1

Basic concepts

1.1 History of the term "plasma"

In the mid nineteenth century the Czech physiologist Jan Evangelista Purkinje introduced use of the Greek word *plasma* (meaning "formed" or "molded") to denote the clear fluid that remains after removal of all the corpuscular material in blood. About half a century later, the American scientist Irving Langmuir proposed in 1922 that the electrons, ions, and neutrals in an ionized gas could similarly be considered as corpuscular material entrained in some kind of fluid medium and called this entraining medium plasma. However it turned out that, unlike blood where there really is a fluid medium carrying the corpuscular material, there actually is no "fluid medium" entraining the electrons, ions, and neutrals in an ionized gas. Ever since, plasma scientists have had to explain to friends and acquaintances that they were not studying blood!

1.2 Brief history of plasma physics

In the 1920s and 1930s a few isolated researchers, each motivated by a specific practical problem, began the study of what is now called plasma physics. This work was mainly directed towards understanding (i) the effect of ionospheric plasma on *long-distance short-wave radio* propagation and (ii) *gaseous electron tubes* used for rectification, switching, and voltage regulation in the pre-semiconductor era of electronics. In the 1940s Hannes Alfvén developed a theory of hydromagnetic waves (now called Alfvén waves) and proposed that these waves would be important in astrophysical plasmas. In the early 1950s large-scale plasma physics based *magnetic fusion energy* research started simultaneously in the USA, Britain, and the then Soviet Union. Since this work was an offshoot of thermonuclear weapon research, it was initially classified but, because of scant progress in each country's effort and the realization that controlled fusion research was unlikely to be of military value, all three countries declassified their efforts in 1958 and have co-operated since. Many other countries now participate in fusion research as well.

Fusion progress was slow through most of the 1960s, but by the end of that decade the empirically developed Russian *tokamak* configuration began producing plasmas with parameters far better than the lackluster results of the previous two decades. By the 1970s and 1980s many tokamaks with progressively improved performance were constructed and at the end of the twentieth century fusion break-even had nearly been achieved in tokamaks. International agreement was reached in the early twenty-first century to build the International Thermonuclear Experimental Reactor (ITER), a break-even tokamak designed to produce 500 megawatts of fusion output power. Non-tokamak approaches to fusion have also been pursued with varying degrees of success; many involve magnetic confinement schemes related to that used in tokamaks. In contrast to fusion schemes based on magnetic confinement, inertial confinement schemes were also developed in which high-power lasers or similarly intense power sources bombard millimeter-diameter pellets of thermonuclear fuel with ultra-short, extremely powerful pulses of strongly focused directed energy. The intense incident power causes the pellet surface to ablate and, in so doing, act like a rocket exhaust pointing radially outwards from the pellet. The resulting radially inwards force compresses the pellet adiabatically, making it both denser and hotter; with sufficient adiabatic compression, fusion ignition conditions are predicted to be achieved.

Simultaneously with the fusion effort, there has been an equally important and extensive study of space plasmas. Measurements of near-Earth space plasmas, such as the aurora and the ionosphere, have been obtained by ground-based instruments since the late nineteenth century. Space plasma research was greatly stimulated when it became possible to use spacecraft to make routine *in situ* plasma measurements of the Earth's *magnetosphere*, the *solar wind*, and the magnetospheres of other planets. Additional interest has resulted from ground-based and spacecraft measurements of topologically complex, dramatic structures sometimes having explosive dynamics in the *solar corona.* Using radio telescopes, optical telescopes, Very Long Baseline Interferometry, and most recently the Hubble and Spitzer spacecraft, large numbers of *astrophysical jets* shooting out from magnetized objects such as stars, active galactic nuclei, and black holes have been observed. Space plasmas often behave in a manner qualitatively similar to laboratory plasmas, but have a much grander scale.

Since the 1960s an important effort has been directed towards using plasmas for *space propulsion*. Plasma thrusters have been developed ranging from small *ion thrusters* for spacecraft attitude correction to powerful *magnetoplasmadynamic thrusters* that – given an adequate power supply – could be used for interplanetary missions. Plasma thrusters are now in use on some spacecraft and are under serious consideration for new and more ambitious spacecraft designs.

