Advanced Dynamics

Advanced Dynamics is a broad and detailed description of the analytical tools of dynamics as used in mechanical and aerospace engineering. The strengths and weaknesses of various approaches are discussed, and particular emphasis is placed on learning through problem solving.

The book begins with a thorough review of vectorial dynamics and goes on to cover Lagrange's and Hamilton's equations as well as less familiar topics such as impulse response, and differential forms and integrability. Techniques are described that provide a considerable improvement in computational efficiency over the standard classical methods, especially when applied to complex dynamical systems. The treatment of numerical analysis includes discussions of numerical stability and constraint stabilization. Many worked examples and homework problems are provided. The book is intended for use in graduate courses on dynamics, and will also appeal to researchers in mechanical and aerospace engineering.

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Preface

This is a dynamics textbook for graduate students, written at a moderately advanced level. Its principal aim is to present the dynamics of particles and rigid bodies in some breadth, with examples illustrating the strengths and weaknesses of the various methods of dynamical analysis. The scope of the dynamical theory includes both vectorial and analytical methods. There is some emphasis on systems of great generality, that is, systems which may have nonholonomic constraints and whose motion may be expressed in terms of quasi-velocities. Geometrical approaches such as the use of surfaces in n-dimensional configuration and velocity spaces are used to illustrate the nature of holonomic and nonholonomic constraints. Impulsive response methods are discussed at some length.

Some of the material presented here was originally included in a graduate course in computational dynamics at the University of Michigan. The ordering of the chapters, with the chapters on dynamical theory presented first followed by the single chapter on numerical methods, is such that the degree of emphasis one chooses to place on the latter is optional. Numerical computation methods may be introduced at any point, or may be omitted entirely.

The first chapter presents in some detail the familiar principles of Newtonian or vectorial dynamics, including discussions of constraints, virtual work, and the use of energy and momentum principles. There is also an introduction to less familiar topics such as differential forms, integrability, and the basic theory of impulsive response.

Chapter 2 introduces methods of analytical dynamics as represented by Lagrange's and Hamilton's equations. The derivation of these equations begins with the Lagrangian form of d'Alembert's principle, a common starting point for obtaining many of the principal forms of dynamical equations of motion. There are discussions of ignorable coordinates, the Routhian method, and the use of integrals of the motion. Frictional and gyroscopic forces are studied, and further material is presented on impulsive systems.

Chapter 3 is concerned with the kinematics and dynamics of rigid body motion. Dyadic and matrix notations are introduced. Euler parameters and axis-and-angle variables are used extensively in representing rigid body orientations in addition to the more familiar Euler angles. This chapter also includes material on constrained impulsive response and input-output methods.

The theoretical development presented in the first three chapters is used as background for the derivations of Chapter 4. Here we present several differential methods which have the advantages of simplicity and computational efficiency over the usual Lagrangian methods in the analysis of general constrained systems or for systems described in terms of quasi-velocities. These methods result in a minimum set of dynamical equations which are computationally efficient. Many examples are included in order to compare and explain the various approaches. This chapter also presents detailed discussions of constraints and energy rates by using velocity space concepts.

Chapter 5 begins with a derivation of Hamilton's principle in its holonomic and nonholonomic forms. Stationarity questions are discussed. Transpositional relations are introduced and there follows a further discussion of integrability including Frobenius' theorem. The central equation and its explicit transpositional form are presented. There is a comparison of integral methods by means of examples.

Chapter 6 presents some basic principles of numerical analysis and explains the use of integration algorithms in the numerical solution of differential equations. For the most part, explicit algorithms such as the Runge–Kutta and predictor–corrector methods are considered. There is an analysis of numerical stability of the integration methods, primarily by solving the appropriate difference equations, but frequency response methods are also used. The last portion of the chapter considers methods of representing kinematic constraints. The one-step method of constraint stabilization is introduced and its advantages over standard methods are explained. There is a discussion of the use of energy and momentum constraints as a means of improving the accuracy of numerical computations.

A principal objective of this book is to improve the problem-solving skills of each student. Problem solving should include not only a proper formulation and choice of variables, but also a directness of approach which avoids unnecessary steps. This requires that the student repeatedly attempt the solution of problems which may be kinematically complex and which involve the application of several dynamical principles. The problems presented here usually have several parts that require more than the derivation of the equations of motion for a given system. Thus, insight is needed concerning other dynamical characteristics. Because of the rather broad array of possible approaches presented here, and due in part to the generally demanding problems, a conscientious student can attain a real perspective of the subject of dynamics and a competence in the application of its principles.

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