Economic Theory and Global Warming

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1. INTRODUCTION

The atmospheric concentration of greenhouse gases, particularly of carbon dioxide, has been increasing since the Industrial Revolution, and this has been occurring at an accelerated rate in the last three decades. As described in detail in the Introduction, it is estimated that, if the emission of carbon dioxide and other greenhouse gases and the disruption of tropical rain forests were to continue at the present pace, global average air surface temperature toward the end of the twenty-first century would be 3–6°C higher than the level prevailing before the Industrial Revolution, resulting in drastic changes in climatic conditions and accompanying disruption of the biological and ecological environments. In view of the significant impacts such climatic changes would exert upon human life, a large number of policy measures and institutional arrangements have been proposed to stabilize atmospheric concentrations of greenhouse gases effectively.

Among them, the institutional arrangements of carbon taxes and markets for tradable emission permits have attracted widespread attention – particularly among economists such as Ingham, Maw, and Ulph (1974), Baumol and Oates (1988), Grubb and Sibenius (1992), Whally and Wigle (1991), Hoel (1991, 1992), Pearce (1991), and Rose and Stevens (1993). Theoretical analyses have been developed, for example, by Bergstrom, Blume, and Varian (1986), Copeland and Taylor (1986, 1995), Poterba (1991), and Uzawa (1991, 1992a, 1993, 1995) of carbon taxes and by Tietenberg (1985, 1992), Barrett (1990), Grubb (1990), Barrett et al. (1992), Bertram (1992), and Larsen and Shah (1992, 1994) of tradable emission permits.

In this chapter and Chapters 2 and 3, we address the theoretical analysis of implications for an allocative mechanism of carbon taxes and the market for tradable emission permits. Our analysis is strictly confined to the realm of static analysis, leaving a full dynamic analysis for later chapters.

In this chapter, we are particularly concerned with the proportional carbon tax schemes under which the tax rate is made proportional either to the level of the national income in the countries where greenhouse gases are emitted or to the sum of the national incomes of all countries in the world. Welfare implications of these institutional arrangements will be examined in detail in Chapter 2.

2. THE MODEL OF GLOBAL WARMING IN THE STATIC CONTEXT

We postulate that each greenhouse gas is so measured as to equate the greenhouse effect with the activity of carbon dioxide (CO_2). Hence, in our model, carbon dioxide is the only chemical agent that has a greenhouse effect. In the static context with which this and Chapters 2 and 3 are concerned, we postulate that the welfare effect of global warming is measured in relation to the total quantity of CO_2 emitted annually into the atmosphere, where the dependency upon the stock of CO_2 accumulated in the atmosphere is not explicitly brought out. This may be regarded as a valid hypothesis because we are concerned with the problem of global warming from the short-run point of view, where the stock of CO_2 accumulations in the atmosphere remains constant.

We consider the world economy to consist of a finite number of individual countries that share the earth's atmosphere as a common environment. Each country is generically denoted by $\nu = 1, \dots, n$.

The behavioral characteristics of individual countries are expressed in the aggregate by two representative economic agents: the consumers, who are concerned with the choice of economic activities related to consumption, on the one hand, and the producers, who are in charge of the choice of technologies and levels of productive activities on the other.

Specifications for Utility Functions

We assume that the economic welfare of each country ν is expressed by a preference ordering that is represented by the utility function

$$u^\nu = u^\nu(c^\nu, a),$$

where $c^\nu = (c_j^\nu)$ is the vector of goods consumed in country ν; j generically refers to consumption goods $(j = 1, \ldots, J)$, and a is the total quantity of CO_2 annually introduced into the atmosphere measured in tons of the carbon content of CO_2 emitted into the atmosphere; that is,

$$a = \sum_\nu a^\nu,$$

where a^ν is the amount of CO_2 emitted into the atmosphere by country ν in relation to its productive activities.

The phenomenon of global warming is expressed by the postulate that the welfare level of each country is influenced by the aggregate of CO_2 emissions of all countries in the world. If a country is relatively small, then its CO_2 emissions may have only a negligible effect on global warming. However, it would be greatly influenced by the CO_2 emissions of large countries such as the United States or Japan.

For each country ν, we assume that the utility function $u^\nu(c^\nu, a)$ satisfies the following neoclassical conditions:

(U1) $u^\nu(c^\nu, a)$ is defined, positive, continuous, and continuously twice-differentiable for all $(c^\nu, a) \geqq (0, 0)$.

