Theory of Financial Risk and Derivative Pricing

From Statistical Physics to Risk Management

Risk control and derivative pricing have become of major concern to financial institutions. The need for adequate statistical tools to measure an anticipate the amplitude of the potential moves of financial markets is clearly expressed, in particular for derivative markets. Classical theories, however, are based on simplified assumptions and lead to a systematic (and sometimes dramatic) underestimation of real risks. *Theory of Financial Risk and Derivative Pricing* summarizes recent theoretical developments, some of which were inspired by statistical physics. Starting from the detailed analysis of market data, one can take into account more faithfully the real behaviour of financial markets (in particular the 'rare events') for asset allocation, derivative pricing and hedging, and risk control. This book will be of interest to physicists curious about finance, quantitative analysts in financial institutions, risk managers and graduate students in mathematical finance.

JEAN-PHILIPPE BOUCHAUD was born in France in 1962. After studying at the French Lycée in London, he graduated from the Ecole Normale Supérieure in Paris, where he also obtained his Ph.D. in physics. He was then appointed by the CNRS until 1992, where he worked on diffusion in random media. After a year spent at the Cavendish Laboratory Cambridge, Dr Bouchaud joined the Service de Physique de l'Etat Condensé (CEA-Saclay), where he works on the dynamics of glassy systems and on granular media. He became interested in theoretical finance in 1991 and co-founded, in 1994, the company Science & Finance (S&F, now Capital Fund Management). His work in finance includes extreme risk control and alternative option pricing and hedging models. He is also Professor at the Ecole de Physique et Chimie de la Ville de Paris. He was awarded the IBM young scientist prize in 1990 and the CNRS silver medal in 1996.

MARC POTTERS is a Canadian physicist working in finance in Paris. Born in 1969 in Belgium, he grew up in Montreal, and then went to the USA to earn his Ph.D. in physics at Princeton University. His first position was as a post-doctoral fellow at the University of Rome La Sapienza. In 1995, he joined Science & Finance, a research company in Paris founded by J.-P. Bouchaud and J.-P. Aguilar. Today Dr Potters is Managing Director of Capital Fund Management (CFM), the systematic hedge fund that merged with S&F in 2000. He directs fundamental and applied research, and also supervises the implementation of automated trading strategies and risk control models for CFM funds. With his team, he has published numerous articles in the new field of statistical finance while continuing to develop concrete applications of financial forecasting, option pricing and risk control. Dr Potters teaches regularly with Dr Bouchaud at the Ecole Centrale de Paris.

Theory of Financial Risk and Derivative Pricing

From Statistical Physics to Risk Management

SECOND EDITION

Jean-Philippe Bouchaud and Marc Potters

CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011–4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 2000
This edition published 2003

Printed in the United Kingdom at the University Press, Cambridge

Typefaces Times 10/13 pt. and Helvetica *System* LaTeX 2_ε [TB]

A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication data
Bouchaud, Jean-Philippe, 1962–
Theory of financial risk and derivative pricing : from statistical physics to risk
management / Jean-Philippe Bouchaud and Marc Potters.–2nd edn
p. cm.
Rev. edn of: Theory of financial risks. 2000.
Includes bibliographical references and index.
ISBN 0 521 81916 4 (hardback)
1. Finance. 2. Financial engineering. 3. Risk assessment. 4. Risk management.
I. Potters, Marc, 1969– II. Bouchaud, Jean-Philippe, 1962– Theory of financial risks.
III. Title.
HG101.B68 2003
658.15′5 – dc21 2003044037

ISBN 0 521 81916 4 hardback

Cambridge University Press
0521819164 - Theory of Financial Risk and Derivative Pricing: From Statistical Physics to Risk Management, Second Edition
Jean-Philippe Bouchaud and Marc Potters
Frontmatter
More information

Contents

Cambridge University Press
0521819164 - Theory of Financial Risk and Derivative Pricing: From Statistical Physics to Risk Management, Second Edition
Jean-Philippe Bouchaud and Marc Potters
Frontmatter
More information

Foreword

Since the 1980s an increasing number of physicists have been using ideas from statistical mechanics to examine financial data. This development was partially a consequence of the end of the cold war and the ensuing scarcity of funding for research in physics, but was mainly sustained by the exponential increase in the quantity of financial data being generated everyday in the world's financial markets.

