

NONLINEAR TIME SERIES ANALYSIS

This book represents a modern approach to time series analysis which is based on the theory of dynamical systems. It starts from a sound outline of the underlying theory to arrive at very practical issues, which are illustrated using a large number of empirical data sets taken from various fields. This book will hence be highly useful for scientists and engineers from all disciplines who study time variable signals, including the earth, life and social sciences.

The paradigm of deterministic chaos has influenced thinking in many fields of science. Chaotic systems show rich and surprising mathematical structures. In the applied sciences, deterministic chaos provides a striking explanation for irregular temporal behaviour and anomalies in systems which do not seem to be inherently stochastic. The most direct link between chaos theory and the real world is the analysis of time series from real systems in terms of nonlinear dynamics. Experimental technique and data analysis have seen such dramatic progress that, by now, most fundamental properties of nonlinear dynamical systems have been observed in the laboratory. Great efforts are being made to exploit ideas from chaos theory wherever the data display more structure than can be captured by traditional methods. Problems of this kind are typical in biology and physiology but also in geophysics, economics and many other sciences.

This revised edition has been significantly rewritten and expanded, including several new chapters. In view of applications, the most relevant novelties will be the treatment of non-stationary data sets and of nonlinear stochastic processes inside the framework of a state space reconstruction by the method of delays. Hence, nonlinear time series analysis has left the rather narrow niche of strictly deterministic systems. Moreover, the analysis of multivariate data sets has gained more attention. For a direct application of the methods of this book to the reader's own data sets, this book closely refers to the publicly available software package TISEAN. The availability of this software will facilitate the solution of the exercises, so that readers now can easily gain their own experience with the analysis of data sets.

HOLGER KANTZ, born in November 1960, received his diploma in physics from the University of Wuppertal in January 1986 with a thesis on transient chaos. In January 1989 he obtained his Ph.D. in theoretical physics from the same place, having worked under the supervision of Peter Grassberger on Hamiltonian many-particle dynamics. During his postdoctoral time, he spent one year on a Marie Curie fellowship of the European Union at the physics department of the University of

Florence in Italy. In January 1995 he took up an appointment at the newly founded Max Planck Institute for the Physics of Complex Systems in Dresden, where he established the research group ‘Nonlinear Dynamics and Time Series Analysis’.

In 1996 he received his *venia legendi* and in 2002 he became adjunct professor in theoretical physics at Wuppertal University. In addition to time series analysis, he works on low- and high-dimensional nonlinear dynamics and its applications. More recently, he has been trying to bridge the gap between dynamics and statistical physics. He has (co-)authored more than 75 peer-reviewed articles in scientific journals and holds two international patents. For up-to-date information see <http://www.mpipks-dresden.mpg.de/mpi-doc/kantzgruppe.html>.

THOMAS SCHREIBER, born 1963, did his diploma work with Peter Grassberger at Wuppertal University on phase transitions and information transport in spatio-temporal chaos. He joined the chaos group of Predrag Cvitanović at the Niels Bohr Institute in Copenhagen to study periodic orbit theory of diffusion and anomalous transport. There he also developed a strong interest in real-world applications of chaos theory, leading to his Ph.D. thesis on nonlinear time series analysis (University of Wuppertal, 1994). As a research assistant at Wuppertal University and during several extended appointments at the Max Planck Institute for the Physics of Complex Systems in Dresden he published numerous research articles on time series methods and applications ranging from physiology to the stock market. His habilitation thesis (University of Wuppertal) appeared as a review in *Physics Reports* in 1999. Thomas Schreiber has extensive experience teaching nonlinear dynamics to students and experts from various fields and at all levels. Recently, he has left academia to undertake industrial research.

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HOLGER KANTZ AND THOMAS SCHREIBER
Max Planck Institute for the Physics of Complex Systems, Dresden



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Preface to the first edition

The paradigm of deterministic chaos has influenced thinking in many fields of science. As mathematical objects, chaotic systems show rich and surprising structures. Most appealing for researchers in the applied sciences is the fact that deterministic chaos provides a striking explanation for irregular behaviour and anomalies in systems which do not seem to be inherently stochastic.

