1

Foundation of radiation theory

In this chapter we review the physical foundation of remote sensing. Except for possible gravitational effects, information accessible to a distant observer must be sensed as electromagnetic radiation, either in the form of reflected or refracted solar or stellar radiation, or in the form of thermal or nonthermal emission. We restrict the discussion to passive techniques. Active methods, involving the generation of electromagnetic radiation (radar, lidar), are not explicitly treated. However, the physical principles discussed in this text are equally applicable to passive and active methods. In either case a discussion of the measurement and interpretation of remotely sensed data must be based on electromagnetic theory. In Section 1.1 we begin with that theory by reviewing Maxwell's equations. The application of the principle of energy conservation to Maxwell's equations leads to the Poynting theorem with the Poynting vector describing radiative energy transport; this is discussed in Section 1.2. However, the Poynting vector does not characterize more complex phenomena, such as reflection, refraction, polarization, or interference; all of these phenomena play significant roles in many aspects of remote sensing. Their study requires, first, a derivation of the wave equation from Maxwell's formulas, and second, finding appropriate solutions for the electric and magnetic field vectors; this is the subject of Section 1.3. Polarization is briefly reviewed in Section 1.4. Effects of electromagnetic waves striking an interface between two media and the conditions that must be satisfied at the boundary are treated in Section 1.5. The derived conditions are then applied to the boundary to find expressions for reflected and refracted waves. These expressions, the Fresnel equations, are discussed in Section 1.6. The same boundary conditions are used again in Section 5.6 to describe the behavior of thin films employed in many ways in remote sensing instruments. The Planck function is introduced in Section 1.7. In Section 1.8, we return to the Poynting vector in a discussion of quantities used in the theory of radiative transfer, such as spectral intensity and radiative flux.

1.1 Maxwell's equations

Electromagnetic radiation between the red limit of the visible spectrum and the microwave region is called the infrared. In round numbers the infrared covers the spectral range from 1 to 1000 μm. Although only the range from 0.35 to 0.75 μm is truly visible to the human eye, the region between 0.75 and 1 μm is often considered as a part of the 'visible' spectrum because many detectors common to that spectral domain, such as conventional photomultipliers, photographic film, and charge-coupled silicon devices, work well up to about 1 μm. At the far end of the infrared spectrum, tuned circuits, waveguides, and other elements associated with radio and microwave technology become the commonly employed detection tools.

Whatever the wavelength, electromagnetic radiation obeys the laws expressed by Maxwell's equations. These equations describe the interrelationship of electric and magnetic quantities by field action, in contrast to action at a distance, which up to Maxwell's time (1873) was the generally accepted point of view. The field concept goes back to Michael Faraday. In all likelihood, the concept suggested itself to him in experiments with magnets and iron filings in which lines of force become almost an observable reality. However, it was left to James Clerk Maxwell to give the field concept a far-reaching and elegant mathematical formulation. Fifteen years after the publication of Maxwell's treatise (1873), Heinrich Hertz (1888) discovered electromagnetic waves, an experimental verification of Maxwell's theory.

In differential form, using the rationalized system and vector notation, the first pair of Maxwell's equations is (e.g. Sommerfeld, 1952):

$$\dot{\mathbf{D}} + \mathbf{J} = \boldsymbol{\nabla} \times \mathbf{H} \tag{1.1.1}$$

and

$$\dot{\mathbf{B}} = -\boldsymbol{\nabla} \times \mathbf{E}, \tag{1.1.2}$$

where $\mathbf{D}$ and $\mathbf{B}$ are the electric displacement and magnetic induction, and $\mathbf{E}$ and $\mathbf{H}$ the electric and magnetic field strengths, respectively; $\mathbf{J}$ is the current density. The dot symbolizes differentiation with respect to time. Definitions of the curl ($\boldsymbol{\nabla}\times$) and the divergence ($\boldsymbol{\nabla}\,\cdot$) operators are given in Appendix 1. The concept of the electric displacement was introduced by Maxwell. The first equation includes Ampère's law and the second represents Faraday's law of induction.