Jean-Philippe Bouchaud and Marc Potters have been important contributors to this literature, and *Theory of Financial Risk and Derivative Pricing*, in this much revised second English-language edition, is an admirable summary of what has been achieved. The authors attain a remarkable balance between rigour and intuition that makes this book a pleasure to read.

To an economist, the most interesting contribution of this literature is a new way to look at the increasingly available high-frequency data. Although I do not share the authors' pessimism concerning long time scales, I agree that the methods used here are particularly appropriate for studying fluctuations that typically occur in frequencies of minutes to months, and that understanding these fluctuations is important for both scientific and pragmatic reasons. As most economists, Bouchaud and Potters believe that models in finance are never 'correct' – the specific models used in practice are often chosen for reasons of tractability. It is thus important to employ a variety of diagnostic tools to evaluate hypotheses and goodness of fit. The authors propose and implement a combination of formal estimation and statistical tests with less rigorous graphical techniques that help inform the data analyst. Though in some cases I wish they had provided conventional standard errors, I found many of their figures highly informative.

The first attempts at applying the methodology of statistical physics to finance dealt with individual assets. Financial economists have long emphasized the importance of correlations across assets returns. One important addition to this edition of *Theory of Financial Risk and Derivative Pricing* is the treatment of the joint behaviour of asset returns, including clustering, extreme correlations and the cross-sectional variation of returns, which is here named *variety*. This discussion plays an important role in risk management.

In this book, as in much of the finance literature inspired by physics, a *model* is typically a set of mathematical equations that 'fit' the data. However, in Chapter 20, Bouchaud and Potters study how such equations may result from the behaviour of economic agents. Much of modern economic theory is occupied by questions of this kind, and here again physicists have much to contribute.

This text will be extremely useful for natural scientists and engineers who want to turn their attention to financial data. It will also be a good source for economists interested in getting acquainted with the very active research programme being pursued by the authors and other physicists working with financial data.

José A. Scheinkman
Theodore Wells '29 Professor of Economics, Princeton University

Preface

> Je vais maintenant commencer à prendre toute la phynance. Après quoi je tuerai tout le monde et je m'en irai.
>
> (A. Jarry, *Ubu roi*.)

Scope of the book

Finance is a rapidly expanding field of science, with a rather unique link to applications. Correspondingly, recent years have witnessed the growing role of financial engineering in market rooms. The possibility of easily accessing and processing huge quantities of data on financial markets opens the path to new methodologies, where systematic comparison between theories and real data not only becomes possible, but mandatory. This perspective has spurred the interest of the statistical physics community, with the hope that methods and ideas developed in the past decades to deal with complex systems could also be relevant in finance. Many holders of PhDs in physics are now taking jobs in banks or other financial institutions.

The existing literature roughly falls into two categories: either rather abstract books from the mathematical finance community, which are very difficult for people trained in natural sciences to read, or more professional books, where the scientific level is often quite poor.[†] Moreover, even in excellent books on the subject, such as the one by J. C. Hull, the point of view on derivatives is the traditional one of Black and Scholes, where the whole pricing methodology is based on the construction of *riskless strategies*. The idea of zero-risk is counter-intuitive and the reason for the existence of these riskless strategies in the Black–Scholes theory is buried in the premises of Ito's stochastic differential rules.

Recently, a handful of books written by physicists, including the present one,[‡] have tried to fill the gap by presenting the physicists' way of approaching scientific problems. The difference lies in priorities: the emphasis is less on rigour than on pragmatism, and no

[†] There are notable exceptions, such as the remarkable book by J. C. Hull, *Futures, Options and Other Derivatives*, Prentice Hall, 1997, or P. Wilmott, *Derivatives, The theory and practice of financial engineering*, John Wiley, 1998.