Starting in the late 1980s a new application of plasma physics appeared – *plasma processing* – a critical aspect of the fabrication of the tiny, complex integrated circuits used in modern electronic devices. This application is now of great economic importance.

In the 1980s investigations began on *non-neutral plasmas*; these mimic the equations of incompressible hydrodynamics and so provide a compelling analog computer for problems in incompressible hydrodynamics. Another application of non-neutral plasmas is as a means to store large quantities of positrons. In the 1990s studies began on *dusty plasmas*. Dust grains immersed in a plasma can become electrically charged and then act as an additional charged particle species. Because dust grains are massive compared to electrons or ions and can be charged to varying amounts, new physical behavior occurs that is sometimes an extension of what happens in a regular plasma and sometimes altogether new. Both non-neutral and dusty plasmas can also form bizarre, strongly coupled collective states where the plasma resembles a solid (e.g., forms quasi-crystalline structures).

In addition to the above activities there have been continuing investigations of industrially relevant plasmas such as *arcs*, *plasma torches*, and *laser plasmas.* In particular, approximately 40% of the steel manufactured in the United States is recycled in huge electric arc furnaces capable of melting over 100 tons of scrap steel in a few minutes. Plasma displays are used for flat-panel televisions and of course there are naturally occurring terrestrial plasmas such as *lightning*.

1.3 Plasma parameters

Three fundamental parameters[1] characterize a plasma:

1. the particle density n (measured in particles per cubic meter),
2. the temperature T of each species (usually measured in eV, where 1 eV=11 605 K),
3. the steady-state magnetic field B (measured in Tesla).

A host of subsidiary parameters (e.g., Debye length, Larmor radius, plasma frequency, cyclotron frequency, thermal velocity) can be derived from these three fundamental parameters. For partially ionized plasmas, the fractional ionization and cross-sections of neutrals are also important.

[1] In older plasma literature, density and magnetic fields are often expressed in cgs units, i.e., densities are given in particles per cubic centimeter, and magnetic fields are given in Gauss. Since the 1990s there has been general agreement to use SI units when possible. SI units have the distinct advantage that electrical units are in terms of familiar quantities such as amps, volts, and ohms and so a model prediction in SI units can much more easily be compared to the results of an experiment than a prediction given in cgs units.

1.4 Examples of plasmas

1.4.1 Non-fusion terrestrial plasmas

It takes considerable resources and skill to make a hot, fully ionized plasma and so, except for the specialized fusion plasmas, most terrestrial plasmas (e.g., arcs, neon signs, fluorescent lamps, processing plasmas, welding arcs, and lightning) have electron temperatures of a few eV and, for reasons given later, have ion temperatures that are colder, often at room temperature. These "everyday" plasmas usually have no imposed steady-state magnetic field and do not produce significant self-magnetic fields. Typically, these plasmas are weakly ionized and dominated by collisional and radiative processes. Densities in these plasmas range from 10^{14} to 10^{22} m^{-3} (for comparison, the density of air at STP is 2.7×10^{25} m^{-3}).

1.4.2 Fusion-grade terrestrial plasmas

Using carefully designed, expensive, and often large plasma confinement systems together with high heating power and obsessive attention to purity, fusion researchers have succeeded in creating fully ionized hydrogen or deuterium plasmas which attain temperatures ranging from tens of eV to *tens of thousands* of eV. In typical magnetic confinement devices (e.g., tokamaks, stellarators, reversed field pinches, mirror devices) an externally produced 1–10 T magnetic field of carefully chosen geometry is imposed on the plasma. Magnetic confinement devices generally have densities in the range 10^{19}–10^{21} m^{-3}. Plasmas used in inertial fusion are much more dense; the goal is to attain for a brief instant densities one or two orders of magnitude larger than solid density ($\sim 10^{28}$ m^{-3}).

1.4.3 Space plasmas

The parameters of these plasmas cover an enormous range. For example, the density of space plasmas varies from 10^{6} m^{-3} in interstellar space to 10^{20} m^{-3} in the solar atmosphere. Most of the astrophysical plasmas that have been investigated have temperatures in the range of 1–100 eV and these plasmas are usually fully ionized.

