to it. A number of students and colleagues have reviewed portions of the manuscript. I owe thanks to Manfred Nermuth for critical advice and to Nelson Altamirano, Elena Bisagni, Peter Reinhard Hansen, Dong Heon Kim, Bernhard Lamel, Martin Meurers, Elena Pesavento, and Heather Rose who helped by catching typographical and technical errors. Cameron Odgers discovered more substantial oversights. Remaining errors are my responsibility. Illustrations were prepared by Nic Paget-Clarke.

It is a pleasure to acknowledge two very special debts. My wife Susan has lived with this book as long as I have; she is an unfailing source of strength. My friend and

mentor Kenneth Arrow is the intellectual father of generations of students; it is an honor to be counted among them. This volume is intended to further communicate some of his contributions.

Ross M. Starr
La Jolla, California
June 1996

Table of notation

$\forall$	"for all"
#	denotes number of elements in a set
$\exists$	"there exists"
$\ni$	"such that" or "includes as an element"
$<$	"less than"
$=$	"equals"
$>$	"greater than"
$\leq$	"less than or equal to," applies coordinatewise to vectors
$\succeq$	quasi-order symbol
$\succeq_i$	preferred or indifferent by household i's preferences
$\preceq_i$	inferior or indifferent by household i's preferences
$\succ_i$	strictly preferred by household i's preferences
$\prec_i$	strictly inferior by household i's preferences
∞	infinity, without bound
$\rightarrow$	approaches as a limit
$\geq$	"greater than or equal to," applies coordinatewise to vectors
∂	partial derivative
$\cdot, \cdot$	space holder for argument of a function
$\cdot$	raised dot, denotes product or scalar product
$\times$	denotes Cartesian product (when placed between the names of two sets)
$\neq$	is not equal to
$\equiv$	is identically (or by definition) equal to
$\|, \|\|$	denotes length measure, written as $\|x\|$ or $\|\|x\|\|$
$\cap$	set intersection
Δ	capital Greek delta, denotes closed ball of radius C (space of possible excess demands, Chapter 24)
$\cup$	set union

$\not\subset$	is not a subset of
ϕ	empty set, null set
Φ	capital Greek phi, denotes price and quantity adjustment correspondence from the set $\Delta \times P$ into itself (Chapter 24)
Γ^i	set of preferred net trades for households of type i (Chapter 22)
Γ	convex hull over all household types i of the sets Γ^i, aggregate average preferred net trade set (Chapter 22)
$\subset, \subseteq$	set inclusion, subset
$\in$	set inclusion, is an element of
$\notin$	is not an element of
ν	Greek nu, running index on sequences
Π	capital Greek pi, denotes multiple product
$\pi^j(p)$	profits of firm j at prices p based on production technology Y^j (Y^j may be unbounded)
$\tilde{\pi}^j(p)$	profits of firm j at prices p based on (bounded) production technology $\mathcal{Y}^j$ or $\tilde{Y}^j$
ρ	price adjustment mapping from Δ to P (Chapter 24)
Ω	capital Greek omega, sum over all households of the union of household preferred net trade set and $\{0\}$ (Chapter 22)
$\Leftrightarrow$	"if and only if," denotes a necessary and sufficient condition
$\sum$	capital Greek sigma, denotes repeated summation
$\{\}$	braces or curly brackets, denote a set or an algebraic quantity
$[\,]$	bracket, denotes algebraic quantity
$(\,)$	parentheses, denotes algebraic quantity
$+$	plus sign, denotes scalar, vector, or set addition
$-$	minus sign, denotes scalar, vector, or set subtraction
$A^i(x)$	upper contour set, set of points in X^i preferred or indifferent to x
$B^i(p)$	budget set of household i at prices p
$\tilde{B}^i(p)$	bounded budget set of household i at prices p
c	large positive real number, chosen to exceed the Euclidean length of any attainable production or consumption bundle, upper bound on length of elements in $\tilde{Y}^j, \tilde{B}^i(p)$
C	very large positive real number, upper bound on length of elements in Δ, strict upper bound on Euclidean length of excess demands in $\tilde{Z}(p)$
$\mathrm{con}(\cdot)$	denotes convex hull
$D^i(p)$	demand function (or correspondence – Chapter 24) of household i evaluated at p
$\tilde{D}^i(p)$	bounded demand function (or correspondence – Chapter 24) of household i at p