(U2) Marginal utilities are positive for the consumption of private goods c^ν, but CO_2 emissions a have a negative marginal utility:

$$u^\nu_{c^\nu}(c^\nu, a) > 0, \quad u^\nu_a(c^\nu, a) < 0, \quad \text{for all } (c^\nu, a) \geqq (0, 0).$$

(U3) Marginal rates of substitution between any pair of consumption goods are diminishing, or, more specifically, $u^\nu(c^\nu, a)$ is strictly quasi-concave with respect to c^ν for any given $a \geqq 0$.

(U4) $u^\nu(c^\nu, a)$ is homogeneous of order 1 with respect to c^ν; that is,

$$u^\nu(tc^\nu, a) = tu^\nu(c^\nu, a), \quad \text{for all } t \geqq 0, \quad c^\nu \geqq 0.$$

We also assume that utility functions $u^\nu(c^\nu, a)$ are strongly separable with respect to c^ν and a; that is,

$$u^\nu(c^\nu, a) = \varphi(a)u^\nu(c^\nu). \tag{1}$$

The concept of separability of utility functions was analyzed in detail by Leontief (1947), Strotz (1957), Gorman (1959), and Houthakker (1960), among others. The concept of separability being used here corresponds to that of strong separability, as introduced in Goldman and Uzawa (1964).

The function $\varphi(a)$ expresses the extent to which people are adversely affected by global warming. It may be referred to as the *impact index* of global warming.

The large number of attempts that have been made to measure the value of environmental quality – particularly by McKenzie (1983), Randall and Stoll (1983), Bishop and Woodward (1995) – also have pertinent implications for the measurement of the impact index of global warming.

In this chapter as well as in the following chapters, except for Chapters 2 and 7, we assume that the impact index of global warming $\varphi(a)$ is identical for all countries involved. We assume that the impact index function $\varphi(a)$ satisfies the following conditions:

$$\varphi(a) > 0, \quad \varphi'(a) < 0, \quad \varphi''(a) < 0 \quad \text{for all } 0 < a < \hat{a},$$

where $\hat{a}$ is the critical level of CO_2 emissions. The critical level of CO_2 emissions $\hat{a}$ is the level of the annual rate of CO_2 emissions that, if CO_2 emissions were continued at a level higher than $\hat{a}$ for a long period, would produce drastic changes in climatic conditions and inflict irrevocable damage on the global environment.

With regard to global warming, the impact index function $\varphi(a)$ of the following form is often postulated, as in Uzawa (1991, 1992a):

$$\varphi(a) = (\hat{a} - a)^\beta, \quad 0 < a < \hat{a}, \tag{2}$$

where $\hat{a}$ is the critical level of CO_2 emissions, and β is the sensitivity parameter, $0 < \beta < 1$.

The relative rate of the marginal change in the impact index due to the marginal increase in the atmospheric emission of CO_2 is defined by

$$\tau(a) = -\frac{\varphi'(a)}{\varphi(a)},$$

which will play a crucial role in our analysis of global warming. It may be referred to as the *impact coefficient* of global warming. It is easily seen that the impact coefficient function $\tau(a)$ satisfies the following conditions:

$$\tau(a) > 0, \quad \tau'(a) - [\tau(a)]^2 > 0.$$

With respect to the impact index function $\varphi(a)$ of the form (2), the impact coefficient $\tau(a)$ is given by

$$\tau(a) = -\frac{\beta}{\hat{a} - a}.$$

The neoclassical conditions (U1–4) for the utility function $u^\nu(c^\nu, a)$ of each country ν are rephrased for the utility function $u^\nu(c^\nu)$ as the following conditions:

(U1′) $u^\nu(c^\nu)$ is defined, positive, continuous, and continuously twice-differentiable, respectively for all $c^\nu \geqq 0$.