Besides the main equations (1.1.1) and (1.1.2), two more expressions are traditionally considered part of Maxwell's equations,

$$\boldsymbol{\nabla} \cdot \mathbf{D} = \rho \tag{1.1.3}$$

and

$$\nabla \cdot \mathbf{B} = 0. \tag{1.1.4}$$

Equation (1.1.3) defines the electric charge density, ρ, while Eq. (1.1.4) states the nonexistence of magnetic charges or monopoles. Strictly from symmetry considerations of Maxwell's equations one may be led to postulate the existence of magnetic charges, but despite many attempts none has been found.

By applying the divergence operator to Eq. (1.1.1) and substituting ρ for $\nabla \cdot \mathbf{D}$, one arrives at the electric continuity equation,

$$\dot{\rho} + \nabla \cdot \mathbf{J} = 0, \tag{1.1.5}$$

which states the conservation of electric charge: a change in the charge density of a volume element must be associated with a current flow across the boundary of that arbitrarily chosen element. The continuity equation in fluid dynamics is an analogous expression of the conservation of mass.

In order to study the interaction of matter with electric and magnetic fields, three material constants are introduced: the electric conductivity, σ,

$$\mathbf{J} = \sigma \mathbf{E}, \tag{1.1.6}$$

the dielectric constant, ε,

$$\mathbf{D} = \varepsilon \mathbf{E}, \tag{1.1.7}$$

and the magnetic permeability, μ,

$$\mathbf{B} = \mu \mathbf{H}. \tag{1.1.8}$$

Equation (1.1.6) is a form of Ohm's law. Since $\mathbf{J}$ is the current density (A m^{-2}) and $\mathbf{E}$ the electric field strength (V m^{-1}), σ is expressed in Ω^{-1} m^{-1}. The inverse conductivity is the resistivity. In the rationalized system the dielectric constant is conveniently written

$$\varepsilon = \varepsilon_0 \varepsilon_{\mathrm{rel}}, \tag{1.1.9}$$

where ε_0 is the dielectric constant of free space (see Appendix 2 for numerical values) and $\varepsilon_{\mathrm{rel}}$ is a dimensionless quantity, which is unity for free space and which has the same value as the dielectric constant in the Gaussian system of units. The permeability is

$$\mu = \mu_0 \mu_{\mathrm{rel}}, \tag{1.1.10}$$

where μ_0 represents the permeability of free space. The relative permeability is unity for free space, larger than unity for paramagnetic materials, and less than unity for diamagnetic substances.

Maxwell's equations are linear. However, the parameters that describe material properties may become nonlinear in exceptionally strong fields, such as in powerful lasers. In these cases nonlinear terms have to be included. The linear material equations, Eqs. (1.1.6) to (1.1.8), are not applicable to ferroelectric or ferromagnetic substances where the relationship between the electric field strength, **E**, and the electric displacement, **D**, or between the magnetic field strength, **H**, and the magnetic induction, **B**, are not only nonlinear, but show hysteresis effects as well. In any case, Maxwell's equations are the foundation of electromagnetism, which includes optics and infrared physics.

1.2 Conservation of energy and the Poynting vector

The Poynting theorem expresses the conservation of energy in electromagnetism. If one takes the scalar product of Eq. (1.1.1) with **E** and of Eq. (1.1.2) with **H**, and adds the results one finds

$$\mathbf{H} \cdot \dot{\mathbf{B}} + \mathbf{E} \cdot \dot{\mathbf{D}} + \mathbf{E} \cdot \mathbf{J} = \mathbf{E} \cdot (\boldsymbol{\nabla} \times \mathbf{H}) - \mathbf{H} \cdot (\boldsymbol{\nabla} \times \mathbf{E}). \tag{1.2.1}$$

