Econometric Exercises, Volume 2

Statistics

Building on the success of Abadir and Magnus' *Matrix Algebra* in the Econometric Exercises series, *Statistics* serves as a bridge between elementary and specialized statistics. Professors Abadir, Heijmans, and Magnus freely use matrix algebra to cover intermediate to advanced material. Each chapter contains a general introduction followed by a series of connected exercises which build up knowledge systematically. The characteristic feature of the book (and indeed the series) is that all exercises are fully solved. The authors present many new proofs of established results, along with new results, often involving shortcuts that resort to statistical conditioning arguments.

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Econometric Exercises

The volumes in Econometric Exercises are intended to be much more than a collection of several hundred solved exercises. Each book has a coherent and well-organized sequence of exercises in a specific field or sub-field of econometrics. Every chapter of a volume begins with a short technical introduction that emphasizes the main ideas and overviews the most relevant theorems and results, including applications and occasionally computer exercises. They are intended for undergraduates in econometrics with an introductory knowledge of statistics, for first and second year graduate students of econometrics, and for students and instructors from neighboring disciplines (e.g., statistics, political science, psychology and communications) with interests in econometric methods.

Titles in the Series (* = planned):

1 Matrix Algebra (K. M. Abadir and J. R. Magnus)
2 Statistics (K. M. Abadir, R. D. H. Heijmans, and J. R. Magnus)
3 Econometric Theory, I (P. Paruolo)*
4 Empirical Applications, I (A. van Soest and M. Verbeek)*
5 Econometric Theory, II*
6 Empirical Applications, II*
7 Bayesian Econometric Methods (G. Koop, D. J. Poirier, and J. L. Tobias)
7 Bayesian Econometric Methods 2nd edition (G. Koop, D. J. Poirier, J. L. Tobias, and J. Chan) *forthcoming*
* Time Series Econometrics, I
* Time Series Econometrics, II
* Microeconometrics
* Panel Data
* Nonlinear Models
* Nonparametrics and Semiparametrics
* Simulation-Based Econometrics
* Computational Methods
* Financial Econometrics
* Robustness
* Econometric Methodology

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Statistics

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To my lovely nephews and nieces: Maher, Sarah, Shahira, Karim, Christine. And to the loving memories of my father, Dr Maher Abadir, and my best friend, Dr Ashraf Mohsen. They used to enjoy a glass of whisky together. They may still do, if the ancients were right.

To Gawein.

To Gideon and Joyce, Hedda and Ralph, and to their amazing children.

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Preface to the Series

The past two decades have seen econometrics grow into a vast discipline. Many different branches of the subject now happily coexist with one another. These branches interweave econometric theory and empirical applications, and bring econometric method to bear on a myriad of economic issues. Against this background, a guided treatment of the modern subject of econometrics in a series of volumes of worked econometric exercises seemed a natural and rather challenging idea.

The present series, *Econometric Exercises*, was conceived in 1995 with this challenge in mind. Now, almost a decade later it has become an exciting reality with the publication of the first installment of a series of volumes of worked econometric exercises. How can these volumes work as a tool of learning that adds value to the many existing textbooks of econometrics? What readers do we have in mind as benefiting from this series? What format best suits the objective of helping these readers learn, practice, and teach econometrics? These questions we now address, starting with our overall goals for the series.

Econometric Exercises is published as an organized set of volumes. Each volume in the series provides a coherent sequence of exercises in a specific field or subfield of econometrics. Solved exercises are assembled together in a structured and logical pedagogical framework that seeks to develop the subject matter of the field from its foundations through to its empirical applications and advanced reaches. As the Schaum Series has done so successfully for mathematics, the overall goal of *Econometric Exercises* is to develop the subject matter of econometrics through solved exercises, providing a coverage of the subject that begins at an introductory level and moves through to more advanced undergraduate and graduate level material.