(U2′) Marginal utilities are positive for the consumption of private goods $c^\nu = (c_j^\nu)$:

$$u_{c^\nu}^\nu(c^\nu) > 0, \quad \text{for all } c^\nu \geqq 0.$$

(U3′) $u^\nu(c^\nu)$ is strictly quasi-concave with respect to $c^\nu \geqq 0$; that is, for any pair of vectors of consumption, c_0^ν, c_1^ν, such that $c_0^\nu \neq c_1^\nu$,

$$u^\nu\left((1-t)c_0^\nu + tc_1^\nu\right) < (1-t)u^\nu\left(c_0^\nu\right) + tu^\nu\left(c_1^\nu\right), \quad \text{for all } 0 < t < 1.$$

(U4′) $u^\nu(c^\nu)$ is homogeneous of order 1 with respect to c^ν:

$$u^\nu(tc^\nu) = tu^\nu(c^\nu), \quad \text{for all } t \geqq 0, c^\nu \geqq 0.$$

We will frequently make use of the Euler identity:

$$u^\nu(c^\nu) = u_{c^\nu}^\nu(c^\nu)c^\nu, \quad \text{for all } c^\nu \geqq 0.$$

NOTE. In the analysis of global warming that will be developed in this book – except for Chapters 2, 5, and 7 – a central role will be played by the impact coefficient function $\tau^\nu(a)$, not by the impact index function $\varphi^\nu(a)$; thus, one may want to work on the premises, that the impact coefficient functions $\tau^\nu(a)$ rather than the impact index functions $\varphi^\nu(a)$ are identical for all countries involved. However, if one assumes that the impact coefficient functions $\tau^\nu(a)$ are identical for all countries involved, then an impact index function $\varphi(a)$ and a set of positive numbers $\beta^\nu > 0$ exist such that

$$\varphi^\nu(a) = \beta^\nu \varphi(a), \quad \text{for all } a > 0 \quad (\nu = 1, \ldots, n).$$

Hence, working with different impact index functions $\varphi^\nu(a)$ with an identical impact coefficient function $\tau(a)$ gives one only a spurious feeling of generality. We might as well assume from the beginning that the impact index function is the same for all countries involved.

The Consumer Optimum

Suppose the world markets for produced goods are perfectly competitive and prices of goods are denoted by an J-dimensional vector $p = (p_1, \ldots, p_j)$. Considering the possibility of zero prices for some goods, one assumes price vectors p to be nonzero, nonnegative: $p \geq 0$; that is, $p_j \geqq 0$ for all j, and $p_j > 0$ for at least one j.

Suppose national income of country ν in units of world prices is given by y^ν. Then, the consumers in country ν would choose the vector of consumption c^ν that maximizes country ν's utility function

$$u^\nu(c^\nu, a) = \varphi(a) u^\nu(c^\nu)$$

subject to the budget constraints

$$pc^\nu = y^\nu, \; c^\nu \geqq 0.$$

The optimum vector of consumption c^ν is characterized by the following marginality conditions:

$$\varphi(a) u^\nu_{c^\nu}(c^\nu) = \lambda^\nu p,$$

where λ^ν is the Lagrangian unknown associated with the budgetary constraint. The Lagrangian unknown λ^ν is nothing but the marginal utility of income y^ν.

Specifications for Production Possibility Sets

The conditions concerning the production of goods in each country ν are specified by the production possibility set T^ν that summarizes the technological possibilities and organizational arrangements for country ν; the endowments of factors of production available in country ν are given.

We assume that there are a finite number of factors of production that are essentially needed in the production of goods. They are generically denoted by $\ell\,(\ell = 1, \ldots, L)$. Without loss of generality, we may assume that the factors of production needed in productive activities are the same for all countries involved.

The endowments of factors of production available in each country ν are expressed by an L-dimensional vector $K^\nu = (K_1^\nu, \ldots, K_L^\nu)$. We assume that each country ν is endowed with a positive quantity of at least one factor of production:

$$K^\nu \geq 0; \text{ that is, } K_j^\nu \geqq 0, \text{ for all } j, \text{ and } K_j^\nu > 0, \quad \text{for at least one } j.$$

In each country ν, the minimum quantities of factors of production required to produce goods by the vector of production $x^\nu = (x_1^\nu, \ldots, x_L^\nu)$ with CO_2 emissions at the level a^ν are specified by an L-dimensional vector-valued function:

$$f^\nu(x^\nu, a^\nu) = \big(f_1^\nu(x^\nu, a^\nu), \ldots, f_L^\nu(x^\nu, a^\nu)\big).$$