[‡] See 'Further reading' below.

theoretical model can ever supersede empirical data. Physicists insist on a detailed comparison between 'theory' and 'experiments' (i.e. empirical results, whenever available), the art of approximations and the systematic use of intuition and simplified arguments.

Indeed, it is our belief that a more intuitive understanding of standard mathematical theories is needed for a better training of scientists and financial engineers in charge of financial risks and derivative pricing. The models discussed in *Theory of Financial Risk and Derivative Pricing* aim at accounting for real markets statistics where the construction of riskless hedges is generally impossible and where the Black–Scholes model is inadequate. The mathematical framework required to deal with these models is however not more complicated, and has the advantage of making the issues at stake, in particular the problem of risk, more transparent.

Much activity is presently devoted to create and develop new methods to measure and control financial risks, to price derivatives and to devise decision aids for trading. We have ourselves been involved in the construction of risk control and option pricing softwares for major financial institutions, and in the implementation of statistical arbitrage strategies for the company Capital Fund Management. This book has immensely benefited from the constant interaction between theoretical models and practical issues. We hope that the content of this book can be useful to all quants concerned with financial risk control and derivative pricing, by discussing at length the advantages and limitations of various statistical models and methods.

Finally, from a more academic perspective, the remarkable stability across markets and epochs of the anomalous statistical features (fat tails, volatility clustering) revealed by the analysis of financial time series begs for a simple, generic explanation in terms of agent based models. This had led in the recent years to the development of the rich and interesting models which we discuss. Although still in their infancy, these models will become, we believe, increasingly important in the future as they might pave the way to more ambitious models of collective human activities.

Style and organization of the book

Despite our efforts to remain simple, certain sections are still quite technical. We have used a smaller font to develop the more advanced ideas, which are not crucial to the understanding of the main ideas. Whole sections marked by a star (*) contain rather specialized material and can be skipped at first reading. Conversely, crucial concepts and formulae are highlighted by 'boxes', either in the main text or in the summary section at the end of each chapter. Technical terms are set in boldface when they are first defined.

We have tried to be as precise as possible, but are sometimes deliberately sloppy and non-rigorous. For example, the idea of probability is not axiomatized: its intuitive meaning is more than enough for the purpose of this book. The notation $P(\cdot)$ means the probability distribution for the variable which appears between the parentheses, and not a well-determined function of a dummy variable. The notation $x \to \infty$ does not necessarily mean that x tends to infinity in a mathematical sense, but rather that x is 'large'. Instead of trying to derive results which hold true in any circumstances, we often compare order of magnitudes of the different effects: small effects are neglected, or included perturbatively.[†]

Finally, we have not tried to be comprehensive, and have left out a number of important aspects of theoretical finance. For example, the problem of interest rate derivatives (swaps, caps, swaptions...) is not addressed. Correspondingly, we have not tried to give an exhaustive list of references, but rather, at the end of each chapter, a selection of books and specialized papers that are directly related to the material presented in the chapter. One of us (J.-P. B.) has been for several years an editor of the *International Journal of Theoretical and Applied Finance* and of the more recent *Quantitative Finance*, which might explain a certain bias in the choice of the specialized papers. Most papers that are still in e-print form can be downloaded from `http://arXiv.org/` using their reference number.