With the vector identity

$$\mathbf{E} \cdot (\boldsymbol{\nabla} \times \mathbf{H}) - \mathbf{H} \cdot (\boldsymbol{\nabla} \times \mathbf{E}) \equiv -\boldsymbol{\nabla} \cdot (\mathbf{E} \times \mathbf{H}) \tag{1.2.2}$$

and the definition

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} \tag{1.2.3}$$

one obtains

$$\mathbf{H} \cdot \dot{\mathbf{B}} + \mathbf{E} \cdot \dot{\mathbf{D}} + \mathbf{E} \cdot \mathbf{J} + \boldsymbol{\nabla} \cdot \mathbf{S} = 0. \tag{1.2.4}$$

This is the Poynting theorem; **S** is the Poynting vector. The first two terms in Eq. (1.2.4) represent rate of change of the magnetic and electric energy densities in the field. The third term, $\mathbf{E} \cdot \mathbf{J}$, describes the energy dissipated by the motion of electric charges. Generally, this motion results in Joule heating and, therefore, in losses to the energy stored in the field. The last term, $\boldsymbol{\nabla} \cdot \mathbf{S}$, represents the net flow of electromagnetic energy across the boundaries of the chosen volume. All terms of Eq. (1.2.4) are measured in J m^{-3} s^{-1}, which is energy per unit volume and unit time. Since the divergence operator corresponds to a differentiation with respect to space coordinates, the units of $\mathbf{S}$ are J m^{-2} s^{-1} or W m^{-2}, thus $\mathbf{S}$ is an energy flux through a surface element.

The definition of the Poynting vector, Eq. (1.2.3), requires that $\mathbf{S}$ be orthogonal to both $\mathbf{E}$ and $\mathbf{H}$. In order to better visualize the relative orientation of these three vectors, we align a Cartesian coordinate system so that the x-axis coincides with the direction of the Poynting vector. The components of $\mathbf{S}$ along the y- and z-axes, as well as the components of $\mathbf{E}$ and $\mathbf{H}$ in the direction of the x-axis, must then be zero: $S_y = S_z = E_x = H_x = 0$. The vectors $\mathbf{E}$ and $\mathbf{H}$ do not have components in the direction of energy transport represented by $\mathbf{S}$. Electromagnetic waves are transverse, in contrast to sound waves, which are longitudinal. To investigate the relative orientation between $\mathbf{E}$ and $\mathbf{H}$, we use the second of Maxwell's equations (Eq. 1.1.2) and the explicit expression of the curl operator (see Appendix 1). With the assumption that μ is constant and E_x and H_x equal zero, one obtains one scalar equation for each of the $\hat{\mathbf{j}}$- and $\hat{\mathbf{k}}$-directions ($\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are the unit vectors in the x-, y-, and z-directions):

$$\mu \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x}; \qquad \mu \frac{\partial H_z}{\partial t} = -\frac{\partial E_y}{\partial x}. \tag{1.2.5}$$

Except for a static field, which is not of interest in this context, Eq. (1.2.5) indicates that H_y must be zero if E_z vanishes and, conversely, H_z must disappear when E_y is zero. These conditions require $\mathbf{E}$ and $\mathbf{H}$ to be at right angles to each other; $\mathbf{E}$, $\mathbf{H}$, and $\mathbf{S}$ form a right-handed, orthogonal system of vectors.