Problem solving and worked exercises play a major role in every scientific subject. They are particularly important in a subject like econometrics where there is a rapidly growing literature of statistical and mathematical technique and an ever-expanding core to the discipline. As students, instructors, and researchers, we all benefit by seeing carefully worked-out solutions to problems that develop the subject and illustrate its methods and workings. Regular exercises and problem sets consolidate learning and reveal applications of textbook material. Clearly laid out solutions, paradigm answers, and alternate routes to solution all develop problem-solving skills. Exercises train students in clear analytical thinking and help them in preparing for tests and exams. Teachers, as well as students, find solved exercises useful in their classroom preparation and in designing problem sets, tests, and examinations. Worked problems and illustrative empirical applications appeal to researchers and professional economists wanting to learn about specific econometric techniques. Our intention for the *Econometric Exercises* series is to appeal to this wide range of potential users.

Each volume of the Series follows the same general template. Chapters begin with a short outline that emphasizes the main ideas and overviews the most relevant theorems and results. The introductions are followed by a sequential development of the material by solved examples and applications, and computer exercises where these are appropriate. All problems are solved and they are graduated in difficulty with solution techniques evolving in a logical, sequential fashion. Problems are asterisked when they require more creative solutions or reach higher levels of technical difficulty. Each volume is self-contained. There is some commonality in material across volumes in the Series in order to reinforce learning and to make each volume accessible to students and others who are working largely, or even completely, on their own.

Content is structured so that solutions follow immediately after the exercise is posed. This makes the text more readable and avoids repetition of the statement of the exercise when it is being solved. More importantly, posing the right question at the right moment in the development of a subject helps to anticipate and address future learning issues that students face. Furthermore, the methods developed in a solution and the precision and insights of the answers are often more important than the questions being posed. In effect, the inner workings of a good solution frequently provide benefit beyond what is relevant to the specific exercise.

Exercise titles are listed at the start of each volume, following the Table of Contents, so that readers may see the overall structure of the book and its more detailed contents. This organization reveals the exercise progression, how the exercises relate to one another, and where the material is heading. It should also tantalize readers with the exciting prospect of advanced material and intriguing applications.

The Series is intended for a readership that includes undergraduate students of econometrics with an introductory knowledge of statistics, first and second year graduate students of econometrics, as well as students and instructors from neighboring disciplines (like statistics, psychology, or political science) with interests in econometric methods. The volumes generally increase in difficulty as the topics become more specialized.

The early volumes in the Series (particularly those covering matrix algebra, statistics, econometric models, and empirical applications) provide a foundation to the study of econometrics. These volumes will be especially useful to students who are following the first year econometrics course sequence in North American graduate schools and need to prepare for graduate comprehensive examinations in econometrics and to write an applied econometrics paper. The early volumes will equally be of value to advanced undergraduates studying econometrics in Europe, to advanced undergraduates and honors students in the Australasian system, and to masters and doctoral students in general. Subsequent volumes will be of interest to professional economists, applied workers, and econometricians who are working with techniques in those areas, as well as students who are taking an advanced course sequence in econometrics and statisticians with interests in those topics.

The *Econometric Exercises* series is intended to offer an independent learning-by-doing program in econometrics and it provides a useful reference source for anyone wanting to learn more about econometric methods and applications. The individual volumes can be used in classroom teaching and examining in a variety of ways. For instance, instructors can work through some of the problems in class to demonstrate methods as they are introduced, they can illustrate theoretical material with some of the solved examples, and they can show real data applications of the methods by drawing on some of the empirical examples. For examining purposes, instructors may draw freely from the solved exercises in test preparation. The systematic development of the subject in individual volumes will make the material easily accessible both for students in revision and for instructors in test preparation.

In using the volumes, students and instructors may work through the material sequentially as part of a complete learning program, or they may dip directly into material where they are experiencing difficulty, in order to learn from solved exercises and illustrations. To promote intensive study, an instructor might announce to a class in advance of a test that some questions in the test will be selected from a certain chapter of one of the volumes. This approach encourages students to work through most of the exercises in a particular chapter by way of test preparation, thereby reinforcing classroom instruction.

As Series Editors, we welcome comments, criticisms, suggestions, and, of course, corrections from all our readers on each of the volumes in the Series as well as on the Series itself. We bid you as much happy reading and problem solving as we have had in writing and preparing this series.