This book is divided into twenty chapters. Chapters 1–4 deal with important results in probability theory and statistics (the central limit theorem and its limitations, the theory of extreme value statistics, the theory of stochastic processes, etc.). Chapter 5 presents some basic notions about financial markets and financial products. The statistical analysis of real data and empirical determination of statistical laws, are discussed in Chapters 6–8. Chapters 9–11 are concerned with the problems of inter-asset correlations (in particular in extreme market conditions) and the definition of risk, value-at-risk and expected shortfall. The theory of optimal portfolio, in particular in the case where the probability of extreme risks has to be minimized, is given in Chapter 12. The problem of forward contracts and options, their optimal hedge and the residual risk is discussed in detail in Chapters 13–15. The standard Black–Scholes point of view is given its share in Chapter 16. Some more advanced topics on options are introduced in Chapters 17 and 18 (such as exotic options, the role of transaction costs and Monte–Carlo methods). The problem of the yield curve, its theoretical description and some empirical properties are addressed in Chapter 19. Finally, a discussion of some of the recently proposed agent based models (in particular the now famous Minority Game) is given in Chapter 20. A short glossary of financial terms, an index and a list of symbols are given at the end of the book, allowing one to find easily where each symbol or word was used and defined for the first time.

Comparison with the previous editions

This book appeared in its first edition in French, under the title: *Théorie des Risques Financiers*, Aléa–Saclay–Eyrolles, Paris (1997). The second edition (or first English edition) was *Theory of Financial Risks*, Cambridge University Press (2000). Compared to this edition, the present version has been substantially reorganized and augmented – from five to twenty chapters, and from 220 pages to 400 pages. This results from the large amount of new material and ideas in the past four years, but also from the desire to make the book

[†] $a \simeq b$ means that a is of order b, $a \ll b$ means that a is smaller than, say, $b/10$. A computation neglecting terms of order $(a/b)^2$ is therefore accurate to 1%. Such a precision is usually enough in the financial context, where the uncertainty on the value of the parameters (such as the average return, the volatility, etc.), is often larger than 1%.

more self-contained and more accessible: we have split the book in shorter chapters focusing on specific topics; each of them ends with a summary of the most important points. We have tried to give explicitly many useful formulas (probability distributions, etc.) or practical clues (for example: how to generate random variables with a given distribution, in Appendix F.)

Most of the figures have been redrawn, often with updated data, and quite a number of new data analysis is presented. The discussion of many subtle points has been extended and, hopefully, clarified. We also added 'Derivative Pricing' in the title, since almost half of the book covers this topic.

More specifically, we have added the following important topics:

- A specific chapter on stochastic processes, continuous time and Ito calculus, and path integrals (see Chapter 3).
- A chapter discussing various aspects of data analysis and estimation techniques (see Chapter 4).
- A chapter describing financial products and financial markets (see Chapter 5).
- An extended description of non linear correlations in financial data (volatility clustering and the leverage effect) and some specific mathematical models, such as the multifractal Bacry–Muzy–Delour model or the Heston model (see Chapters 7 and 8).
- A detailed discussion of models of inter-asset correlations, multivariate statistics, clustering, extreme correlations and the notion of 'variety' (see Chapters 9 and 11).
- A detailed discussion of the influence of drift and correlations in the dynamics of the underlying on the pricing of options (see Chapter 15).
- A whole chapter on the Black-Scholes way, with an account of the standard formulae (see Chapter 16).
- A new chapter on Monte-Carlo methods for pricing and hedging options (see Chapter 18).
- A chapter on the theory of the yield curve, explaining in (hopefully) transparent terms the Vasicek and Heath–Jarrow–Morton models and comparing their predictions with empirical data. (See Chapter 19, which contains some material that was not previously published.)
- A whole chapter on herding, feedback and agent based models, most notably the minority game (see Chapter 20).

Many chapters now contain some new material that have never appeared in press before (in particular in Chapters 7, 9, 11 and 19). Several more minor topics have been included or developed, such as the theory of progressive dominance of extremes (Section 2.4), the anomalous time evolution of 'hypercumulants' (Section 7.2.1), the theory of optimal portfolios with non linear constraints (Section 12.2.2), the 'worst fluctuation' method to estimate the value-at-risk of complex portfolios (Section 12.4) and the theory of value-at-risk hedging (Section 14.5).