1.5 Logical framework of plasma physics

Plasmas are complex and exist in a wide variety of situations differing by many orders of magnitude. An important situation where plasmas do not normally exist is ordinary human experience. Consequently, people do not have the sort of intuition for plasma behavior that they have for solids, liquids, or gases. Although plasma behavior seems non- or counter-intuitive at first, with suitable effort a good intuition for plasma behavior can be developed. This intuition can be helpful for making initial predictions about plasma behavior in a new situation, because plasmas have the remarkable property of being extremely *scalable*; i.e., the same qualitative phenomena often occur in plasmas differing by many orders of magnitude. Plasma physics is usually not a precise science. It is rather a web of overlapping points of view, each modeling a limited range of behavior. Understanding of plasmas is developed by studying these various points of view, all the while keeping in mind the linkages between the points of view.

Plasma dynamics is determined by the *self-consistent interaction between electromagnetic fields and statistically large numbers of charged particles* as shown schematically in Fig. 1.1. In principle, the time evolution of a plasma can be calculated as follows:

1. given the trajectory $\mathbf{x}_j(t)$ and velocity $\mathbf{v}_j(t)$ of each and every particle j, the electric field $\mathbf{E}(\mathbf{x},t)$ and magnetic field $\mathbf{B}(\mathbf{x},t)$ can be evaluated using Maxwell's equations and simultaneously;
2. given the instantaneous electric and magnetic fields $\mathbf{E}(\mathbf{x},t)$ and $\mathbf{B}(\mathbf{x},t)$, the forces on each and every particle j can be evaluated using the Lorentz equation and then used to update the trajectory $\mathbf{x}_j(t)$ and velocity $\mathbf{v}_j(t)$ of each particle.

While this approach is conceptually easy to understand, it is normally impractical to implement because of the extremely large number of particles and, to a lesser extent, because of the complexity of the electromagnetic field. To gain a practical understanding, we therefore do not attempt to evaluate the entire complex behavior all at once but, instead, study plasmas by considering specific phenomena. For each phenomenon under immediate consideration, appropriate simplifying approximations are made, leading to a more tractable problem and hopefully revealing the essence of what is going on. A situation where a certain set of approximations is valid and provides a self-consistent description is called a regime. There are a number of general categories of simplifying approximations, namely:

1. *Approximations involving the electromagnetic field:*
   (a) assuming the magnetic field is zero (unmagnetized plasma);
   (b) assuming there are no inductive electric fields (electrostatic approximation);

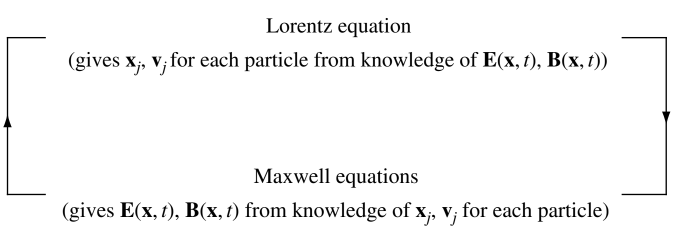

Fig. 1.1 Interrelation between Maxwell's equations and the Lorentz equation.

(c) neglecting the displacement current in Ampère's law (suitable for phenomena having characteristic velocities much slower than the speed of light);
(d) assuming that all magnetic fields are produced by conductors external to the plasma;
(e) various assumptions regarding geometric symmetry (e.g., spatially uniform, uniform in a particular direction, azimuthally symmetric about an axis).

