Introductory Quantum Optics

This book provides an elementary introduction to the subject of quantum optics, the study of the quantum-mechanical nature of light and its interaction with matter.

The presentation is almost entirely concerned with the quantized electromagnetic field. Topics covered include single-mode field quantization in a cavity, quantization of multimode fields, quantum phase, coherent states, quasi-probability distribution in phase space, atom-field interactions, the Jaynes–Cummings model, quantum coherence theory, beam splitters and interferometers, nonclassical field states with squeezing etc., tests of local realism with entangled photons from down-conversion, experimental realizations of cavity quantum electrodynamics, trapped ions, decoherence, and some applications to quantum information processing, particularly quantum cryptography. The book contains many homework problems and a comprehensive bibliography.

This text is designed for upper-level undergraduates taking courses in quantum optics who have already taken a course in quantum mechanics, and for first- and second-year graduate students.

A solutions manual is available to instructors via solutions@cambridge.org.

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Introductory Quantum Optics

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C. C. G. dedicates this book to his son, Eric.
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