We hope that on the whole, this clarified and extended edition will be of interest both to newcomers and to those already acquainted with the previous edition of our book.

Acknowledgements†

This book owes much to discussions that we had with our colleagues and friends at Science and Finance/CFM: Jelle Boersma, Laurent Laloux, Andrew Matacz, and Philip Seager. We want to thank in particular Jean-Pierre Aguilar, who introduced us to the reality of financial markets, suggested many improvements, and supported us during the many years that this project took to complete.

We also had many fruitful exchanges over the years with Alain Arnéodo, Erik Aurell, Marco Avellaneda, Elie Ayache, Belal Baaquie, Emmanuel Bacry, François Bardou, Martin Baxter, Lisa Borland, Damien Challet, Pierre Cizeau, Rama Cont, Lorenzo Cornalba, Michel Dacorogna, Michael Dempster, Nicole El Karoui, J. Doyne Farmer, Xavier Gabaix, Stefano Galluccio, Irene Giardina, Parameswaran Gopikrishnan, Philippe Henrotte, Giulia Iori, David Jeammet, Paul Jefferies, Neil Johnson, Hagen Kleinert, Imre Kondor, Jean-Michel Lasry, Thomas Lux, Rosario Mantegna,‡ Matteo Marsili, Marc Mézard, Martin Meyer, Aubry Miens, Jeff Miller, Jean-François Muzy, Vivienne Plerou, Benoît Pochart, Bernd Rosenow, Nicolas Sagna, José Scheinkman, Farhat Selmi, Dragan Šestović, Jim Sethna, Didier Sornette, Gene Stanley, Dietrich Stauffer, Ray Streater, Nassim Taleb, Robert Tompkins, Johaness Voït, Christian Walter, Mark Wexler, Paul Wilmott, Matthieu Wyart, Tom Wynter and Karol Życzkowski.

We thank Claude Godrèche, who was the editor for the French version of this book, and Simon Capelin, in charge of the two English editions at C.U.P., for their friendly advice and support, and José Scheinkman for kindly accepting to write a foreword. M. P. wishes to thank Karin Badt for not allowing any compromises in the search for higher truths. J.-P. B. wants to thank J. Hammann and all his colleagues from Saclay and elsewhere for providing such a free and stimulating scientific atmosphere, and Elisabeth Bouchaud for having shared so many far more important things.

This book is dedicated to our families and children and, more particularly, to the memory of Paul Potters.

Further reading

• Econophysics and 'phynance'

J. Baschnagel, W. Paul, *Stochastic Processes, From Physics to Finance*, Springer-Verlag, 2000.

J.-P. Bouchaud, K. Lauritsen, P. Alstrom (Edts), *Proceedings of "Applications of Physics in Financial Analysis"*, held in Dublin (1999), Int. J. Theo. Appl. Fin., **3**, (2000).

J.-P. Bouchaud, M. Marsili, B. Roehner, F. Slanina (Edts), *Proceedings of the Prague Conference on Application of Physics to Economic Modelling*, Physica A, **299**, (2001).

A. Bunde, H.-J. Schellnhuber, J. Kropp (Edts), *The Science of Disaster*, Springer-Verlag, 2002.

J. D. Farmer, *Physicists attempt to scale the ivory towers of finance*, in Computing in Science and Engineering, November 1999, reprinted in Int. J. Theo. Appl. Fin., **3**, 311 (2000).

† Funding for this work was made available in part by market inefficiencies.
‡ Who kindly gave us the permission to reproduce three of his graphs.

M. Levy, H. Levy, S. Solomon, *Microscopic Simulation of Financial Markets*, Academic Press, San Diego, 2000.

R. Mantegna, H. E. Stanley, *An Introduction to Econophysics*, Cambridge University Press, Cambridge, 1999.

B. Roehner, *Patterns of Speculation: A Study in Observational Econophysics*, Cambridge University Press, 2002.

J. Voït, *The Statistical Mechanics of Financial Markets*, Springer-Verlag, 2001.