We assume that marginal rates of substitution between the production of goods and the emission of CO_2 are smooth and diminishing, that there are always trade-offs between the production of goods and the emission of CO_2, and that the conditions of constant returns to scale prevail. That is, we assume

(T1) $f^\nu(x^\nu, a^\nu)$ are defined, positive, continuous, and continuously twice-differentiable for all $(x^\nu, a^\nu) \geqq 0$;

(T2) $f_{x^\nu}^\nu(x^\nu, a^\nu) > 0,\ f_{a^\nu}^\nu(x^\nu, a^\nu) \leqq 0, \quad$ for all $(x^\nu, a^\nu) \geqq 0$;

(T3) $f^\nu(x^\nu, a^\nu)$ are strictly quasi-convex with respect to (x^ν, a^ν) for all $(x^\nu, a^\nu) \geqq 0$;

(T4) $f^\nu(x^\nu, a^\nu)$ are homogeneous of order 1 with respect to (x^ν, a^ν); that is,

$$f^\nu(tx^\nu, ta^\nu) = tf^\nu(x^\nu, a^\nu), \quad \text{for all } t \geqq 0, (x^\nu, a^\nu) \geqq 0.$$

From the constant returns-to-scale conditions (T4), we have the Euler identity

$$f^\nu(x^\nu, a^\nu) = f_{x^\nu}^\nu(x^\nu, a^\nu)x^\nu + f_{a^\nu}^\nu(x^\nu, a^\nu)a^\nu, \quad \text{for all } (x^\nu, a^\nu) \geqq 0.$$

The production possibility set of each country ν, T^ν, is composed of all combinations (x^ν, a^ν) of vectors of production x^ν and CO_2 emissions a^ν that are possibly produced with the organizational arrangements and technological conditions in country ν and the given endowments of factors of production K^ν of country ν. Hence, it may be expressed as

$$T^\nu = \{(x^\nu, a^\nu): (x^\nu, a^\nu) \geqq 0,\ f^\nu(x^\nu, a^\nu) \leqq K^\nu\}.$$

Postulates (T1–3) imply that the production possibility set T^ν is a closed, convex set of $J+1$-dimensional vectors (x^ν, a^ν).

The Producer Optimum

As in the case of the consumer optimum, prices of goods on the world market are denoted by a price vector $p = (p_1, \ldots, p_J)$. Suppose that the carbon taxes at the rate of θ^ν are levied on the emission of CO_2 in country ν. Carbon tax rate θ^ν is assumed to be nonnegative: $\theta^\nu \geqq 0$; thus the case of the laissez faire regime ($\theta^\nu = 0$) is not excluded.

Then the producers in country ν would choose those combinations (x^ν, a^ν) of vectors of production x^ν and CO_2 emissions a^ν that maximize net profits

$$px^\nu - \theta^\nu a^\nu$$

over $(x^\nu, a^\nu) \in T^\nu$.

Conditions (T1–3) postulated above ensure that, for any combination of price vector p and carbon tax rate θ^ν, the optimum combination (x^ν, a^ν) of vector of production x^ν and CO_2 emissions a^ν always exists and is uniquely determined. We may denote them by the functional form

$$x^\nu = x^\nu(p, \theta^\nu), \quad a^\nu = a^\nu(p, \theta^\nu).$$

The optimum production plan $(x^\nu(p, \theta^\nu), a^\nu(p, \theta^\nu))$ may be characterized by the following conditions:

(i) $(x^\nu(p, \theta^\nu),\ a^\nu(p, \theta^\nu)) \in T^\nu$

(ii) $px^\nu(p, \theta^\nu) - \theta^\nu a^\nu(p, \theta^\nu) > px^\nu - \theta^\nu a^\nu,$

for all $(x^\nu, a^\nu) \in T^\nu$, $(x^\nu, a^\nu) \neq (x^\nu(p, \theta^\nu), a^\nu(p, \theta^\nu))$.