The most direct link between chaos theory and the real world is the analysis of time series from real systems in terms of nonlinear dynamics. On the one hand, experimental technique and data analysis have seen such dramatic progress that, by now, most fundamental properties of nonlinear dynamical systems have been observed in the laboratory. On the other hand, great efforts are being made to exploit ideas from chaos theory in cases where the system is not necessarily deterministic but the data displays more structure than can be captured by traditional methods. Problems of this kind are typical in biology and physiology but also in geophysics, economics, and many other sciences.

In all these fields, even simple models, be they microscopic or phenomenological, can create extremely complicated dynamics. How can one verify that one's model is a good counterpart to the equally complicated signal that one receives from nature? Very often, good models are lacking and one has to study the system just from the observations made in a single time series, which is the case for most non-laboratory systems in particular. The theory of nonlinear dynamical systems provides new tools and quantities for the characterisation of irregular time series data. The scope of these methods ranges from invariants such as Lyapunov exponents and dimensions which yield an accurate description of the structure of a system (provided the data are of high quality) to statistical techniques which allow for classification and diagnosis even in situations where determinism is almost lacking.

This book provides the experimental researcher in nonlinear dynamics with methods for processing, enhancing, and analysing the measured signals. The theorist will be offered discussions about the practical applicability of mathematical results. The

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Preface to the first edition

time series analyst in economics, meteorology, and other fields will find inspiration for the development of new prediction algorithms. Some of the techniques presented here have also been considered as possible diagnostic tools in clinical research. We will adopt a critical but constructive point of view, pointing out ways of obtaining more meaningful results with limited data. We hope that everybody who has a time series problem which cannot be solved by traditional, linear methods will find inspiring material in this book.

Dresden and Wuppertal
November 1996

Preface to the second edition

In a field as dynamic as nonlinear science, new ideas, methods and experiments emerge constantly and the focus of interest shifts accordingly. There is a continuous stream of new results, and existing knowledge is seen from a different angle after very few years. Five years after the first edition of “Nonlinear Time Series Analysis” we feel that the field has matured in a way that deserves being reflected in a second edition.

The modification that is most immediately visible is that the program listings have been replaced by a thorough discussion of the publicly available software TISEAN. Already a few months after the first edition appeared, it became clear that most users would need something more convenient to use than the bare library routines printed in the book. Thus, together with Rainer Hegger we prepared stand-alone routines based on the book but with input/output functionality and advanced features. The first public release was made available in 1998 and subsequent releases are in widespread use now. Today, TISEAN is a mature piece of software that covers much more than the programs we gave in the first edition. Now, readers can immediately apply most methods studied in the book on their own data using TISEAN programs. By replacing the somewhat terse program listings by minute instructions of the proper use of the TISEAN routines, the link between book and software is strengthened, supposedly to the benefit of the readers and users. Hence we recommend a download and installation of the package, such that the exercises can be readily done by help of these ready-to-use routines.

The current edition has been extended in view of enlarging the class of data sets to be treated. The core idea of phase space reconstruction was inspired by the analysis of deterministic chaotic data. In contrast to many expectations, purely deterministic and low-dimensional data are rare, and most data from field measurements are evidently of different nature. Hence, it was an effort of our scientific work over the past years, and it was a guiding concept for the revision of this book, to explore the possibilities to treat other than purely deterministic data sets.

There is a whole new chapter on non-stationary time series. While detecting non-stationarity is still briefly discussed early on in the book, methods to deal with manifestly non-stationary sequences are described in some detail in the second part. As an illustration, a data source of lasting interest, human speech, is used. Also, a new chapter deals with concepts of synchrony between systems, linear and nonlinear correlations, information transfer, and phase synchronisation.

Recent attempts on modelling nonlinear stochastic processes are discussed in Chapter 12. The theoretical framework for fitting Fokker–Planck equations to data will be reviewed and evaluated. While Chapter 9 presents some progress that has been made in modelling input–output systems with stochastic but observed input and on the embedding of time delayed feedback systems, the chapter on modelling considers a data driven phase space approach towards Markov chains. Wind speed measurements are used as data which are best considered to be of nonlinear stochastic nature despite the fact that a physically adequate mathematical model is the deterministic Navier–Stokes equation.

In the chapter on invariant quantities, new material on entropy has been included, mainly on the ϵ - and continuous entropies. Estimation problems for stochastic versus deterministic data and data with multiple length and time scales are discussed.