$f(\,)$	typical functional notation
F	set of firms (finite)
$G^i(x)$	lower contour set, set of elements of X^i inferior or indifferent to x under i's preferences
h, i	representative households, elements of H
H	set of households (finite)
j	representative firm, element of F
k	representative commodity, $k = 1, 2, \ldots, N$
M	maximum over all commodities of the sum of the N largest household initial endowments of each commodity (Chapter 22)
$M^i(p)$	value of budget of household i at prices p in an economy with technology sets Y^j
$\tilde{M}^i(p)$	value of budget of household i at prices p in an economy with technology sets $\mathcal{Y}^j$ or $\tilde{Y}^j$
N	number of commodities, finite positive integer
n	running index on a sequence or commodities, $n = 1, 2, 3, \ldots$
p	price vector
P	price space, unit simplex in $\mathbf{R}^N$
q	running index on individuals in a replica economy (Chapter 22)
Q	number of replications in a replica economy (Chapter 22)
$\mathbf{R}$	set of real numbers
$\mathbf{R}^N$	real N-dimensional Euclidean space
$\mathbf{R}^N_+$	nonnegative quadrant (orthant) of $\mathbf{R}^N$
$\mathbf{R}^N_{++}$	strictly positive quadrant (orthant) of $\mathbf{R}^N$
$\mathbf{R}^N_-$	nonpositive quadrant (orthant) of $\mathbf{R}^N$
S	N-simplex
S, T	representative sets
$S^j(p)$	supply function (or correspondence – Chapter 24) of firm j based on technology set Y^j
$\tilde{S}^j(p)$	supply function (or correspondence – Chapter 24) of firm j based on (bounded) technology set $\mathcal{Y}^j$ or $\tilde{Y}^j$
$u^i()$	household i's utility function
x	representative commodity bundle
X^i	household i's possible consumption set
X	aggregate possible consumption set, sum of sets X^i
$\mathcal{Y}^j$	firm j's production technology in a model of bounded firm technology sets (Chapters 11 to 14)
$\mathcal{Y}$	aggregate (sum of individual firm sets) technology set in a model of bounded firm technology sets (Chapters 11 to 14)

Y^j	firm j's technology set (may be unbounded; Chapters 15 to 18 and 24)
Y	aggregate (possibly unbounded) technology set, sum of Y^js
$\tilde{Y}^j$	firm j's artificially bounded technology set; intersection of Y^j with a closed ball of radius c (Chapters 15 to 18 and 24)
$\tilde{Y}$	aggregate artificially bounded technology set; sum of $\tilde{Y}^j$s (Chapters 15 to 18 and 24)
$Z(p)$	excess demand function (or correspondence – Chapter 24) of an unbounded economy (Chapters 15 to 18 and 24)
$\tilde{Z}(p)$	excess demand function (or correspondence – Chapter 24) of an economy subject to exogenous or artificial bounds on demand and supply functions and correspondences

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(P.I)	$\mathcal{Y}^j$ *is convex for each* $j \in F$.
(P.II)	$0 \in \mathcal{Y}^j$ *for each* $j \in F$.
(P.III)	$\mathcal{Y}^j$ *is closed for each* $j \in F$.
(P.IV)	(a) *if* $y \in Y$ *and* $y \neq 0$, *then* $y_k < 0$ *for some* k. (b) *if* $y \in Y$ *and* $y \neq 0$, *then* $-y \notin Y$.
(P.V)	*For each* $j \in F$, $\mathcal{Y}^j$ *is strictly convex.*
(P.VI)	$\mathcal{Y}^j$ *is a bounded set for each* $j \in F$.
(C.I)	X^i *is closed and nonempty.*
(C.II)	$X^i \subseteq \mathbf{R}^N_+$. X^i *is unbounded above, that is, for any* $x \in X^i$ *there is* $y \in X^i$ *so that* $y > x$, *that is, for* $n = 1, 2, \ldots, N$, $y_n \geq x_n$ *and* $y \neq x$.
(C.III)	X^i *is convex.*
(C.IV)	*(Non-Satiation) Let* $x \in X^i$. *Then there is* $y \in X^i$ *so that* $y \succ_i x$.
(C.IV*)	*(Weak Monotonicity) Let* $x, y \in X^i$ *and* $x >> y$. *Then* $x \succ_i y$.
(C.V)	*(Continuity) For every* $x^\circ \in X^i$, *the sets* $A^i(x^\circ) = \{x \mid x \in X^i, x \succeq_i x^\circ\}$ *and* $G^i(x^\circ) = \{x \mid x \in X^i, x^\circ \succeq_i x\}$ *are closed.*
(C.VI)(C)	*(Convexity of Preferences)* $x \succ_i y$ *implies* $((1-\alpha)x + \alpha y) \succ_i y$, *for* $0 < \alpha < 1$.
(C.VI)(SC)	*(Strict Convexity of Preferences): Let* $x \succeq_i y$ *(note that this includes* $x \sim_i y$*),* $x \neq y$, *and let* $0 < \alpha < 1$. *Then,* $\alpha x + (1-\alpha)y \succ_i y$.
(C.VII)	*For all* $i \in H$, $\tilde{M}^i(p) > \inf_{x \in X^i \cap \{x \mid \lvert x \rvert \leq c\}} p \cdot x$ *for all* $p \in P$.