To see how the optimum levels of production and CO_2 emissions are determined, let us denote the vector of imputed rental prices of factors of production by $r^\nu = (r_\ell^\nu)$, $[r_\ell^\nu \geqq 0]$. Then the optimum conditions are

$$p \leqq r^\nu f_{a^\nu}^\nu(x^\nu, a^\nu) \qquad (\text{mod.}\, x^\nu) \tag{3}$$

$$\theta^\nu \geqq r^\nu\left[-f_{a^\nu}^\nu(x^\nu, a^\nu)\right] \quad (\text{mod.}\, a^\nu) \tag{4}$$

$$f^\nu(x^\nu, a^\nu) \leqq K^\nu \qquad (\text{mod.}\, r^\nu). \tag{5}$$

The first condition (3) means that

$$p_j = \sum_\ell r_\ell^\nu f_{jx_\ell^\nu}^\nu(x^\nu, a^\nu) \quad \text{(with equality when } x_j^\nu > 0\text{)},$$

which expresses the familiar principle that the choice of production technologies and levels of production are so adjusted as to equate marginal factor costs with output prices.

The second condition (4) similarly means that CO_2 emissions are so controlled that the marginal loss due to the marginal increase in CO_2 emissions is equal to carbon tax rate θ^ν when $a^\nu > 0$ and is not larger than θ^ν when $a^\nu = 0$.

The third condition (5) means that the employment of factors of production does not exceed the endowments, and the conditions of full employment are satisfied whenever rental price r_ℓ^ν is positive.

In what follows, for the sake of expository brevity, marginality conditions are often assumed to be satisfied by equality.

We have assumed that the technologies are subject to constant returns to scale (T4), and thus, in view of the Euler identity, conditions (3), (4), and (5) imply that

$$\begin{aligned} px^\nu - \theta^\nu a^\nu &= r^\nu\left[f_{x^\nu}^\nu(x^\nu, a^\nu)x^\nu + f_{a^\nu}^\nu(x^\nu, a^\nu)a^\nu\right] \\ &= r^\nu f^\nu(x^\nu, a^\nu) = r^\nu K^\nu. \end{aligned}$$

That is, the net evaluation of output is equal to the sum of the rental payments to all factors of production.

Suppose all factors of production are owned by individual members of the country ν. Then, national income y^ν of country ν is equal to the sum of the rental payments $r^\nu K^\nu = \sum_\ell r_\ell^\nu K_\ell^\nu$ and the tax payments $\theta^\nu a^\nu$ made by the producers for the emission of CO_2 in country ν;

that is,

$$y^\nu = r^\nu K^\nu + \theta^\nu a^\nu = (px^\nu - \theta^\nu a^\nu) + \theta^\nu a^\nu = px^\nu.$$

Hence, national income y^ν of country ν is equal to the value of outputs in units of market prices px^ν, thus conforming with the standard practice in national income accounting.

The producer optimum is similarly characterized when a perfectly competitive world market for tradable emission permits exists. Suppose prices on the market are given by the pair of $p = (p_j)$ and q. Then the vector (x^ν, a^ν) that maximizes net profits

$$px^\nu - qa^\nu, \quad (x^\nu, a^\nu) \in T^\nu$$

is uniquely determined and continuously twice-differentiable in (p, q); we may also denote them by the functional form

$$x^\nu = x^\nu(p, q), \quad a^\nu = a^\nu(p, q).$$

The optimum production plan $(x^\nu(p, q), a^\nu(p, q))$ may also be characterized by the following conditions:

(i) $(x^\nu(p, q), a^\nu(p, q)) \in T^\nu$

(ii) $px^\nu(p, q) - qa^\nu(p, q) > px^\nu - qa^\nu$, for all $(x^\nu, a^\nu) \in T^\nu$, $(x^\nu, a^\nu) \neq (x^\nu(p, q), a^\nu(p, q))$.

Activity Analysis and Technological Possibility Sets

The specifications of technological possibility sets introduced earlier in this section contain certain ambiguities when more than one factor of production is involved. The quantities of factors of production required to produce goods by the vector $x^\nu = (x_1^\nu, \ldots, x_L^\nu)$ with the CO_2 emission at the level a^ν are determined by the choice of technologies and levels of production activities; thus, the quantities of factors of production required for (x^ν, a^ν) are mutually dependent, and the minimum quantity required for each type of factors of production may generally not be uniquely defined independently of the employment of other factors of production.

To explicitly examine the relationships between the choice of technologies and the quantity of CO_2 emissions, we may carry out the discussion better within the framework of the theory of activity analysis. Let us denote the vector of activity levels by $\xi^\nu = (\xi_s^\nu)$, $\xi_s^\nu \geqq 0$, where ξ_s^ν stands for the level of activity s. We assume that activities $\{s\}$ comprise all possible production activities carried out by the producers in country ν.

