

## An Introduction to the Philosophy of Mathematics

This introduction to the philosophy of mathematics focuses on contemporary debates in an important and central area of philosophy. The reader is taken on a fascinating and entertaining journey through some intriguing mathematical and philosophical territory. Topics include the realism/anti-realism debate in mathematics, mathematical explanation, the limits of mathematics, the significance of mathematical notation, inconsistent mathematics, and the applications of mathematics. Each chapter has a number of discussion questions and recommended further reading from both the contemporary literature and older sources. Very little mathematical background is assumed, and all of the mathematics encountered is clearly introduced and explained using a wide variety of examples. The book is suitable for an undergraduate course in philosophy of mathematics and, more widely, for anyone interested in philosophy and mathematics.

MARK COLYVAN is Professor of Philosophy and Director of the Sydney Centre for the Foundations of Science at the University of Sydney. He is the co-author (with Lev Ginzburg) of *Ecological Orbits: How Planets Move and Populations Grow* (2004) and author of *The Indispensability of Mathematics* (2001).

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