2. *Approximations involving the particle description:*
   (a) Averaging of the Lorentz force over some sub-group of particles:
      i. Vlasov theory: average over all particles of a given species (electrons or ions) having the same velocity at a given location and characterize the plasma using the distribution function $f_\sigma(\mathbf{x}, \mathbf{v}, t)$, which gives the density of particles of species σ having velocity $\mathbf{v}$ at position $\mathbf{x}$ at time t;
      ii. Two-fluid theory: average velocities over all particles of a given species at a given location and characterize the plasma using the species density $n_\sigma(\mathbf{x}, t)$, mean velocity $\mathbf{u}_\sigma(\mathbf{x}, t)$, and pressure $P_\sigma(\mathbf{x}, t)$ defined relative to the species mean velocity.
      iii. Magnetohydrodynamic theory: average momentum over all particles of all species and characterize the plasma using the center-of-mass density $\rho(\mathbf{x}, t)$, center-of-mass velocity $\mathbf{U}(\mathbf{x}, t)$, and pressure $P(\mathbf{x}, t)$ defined relative to the center-of-mass velocity.
   (b) Assumptions about time (e.g., assume the phenomenon under consideration is fast or slow compared to some characteristic frequency of the particles such as the cyclotron frequency).
   (c) Assumptions about space (e.g., assume the scale length of the phenomenon under consideration is large or small compared to some characteristic plasma length such as the cyclotron radius).
   (d) Assumptions about velocity (e.g., assume the phenomenon under consideration is fast or slow compared to the thermal velocity $v_{T\sigma}$ of a particular species σ).

The large number of possible permutations and combinations that can be constructed from the above list means that there will be a large number of regimes. Since developing an intuitive understanding requires making approximations of the sort listed above and since these approximations lack an obvious hierarchy, it is not clear where to begin. In fact, as sketched in Fig. 1.2, the models for particle motion (Vlasov, two-fluid, MHD) involve a circular argument. Wherever we start on this circle, we are always forced to take at least one new concept on trust and hope that its validity will be established later. The reader is encouraged to refer to Fig. 1.2 as its various components are examined so that the logic of this circle will eventually become clear.

Because the argument is circular, the starting point is at the author's discretion, and for good (but not overwhelming) reasons, this author has decided that the optimum starting point on Fig. 1.2 is the subject of *Debye shielding*. Debye concepts, the Rutherford model for how charged particles scatter from each other,

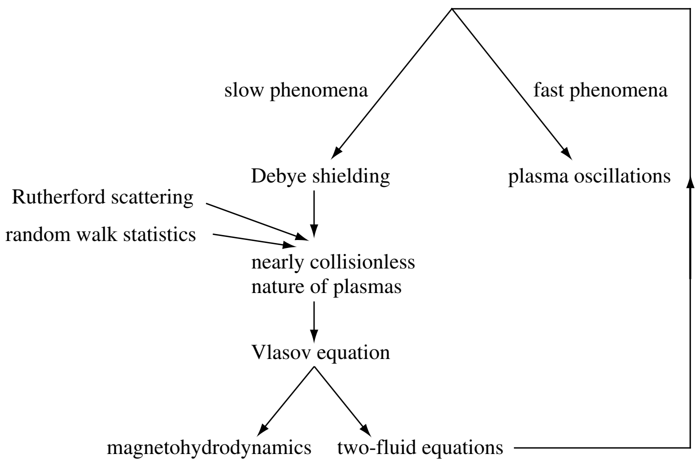

Fig. 1.2 Hierarchy of models of plasmas showing circular nature of logic.

and some elementary statistics will be combined to construct an argument showing that plasmas are *weakly collisional*. We will then discuss *phase-space* concepts and introduce the *Vlasov equation* for the phase-space density. Averages of the Vlasov equation will provide *two-fluid equations* and also the *magnetohydrodynamic* (MHD) equations. Having established this framework, we will then return to study features of these points of view in more detail, often tying up loose ends that occurred in our initial derivation of the framework. Somewhat separate from the study of Vlasov, two-fluid, and MHD equations (which all attempt to give a self-consistent picture of the plasma) is the study of *single particle orbits in prescribed fields*. This provides useful intuition on the behavior of a typical particle in a plasma, and can provide important inputs or constraints for the self-consistent theories.

1.6 Debye shielding

We begin our study of plasmas by examining Debye shielding, a concept originating from the theory of liquid electrolytes (Debye and Hückel 1923). Consider a finite-temperature plasma consisting of a statistically large number of electrons and ions and assume that the ion and electron densities are initially equal and spatially uniform. As will be seen later, the ions and electrons need not be in thermal equilibrium with each other, and so the ions and electrons will be allowed to have separate temperatures denoted by T_i, T_e.

