RATIONAL HERDS

Economic Models of Social Learning

Penguins jumping off a cliff, economic forecasters predicting a recovery in the business cycle, financial advisors for the stock market speculating against a currency, and farmers using new seeds in India are all practicing social learning. Such learning from the behavior of others can lead to herds, crashes, and booms. These issues have become, over the last ten years, an exciting field of research in theoretical and applied economics, finance, and other social sciences. This book provides both an informal introduction and in-depth insights into the most recent advances.

The properties of social learning depend on the context in which learning and actions take place. Each chapter is devoted to a separate issue: Individuals learn from the observations of actions, from the outcomes of these actions, and from what others say. They may delay or make an immediate decision; they may compete against others or gain from cooperation; they make decisions about capital investment, crop choices, and financial investments. The book highlights the similarities and the differences between the various cases. A recurrent theme is that society may learn more if individuals are less than perfectly rational in their interpretation of others’ behavior.

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Rational Herds

ECONOMIC MODELS OF SOCIAL LEARNING

CHRISTOPHE P. CHAMLEY
To the memory of my father
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Preface

Learning by individuals from the behavior of others and imitation pervade the social life. Issues related to such learning have been debated since the beginning of the social sciences, more than a century ago. However, in the last ten years or so they have stimulated a revival and very active research in economics, with extensions to other “human sciences” (sociology, psychology, political science). The purpose of this book is to give an account of these studies. Perhaps it will induce others to enter the field and provide them with some training.

The setting is one of rational agents with limited information who share that information with others through their actions. The properties of the learning process are analyzed from a theoretical point of view, but some empirical studies are discussed in relation to the theoretical results.

Special attention is devoted to the pathologies of social learning by rational agents. Herds appear to be obvious examples of failures of social learning. Indeed, herds, fads, bubbles, crashes, and booms are cited as proofs of the irrationality of individuals. However, most of these colorful events will appear in the models of rational agents studied in this book.

The assumption of rationality may seem a bit narrow. Indeed, at this stage of the evolution of research, the concept of rationality itself is beginning to be seriously investigated. In this book, the usefulness of the assumption goes beyond the standard “benchmark” justification: a recurrent issue will be that despite the rationality of individual behavior, and often because of that rationality, the process of social learning may be inefficient or fail completely. The results hint at some social benefits of nonrational behavior by individuals, but that topic is beyond the scope of the present work.

Readers

The book can be read at two levels: the first, nontechnical and the second, more formal. Both levels will demand some intellectual concentration, however.

Each chapter is devoted to a specific issue. Examples are the various channels for the transmission of information (actions, outcomes of the actions, words, and so on), the coordination of agents, and price fluctuations in a financial market. For each chapter, the results and the methodology are described in an informal introduction.
Preface

In some of the main chapters, a first section presents a reduced model that exhibits most of the essential properties. These parts of the book should be accessible to a wide audience of readers who are interested in the issues and are prepared to follow logical arguments, sometimes with a bit of formalism.

For graduate students and researchers in social sciences (mainly economics and finance, but also other social sciences), the book provides an introduction to the technical literature. The main subjects have been selected with a personal bias, and are presented in their essence. The models are analyzed rigorously without some of the baggage that is sometimes required by professional journals. In a number of cases, the analysis had to be adapted, or even rewritten, for that purpose. The techniques do not use highbrow mathematics. Most of the model manipulations use first principles.

The models are simple, but a major goal is to give the student sufficient understanding of the internal structure of these models to develop his own intuition about their “deep” properties. A model is not an exercise with cute results or a quick “validation” of some story, but it is a tool to make an argument that goes beyond its technical boundaries. It is my view that the understanding of these properties cannot be grasped from a survey, and that, in the field of social learning, it takes a considerable amount of time to develop this understanding if one has to read the technical literature. The purpose of the book is to shorten that time for the student before he goes to the frontline papers and does research about theoretical or empirical topics.

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