The vector of produced quantities of goods, the quantity of CO_2 emissions, and the quantities of factors of production required when production activities are carried out at $\xi^\nu = (\xi_s^\nu)$ are, respectively, represented by the functional form

$$x^\nu(\xi^\nu) = \big(x_j^\nu(\xi^\nu)\big),\ a^\nu(\xi^\nu),\ K^\nu(\xi^\nu) = \big(K_\ell^\nu(\xi^\nu)\big).$$

We assume that functions $x^\nu(\xi^\nu)$, $a^\nu(\xi^\nu)$, $K^\nu(\xi^\nu)$ satisfy the following conditions:

(T1′) Substitution between outputs and various factors of production are smooth; that is, $x^\nu(\xi^\nu)$, $a^\nu(\xi^\nu)$, $K^\nu(\xi^\nu)$ are defined, continuous, and continuously twice-differentiable for all $\xi^\nu \geqq 0$.

(T2′) Marginal rates of substitution are diminishing; that is, $x^\nu(\xi^\nu)$ is strictly quasi-concave with respect to $\xi^\nu \geqq 0$, whereas $a^\nu(\xi^\nu)$ and $K^\nu(\xi^\nu)$ are strictly quasi-convex with respect to $\xi^\nu \geqq 0$.

(T3′) Constant returns to scale prevail; that is, $x^\nu(\xi^\nu)$, $a^\nu(\xi^\nu)$, $K^\nu(\xi^\nu)$ are homogeneous of order 1 with respect to $\xi^\nu \geqq 0$.

The production possibility set T^ν of country ν may now be defined by

$$T^\nu = \{(x^\nu, a^\nu)\colon 0 \leqq x^\nu \leqq x^\nu(\xi^\nu),\ a^\nu \geqq a^\nu(\xi^\nu),\ K^\nu(\xi^\nu) \leqq K^\nu,\ \xi^\nu \geqq 0\}.$$

The production possibility set T^ν thus defined is a nonempty set of $J + 1$-dimensional vectors (x^ν, a^ν) that describes the technologically possible combinations of the vectors $x^\nu = (x_j^\nu)$ specifying the aggregate quantities x_j^ν of goods produced in country ν and the amount a^ν of CO_2 emitted into the atmosphere in country ν. Postulates (T1′–3′) imply that the production possibility set T^ν is a closed, convex set in the space of $J + 1$-dimensional vectors (x^ν, a^ν).

The Producer Optimum

Suppose prices on a perfectly competitive market are given by price vector $p = (p_j)$, and carbon taxes at the rate θ^ν are levied. Then the producers in country ν would choose those vectors of activity levels $\xi^\nu = (\xi_s^\nu)$ and combinations (x^ν, a^ν) of production vector x^ν and CO_2 emissions a^ν that maximize net profits

$$px^\nu - \theta^\nu a^\nu$$

over $(x^\nu, a^\nu) \in T^\nu$.

Postulates (T1–3) guarantee that, for any combination of price vector $p = (p_j)$ and carbon tax rate θ^ν, the optimum combination (x^ν, a^ν) of production vector x^ν and CO_2 emissions a^ν always exists and is uniquely determined.

For any combination of price vector $p = (p_j)$ and carbon tax rate θ^ν, the vector of the optimum activity levels $\xi^\nu = (\xi_s^\nu)$ may be characterized by the following marginality conditions, where $r^\nu = (r_\ell^\nu)$ denotes the vector of the imputed rental prices of factors of production:

(i)′ For each activity s, marginal net profits $px^\nu_{\xi_s^\nu}(\xi^\nu) - \theta^\nu a^\nu_{\xi_s^\nu}(\xi^\nu)$ are less than or equal to marginal factor costs $r^\nu K^\nu_{\xi_s^\nu}(\xi^\nu)$:

$$px^\nu_{\xi_s^\nu}(\xi^\nu) - \theta^\nu a^\nu_{\xi_s^\nu}(\xi^\nu) \leqq r^\nu K^\nu_{\xi_s^\nu}(\xi^\nu) \quad (\text{mod.}\, \xi^\nu) \tag{6}$$

with equality when activity s is operated at a positive level $\xi_s^\nu > 0$.