Since more and more experiments now yield good multivariate data, alternatives to time delay embedding using multiple probe measurements are considered at various places in the text. This new development is also reflected in the functionality of the TISEAN programs. A new multivariate data set from a nonlinear semiconductor electronic circuit is introduced and used in several places. In particular, a differential equation has been successfully established for this system by analysing the data set.

Among other smaller rearrangements, the material from the former chapter “Other selected topics”, has been relocated to places in the text where a connection can be made more naturally. High dimensional and spatio-temporal data is now discussed in the context of embedding. We discuss multi-scale and self-similar signals now in a more appropriate way right after fractal sets, and include recent techniques to analyse power law correlations, for example detrended fluctuation analysis.

Of course, many new publications have appeared since 1997 which are potentially relevant to the scope of this book. At least two new monographs are concerned with the same topic and a number of review articles. The bibliography has been updated but remains a selection not unaffected by personal preferences.

We hope that the extended book will prove its usefulness in many applications of the methods and further stimulate the field of time series analysis.

Dresden
 December 2002

Acknowledgements

If there is any feature of this book that we are proud of, it is the fact that almost all the methods are illustrated with real, experimental data. However, this is anything but our own achievement – we exploited other people’s work. Thus we are deeply indebted to the experimental groups who supplied data sets and granted permission to use them in this book. The production of every one of these data sets required skills, experience, and equipment that we ourselves do not have, not forgetting the hours and hours of work spent in the laboratory. We appreciate the generosity of the following experimental groups:

- NMR laser.** Our contact persons at the Institute for Physics at Zürich University were Leci Flepp and Joe Simonet; the head of the experimental group is E. Brun. (See Appendix B.2.)
- Vibrating string.** Data were provided by Tim Molteno and Nick Tufillaro, Otago University, Dunedin, New Zealand. (See Appendix B.3.)
- Taylor–Couette flow.** The experiment was carried out at the Institute for Applied Physics at Kiel University by Thorsten Buzug and Gerd Pfister. (See Appendix B.4.)
- Atrial fibrillation.** This data set is taken from the MIT-BIH Arrhythmia Database, collected by G. B. Moody and R. Mark at Beth Israel Hospital in Boston. (See Appendix B.6.)
- Human ECG.** The ECG recordings we used were taken by Petr Saporin at Saratov State University. (See Appendix B.7.)
- Foetal ECG.** We used noninvasively recorded (human) foetal ECGs taken by John F. Hofmeister as the Department of Obstetrics and Gynecology, University of Colorado, Denver CO. (See Appendix B.7.)
- Phonation data.** This data set was made available by Hanspeter Herzel at the Technical University in Berlin. (See Appendix B.8.)
- Human posture data.** The time series was provided by Steven Boker and Bennett Bertenthal at the Department of Psychology, University of Virginia, Charlottesville VA. (See Appendix B.9.)

Autonomous CO₂ laser with feedback. The data were taken by Riccardo Meucci and Marco Ciofini at the INO in Firenze, Italy. (See Appendix B.10.)

Nonlinear electric resonance circuit. The experiment was designed and operated by M. Diestelhorst at the University of Halle, Germany. (See Appendix B.11.)

Nd:YAG laser. The data we use were recorded in the University of Oldenburg, where we wish to thank Achim Kittel, Falk Lange, Tobias Letz, and Jürgen Parisi. (See Appendix B.12.)

We used the following data sets published for the Santa Fe Institute Time Series Contest, which was organised by Neil Gershenfeld and Andreas Weigend in 1991:

NH₃ laser. We used data set A and its continuation, which was published after the contest was closed. The data was supplied by U. Hübner, N. B. Abraham, and C. O. Weiss. (See Appendix B.1.)

Human breath rate. The data we used is part of data set B of the contest. It was submitted by Ari Goldberger and coworkers. (See Appendix B.5.)

During the composition of the text we asked various people to read all or part of the manuscript. The responses ranged from general encouragement to detailed technical comments. In particular we thank Peter Grassberger, James Theiler, Daniel Kaplan, Ulrich Parlitz, and Martin Wiesenfeld for their helpful remarks. Members of our research groups who either contributed by joint work to our experience and knowledge or who volunteered to check the correctness of the text are Rainer Hegger, Andreas Schmitz, Marcus Richter, Mario Ragwitz, Frank Schmüser, Rathinaswamy Bhavanan Govindan, and Sharon Sessions. We have also considerably profited from comments and remarks of the readers of the first edition of the book. Their effort in writing to us is gratefully appreciated.

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