Since the ions and electrons have random thermal motion, thermally induced perturbations about the equilibrium will cause small, transient spatial variations of the electrostatic potential ϕ. In the spirit of circular argument the following assumptions are now invoked without proof:

1. The plasma is assumed to be nearly collisionless so that collisions between particles may be neglected to first approximation.
2. Each species, denoted as σ, may be considered as a "fluid" having a density n_σ, a temperature T_σ, a pressure $P_\sigma = n_\sigma \kappa T_\sigma$ (κ is Boltzmann's constant), and a mean velocity $\mathbf{u}_\sigma$ so that the collisionless equation of motion for each fluid is

$$m_\sigma \frac{\mathrm{d}\mathbf{u}_\sigma}{\mathrm{d}t} = q_\sigma \mathbf{E} - \frac{1}{n_\sigma}\nabla P_\sigma, \tag{1.1}$$

where m_σ is the particle mass, q_σ is the charge of a particle, and $\mathbf{E}$ is the electric field.

Now consider a perturbation with a sufficiently *slow* time dependence to allow the following assumptions:

1. The inertial term $\sim \mathrm{d}/\mathrm{d}t$ on the left-hand side of Eq. (1.1) is negligible and may be dropped.
2. Inductive electric fields are negligible so the electric field is almost entirely electrostatic, i.e., $\mathbf{E} \sim -\nabla\phi$.
3. All temperature gradients are smeared out by thermal particle motion so that the temperature of each species is spatially uniform.
4. The plasma remains in thermal equilibrium throughout the perturbation (i.e., it can always be characterized by a temperature).

Invoking these approximations, Eq. (1.1) reduces to

$$0 \approx -n_\sigma q_e \nabla\phi - \kappa T_\sigma \nabla n_\sigma, \tag{1.2}$$

a simple balance between the force due to the electrostatic electric field and the force due to the isothermal pressure gradient. Equation (1.2) is readily solved to give the *Boltzmann relation*

$$n_\sigma = n_{\sigma 0} \exp\left(-q_\sigma \phi / \kappa T_\sigma\right), \tag{1.3}$$

where $n_{\sigma 0}$ is a constant. It is important to emphasize that the Boltzmann relation results from the assumption that the perturbation is *very slow;* if this is not the case, then inertial effects, inductive electric fields, or temperature gradient effects will cause the plasma to have a completely different behavior from the Boltzmann relation. Situations exist where this "slowness" assumption is valid for electron dynamics but not for ion dynamics, in which case the Boltzmann condition will apply only to the electrons but not to the ions (the converse situation does not normally occur because ions, being heavier, are always more sluggish than electrons and so it is only possible for a phenomenon to appear slow to electrons but not to ions).

Let us now imagine slowly inserting a single additional particle (so-called "test" particle) with charge q_T into an initially unperturbed, spatially uniform, neutral plasma. To keep the algebra simple, we define the origin of our coordinate system to be at the location of the test particle. Before insertion of the test particle, the plasma potential was $\phi = 0$ everywhere because the ion and electron densities were spatially uniform and equal, but now the ions and electrons will be perturbed because of their interaction with the test particle. Particles having the same polarity as q_T will be slightly repelled whereas particles of opposite polarity will be slightly attracted. The slight displacements resulting from these repulsions and attractions will result in a small, but finite, potential in the plasma. This potential will be the superposition of the test particle's own potential and the potential of the plasma particles that have moved slightly in response to the test particle.

This slight displacement of plasma particles is called *shielding* or *screening* of the test particle because the displacement tends to reduce the effectiveness of the test particle field. To see this, suppose the test particle is a positively charged ion. When immersed in the plasma it will attract nearby electrons and repel nearby ions; the net result is an effectively negative charge cloud surrounding the test particle. An observer located far from the test particle and its surrounding cloud would see the combined potential of the test particle and its associated cloud. Because the cloud has the opposite polarity to the test particle, the cloud potential will partially cancel (i.e., *shield* or *screen*) the test particle potential.