(ii)′ For each factor of production ℓ, the required employment $K_\ell^\nu(\xi^\nu)$ does not exceed the endowments K_ℓ^ν:

$$K_\ell^\nu(\xi^\nu) \leqq K_\ell^\nu \quad (\text{mod.}\, r^\nu) \tag{7}$$

with equality when the rental price of factor r^ν of production ℓ is positive: $r_\ell^\nu > 0$.

We multiply both sides of (6) by ξ_s^ν and sum over s to obtain

$$px^\nu_{\xi^\nu}(\xi^\nu)\xi^\nu - \theta^\nu a^\nu_{\xi^\nu}(\xi^\nu)\xi^\nu = r^\nu K^\nu_{\xi^\nu}(\xi^\nu)\xi^\nu,$$

which, in view of the constant-returns-to-scale conditions, yields

$$px^\nu(\xi^\nu) - \theta^\nu a^\nu(\xi^\nu) = r^\nu K^\nu(\xi^\nu).$$

Hence, in view of condition (7), we have

$$px^\nu - \theta^\nu a^\nu = r^\nu K^\nu.$$

That is, the net evaluation of output is equal to the sum of the rental payments to the factors of production.

The Case of Simple Linear Technologies

In the simplest case, the vector of activity levels $\xi^\nu = (\xi^\nu_s)$ may be identified with the vector of produced quantities of goods $x^\nu = (x^\nu_j)$, and all technological coefficients are assumed to be constant. Then the CO_2 emissions $a^\nu(x^\nu)$ and the quantities of factors of production $K^\nu(x^\nu)$ required to produce $x^\nu = (x^\nu_j)$ are, respectively, represented by

$$a^\nu(x^\nu) = \alpha^\nu x^\nu, \quad K^\nu(x^\nu) = A^\nu x^\nu,$$

where $\alpha^\nu = (\alpha^\nu_j)$ is the vector of technological coefficients specifying the amount of CO_2 emissions associated with the production of goods, and $A^\nu = (A^\nu_{\ell j})$ is the matrix of technological coefficients specifying the quantities of factors of production required in the production of goods. Then the production possibility set T^ν of country ν may be given by

$$T^\nu = \{(x^\nu, a^\nu)\colon x^\nu \geqq 0,\ A^\nu x^\nu \leqq K^\nu,\ a^\nu \geqq \alpha^\nu x^\nu\}.$$

In this simple linear case, the marginality conditions for the producer optimum are given by

$$p_j - \theta^\nu \alpha^\nu_j \leqq \sum_\ell r^\nu_\ell A^\nu_{\ell j}$$

with equality when $x^\nu_j > 0$, and

$$\sum_j A^\nu_{\ell j} x^\nu_j \leqq K^\nu_\ell$$

with equality when $r^\nu_\ell > 0$.

NOTE. For any given combination (x^ν, a^ν) of production vector x^ν and CO_2 emissions a^ν, let us define the set of quantities of factors of production $T^\nu(x^\nu, a^\nu)$ by

$$T^\nu(x^\nu, a^\nu) = \{R^\nu\colon R^\nu \geqq K^\nu(\xi^\nu),\ x^\nu(\xi^\nu) \geqq x^\nu,\ a^\nu(\xi^\nu) \leqq a^\nu,\ \text{for some } \xi^\nu \geqq 0\}.$$

The set $T^\nu(x^\nu, a^\nu)$ thus defined is a closed convex set in the L-dimensional vector space of the vectors of quantities of factors of production. When the number of factors of production is more than one ($L > 1$), the functions specifying the minimum quantities of factors of production required to produce (x^ν, a^ν) as introduced earlier in this section,

$$f^\nu(x^\nu, a^\nu) = \left(f_1^\nu(x^\nu, a^\nu), \ldots, f_L^\nu(x^\nu, a^\nu)\right),$$

are generally not well defined.

However, all the analyses developed in this book remain valid for the general case formulated in terms of activity analysis. For the sake of expository simplicity and intuitive reasoning, our discussion will be carried out in terms of the functional approach.

3. GLOBAL WARMING AND DEFERENTIAL EQUILIBRIUM

The phenomenon of global warming exhibits the basic features of public goods in the sense introduced by Paul Samuelson in his classic paper (Samuelson 1954). Thus, it is necessary to introduce some sort of institutional arrangements to realize socially acceptable patterns of resource allocation and income distribution. This is particularly relevant in our discussion of global warming because the participants in the global warming problem are not individual members of society, but rather the principal agents are the nations in the world. No definite rules or established customs concerning global warming exist that are binding on the nations involved.