York, Tilburg, New Haven
July 2004

Karim M. Abadir
Jan R. Magnus
Peter C. B. Phillips

Preface

This volume contains exercises in distribution theory, estimation, and inference. The abbreviated name of the volume should be taken in the context of the Series into which it fits. Since statistics is a very large subject, we expect that the reader has already followed an introductory statistics course. This volume covers intermediate to advanced material.

There are many outstanding books on second courses in statistics, or introductory statistical theory, as well as excellent advanced texts; see our reference list. However, the level between them is less well represented. Furthermore, the use of matrix algebra is typically relegated to some of the advanced texts. These are two of the gaps we aim to fill. We also present many new proofs of established results, in addition to new results, often involving shortcuts that resort to statistical conditioning arguments.

Along with *Matrix Algebra*, the first volume of the Series, this volume fulfills two different functions. It is of interest in its own right, but it also forms the basis on which subsequent, more specialized, volumes can build. As a consequence, not all the material of Part A is used in Part B, because the former contains many results in the important toolkit of distribution theory, which will be of use for later volumes in the Series.

In deciding which topic (and how much of it) to include, we have tried to balance the need for cohesion within one volume with the need for a wide foundation. There are inevitable omissions and incomplete coverage of more specialized material. Such topics are covered in later volumes.

At the beginning of each chapter, we introduce a topic and then follow with exercises on it. These introductions contain the basic concepts laying the ground for the exercises and briefly sketching how they hang together. The introduction does not attempt to list all the results from the exercises; instead, we try to give a broad flavor of the topic. At the end of each chapter we provide Notes, which contain some pointers to the literature and some comments about generalizations. They should be of interest even if the reader has not attempted all the exercises. We occasionally avoid formal details in an effort to stress ideas and methods, and we give references to details in the Notes.

We have chosen to pitch the standard level at readers who are not necessarily familiar with the elements of complex analysis. We have therefore added a star (∗) more readily to exercises containing complex variables. Some introductory material on complex variables and other mathematical techniques (such as the Stieltjes integral) is collected in Appendix A. Readers intending to cover much of the book may find it useful to start with Appendix A as a background.

We have also given more hints to solutions in Part B, which is more advanced than Part A. As for specifics on coverage and course selections:

- The same chapter can be done at differing levels, leaving more difficult topics and exercises to further courses. This is particularly true for Part A, which is more encyclopedic than is needed to work through Part B.
- Chapter 10 (especially Section 10.3) is rather specialized.
- Sections 11.2 and 11.3 (sufficiency, ancillarity, et cetera) are not needed to proceed with much of the subsequent material, although they clarify choices that would otherwise seem arbitrary. The corresponding parts of the introduction to Chapter 11 may be skipped at a first reading, starting with the definition of sufficiency and ending with Basu's theorem.
- Section 14.3 and the corresponding parts of the introduction to Chapter 14 can also be skipped at a first reading, with the exception of the last exercise of that section, namely Exercise 14.39 on the asymptotic relative efficiency of tests, which is needed for Section 14.4.

We are grateful for constructive comments and assistance from Hendri Adriaens, Ramon van den Akker, Paul Bekker, Adel Beshai, Giovanni Caggiano, Pavel Čížek, Adriana Cornea-Madeira, Dmitry Danilov, Walter Distaso, Malena García Reyes, Liudas Giraitis, Angelica Gonzalez, David Hendry, Steve Lawford, Michel Lubrano, William Mikhail, Peter C. B. Phillips, Gaurav Saroliya, Ashoke Sinha, George Styan, Gabriel Talmain, Yubo Tao, Andrey Vasnev, Anna Woźniak, and the anonymous referees/readers. Special thanks go to Tassos Magdalinos and Paolo Paruolo for their many helpful comments. Karim is grateful to his former students on the course "Statistical Theory" at Exeter and York for their patience while some of these exercises were being tried out on them. Their constructive feedback has certainly made a difference. We also thank Susan Parkinson for her meticulous reading of our book, and Nicola Chapman, Karen Maloney, and their team at CUP.

To our great sadness our friend, colleague, and coauthor Risto Heijmans passed away in July 2014. We will miss his erudite wisdom in statistics and many other subjects, and above all his humour and zest for life.

London, Amsterdam Karim M. Abadir
September 2017 Jan R. Magnus