Screening is calculated using Poisson's equation with the source terms being the test particle and its associated cloud. The cloud contribution is determined using the Boltzmann relation for the particles that participate in the screening. This is a "self-consistent" calculation for the potential because the shielding cloud is affected by its self-potential.

Thus, Poisson's equation becomes

$$\nabla^2 \phi = -\frac{1}{\varepsilon_0}\left[q_T \delta(\mathbf{r}) + \sum_\sigma n_\sigma(\mathbf{r}) q_\sigma\right], \tag{1.4}$$

where the term $q_T\delta(\mathbf{r})$ on the right-hand side represents the charge density due to the test particle and the term $\sum_\sigma n_\sigma(\mathbf{r})q_\sigma$ represents the charge density of all plasma particles that participate in the screening (i.e., everything except the test particle). Before the test particle was inserted $\sum_{\sigma=i,e} n_\sigma(\mathbf{r})q_\sigma$ vanished because the plasma was assumed to be initially neutral.

Since the test particle was inserted slowly, the plasma response will be Boltzmann-like and we may substitute for $n_\sigma(\mathbf{r})$ using Eq. (1.3). Furthermore, because the perturbation due to a single test particle is infinitesimal, we can safely assume that $|q_\sigma \phi| \ll \kappa T_\sigma$, in which case Eq. (1.3) becomes simply $n_\sigma/n_{\sigma 0} \approx 1 - q_\sigma \phi/\kappa T_\sigma$ and so Eq. (1.4) becomes

$$\nabla^2 \phi = -\frac{1}{\varepsilon_0}\left[q_T \delta(\mathbf{r}) + \left(1 - \frac{q_e \phi}{\kappa T_e}\right) n_{e0} q_e + \left(1 - \frac{q_i \phi}{\kappa T_i}\right) n_{i0} q_i\right]. \tag{1.5}$$

The assumption of initial neutrality means that $n_{e0} q_e + n_{i0} q_i = 0$, in which case Eq. (1.5) reduces to

$$\nabla^2 \phi - \frac{1}{\lambda_D^2} \phi = -\frac{q_T}{\varepsilon_0} \delta(\mathbf{r}), \tag{1.6}$$

where the effective Debye length is defined by

$$\frac{1}{\lambda_D^2} = \sum_\sigma \frac{1}{\lambda_\sigma^2} \tag{1.7}$$

and the species Debye length λ_σ is

$$\lambda_\sigma^2 = \frac{\varepsilon_0 \kappa T_\sigma}{n_{\sigma 0} q_\sigma^2}. \tag{1.8}$$

The second term on the left-hand side of Eq. (1.6) is just the negative of the shielding cloud charge density. The summation in Eq. (1.7) is over all species that participate in the shielding. Since ions cannot move fast enough to keep up with an electron test charge, which would be moving at the nominal electron thermal velocity, the shielding of electrons is only by other electrons, whereas the shielding of ions is by both ions and electrons.

Equation (1.6) can be solved using standard mathematical techniques (cf. assignments) to give

$$\phi(\mathbf{r}) = \frac{q_T}{4\pi\varepsilon_0 r} \mathrm{e}^{-r/\lambda_D}; \tag{1.9}$$

this is sometimes called the Yukawa potential. For $r \ll \lambda_D$ the potential $\phi(\mathbf{r})$ is identical to the potential of a test particle in vacuum, whereas for $r \gg \lambda_D$ the test charge is completely screened by its surrounding shielding cloud. The nominal radius of the shielding cloud is λ_D. Because the test particle is completely screened for $r \gg \lambda_D$, the total shielding cloud charge is equal in magnitude to the charge on the test particle and opposite in sign. This test-particle/shielding-cloud analysis makes sense only if there is a macroscopically large number of plasma particles in the shielding cloud; i.e., the analysis makes sense only if $4\pi n_0 \lambda_D^3/3 \gg 1$. This will be seen later to be the condition for the plasma to be nearly collisionless and so validate assumption 1 at the top of p. 8.

In order for shielding to be a relevant issue, the Debye length must be small compared to the overall dimensions of the plasma, because otherwise no point in the plasma could be outside the shielding cloud. Finally, it should be realized