To begin with, we explore the implications of market solutions for global warming. We assume that produced goods are freely traded between the countries in the world, whereas factors of production are not traded between the countries. We confine ourselves to the static circumstances under which the endowments of factors of production in individual countries and the stock of CO_2 accumulated in the atmosphere all remain constant. All the notations and postulates introduced in the previous sections are retained.

Global Warming and Competitive Equilibrium

Prices of goods on the world market are denoted by price vector $p = (p_j)$. The producers in country ν choose those combinations (x^ν, a^ν) of production vectors and CO_2 emissions that maximize net profits

$$px^\nu = px^\nu - 0a^\nu$$

over $(x^\nu, a^\nu) \in T^\nu$. Note that, in the case of perfectly competitive markets, no carbon taxes would be levied for CO_2 emissions; that is, $\theta^\nu = 0$.

In each country ν, the optimum levels of production x^ν and CO_2 emissions a^ν are determined at the levels at which, marginal factor costs for the production of goods are equated to the prices on the world market with the highest CO_2 emissions technologically possible; that is, in the case of perfectly competitive markets, the producer optimum is characterized by conditions (3), (4), and (5) with $\theta^\nu = 0$.

The consumer optimum, on the other hand, is obtained by maximizing utility function

$$u^\nu(c^\nu, a) = \varphi(a) u^\nu(c^\nu)$$

subject to the budget constraints

$$pc^\nu = y^\nu, \; c^\nu \geqq 0,$$

where y^ν is national income of country ν to be given by

$$y^\nu = r^\nu K^\nu = px^\nu.$$

The optimum vector of consumption c^ν is obtained by solving the following marginality conditions:

$$\varphi(a) u^\nu_{c^\nu}(c^\nu) = \lambda^\nu p,$$

where λ^ν is the Lagrange unknown associated with the budgetary constraint.

Total CO_2 emissions a are given by

$$a = \sum_\nu a^\nu.$$

Competitive market equilibrium for the world economy is obtained if we find a vector of prices p at which total demand for goods and services is equal to total supply

$$\sum_\nu c^\nu = \sum_\nu x^\nu.$$

The emission of CO_2 plays the role of a public "bad" provided to the producers in each country "free of charge"; thus, an excess amount of CO_2 is emitted into the atmosphere with a negative impact on world welfare. However, it is not immediately apparent that this is indeed the case. Instead we consider market equilibrium under a different behavioral postulate. We term such a market equilibrium *deferential equilibrium* because we believe that a certain sense of decency and awe to nature are embodied in the behavioral postulates of the model.

Deferential Equilibrium and Proportional Carbon Taxes

Deferential equilibrium is obtained if, when each country decides the levels of production activities, it takes into account the negative impact on its own utility level brought about by its CO_2 emissions. Formally, *deferential equilibrium* is defined as follows:

Consider the situation in which a combination (x^ν, a^ν) of production vector x^ν and CO_2 emissions a^ν is chosen in country ν. Suppose CO_2 emissions in country ν are increased by the marginal quantity. This would imply a marginal decrease in country ν's utility. Deferential equilibrium is obtained if this marginal decrease in country ν's utility is balanced with its marginal increase in utility caused by the accompanying marginal increase in the levels of production due to the marginal increase in CO_2 emissions in country ν. One can easily see that deferential equilibrium precisely corresponds to the Nash solution in game theory.

Hence, the choice of the levels of consumption, production, and CO_2 emissions under the deferential behavioristic postulate may be regarded as the optimum solution to the following maximum problem for country ν:

Maximum Problem for Deferential Equilibrium. Find the combination (c^ν, x^ν, a^ν, a) of consumption vector c^ν, production vector x^ν, CO_2 emissions a^ν, and virtual level of total CO_2 emissions a that maximizes country ν's utility

$$u^\nu(c^\nu, a) = \varphi(a)u^\nu(c^\nu)$$

subject to the constraints that

(i) Consumption expenditures are equal to national income $y^\nu = px^\nu$:

$$pc^\nu = y^\nu, \; c^\nu \geqq 0. \tag{8}$$