1.3 Wave propagation

In an isotropic, stationary medium, the material constants σ, ε, and μ are uniform and constant scalars. The first pair of Maxwell's equations may then be stated:

$$\varepsilon \dot{\mathbf{E}} + \sigma \mathbf{E} = \boldsymbol{\nabla} \times \mathbf{H} \tag{1.3.1}$$

and

$$\mu \dot{\mathbf{H}} = -\boldsymbol{\nabla} \times \mathbf{E}. \tag{1.3.2}$$

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If one differentiates Eq. (1.3.1) with respect to time and multiplies by μ, one obtains

$$\varepsilon\mu\ddot{\mathbf{E}} + \sigma\mu\dot{\mathbf{E}} = \mu\frac{\partial}{\partial t}(\boldsymbol{\nabla}\times\mathbf{H}). \tag{1.3.3}$$

Application of the curl operator to Eq. (1.3.2) yields

$$\mu\boldsymbol{\nabla}\times\dot{\mathbf{H}} = -\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{E}). \tag{1.3.4}$$

For a medium at rest the order of differentiation with respect to space and time may be interchanged. Applying the vector identity

$$\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{E}) \equiv \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{E}) - \boldsymbol{\nabla}^2\mathbf{E} \tag{1.3.5}$$

and assuming the medium to be free of electric charges $[(\boldsymbol{\nabla}\cdot\mathbf{E}) = 0]$ leads to

$$\varepsilon\mu\ddot{\mathbf{E}} + \sigma\mu\dot{\mathbf{E}} = \nabla^2\mathbf{E}. \tag{1.3.6}$$

The Laplace operator, ∇^2, is defined in Appendix 1. This partial differential equation characterizes wave and relaxation phenomena. Again, we assume the x-axis to be aligned with the Poynting vector, so that $E_x = 0$. To simplify matters further, we rotate the coordinate system around the x-axis until the y-axis coincides with the direction of the electric field strength, so that $E_z = 0$ also. Only the y-component of $\mathbf{E}$ remains and Eq. (1.3.6) becomes a scalar equation for the unknown $E_y(x, t)$,

$$\varepsilon\mu\ddot{E}_y + \sigma\mu\dot{E}_y = E''_y. \tag{1.3.7}$$

We denote differentiation with respect to time by a dot and with respect to a space coordinate (in this case with respect to x) by a prime. The assumption $E_y = T(t)X(x)$ separates the variables,

$$\varepsilon\mu\frac{\ddot{T}}{T} + \sigma\mu\frac{\dot{T}}{T} = \frac{X''}{X} = -k^2. \tag{1.3.8}$$

Since the left side depends only on the variable t and the middle part only on the variable x, Eq. (1.3.8) can only be satisfied if the left and the middle part equal a constant, $-k^2$. The reason for choosing a negative square and the physical meaning of k will become apparent later. With the introduction of k, Eq. (1.3.8) yields two ordinary differential equations:

$$\varepsilon\mu\ddot{T} + \sigma\mu\dot{T} + k^2 T = 0 \tag{1.3.9}$$

and

$$X'' + k^2 X = 0. \tag{1.3.10}$$

A solution of Eq. (1.3.10) is readily shown to be

$$X = A\,\mathrm{e}^{\pm \mathrm{i}kx}. \tag{1.3.11}$$

The amplitude A is not defined by Eq. (1.3.10); it is determined by boundary conditions. For convenience we use notation with complex arguments in the treatment of wave phenomena. To simplify notation we omit the amplitudes but reintroduce them when needed. To solve Eq. (1.3.9) one may assume a solution of exponential form,

$$T = \mathrm{e}^{pt}, \tag{1.3.12}$$

which yields a characteristic equation for p,

$$\varepsilon\mu p^2 + \sigma\mu p + k^2 = 0. \tag{1.3.13}$$

We make two choices for p. In the first case we find the roots of Eq. (1.3.13) for p, assuming the coefficients ε, μ, σ, and k to be real quantities. Later, we will be interested in periodic solutions of Eq. (1.3.12), which imply $p = \pm\,\mathrm{i}\omega$. In that case, if $\sigma \neq 0$, at least one of the coefficients must be complex. The roots of Eq. (1.3.13) for p are

$$p = -\frac{\sigma}{2\varepsilon} \pm \left(\frac{\sigma^2}{4\varepsilon^2} - \frac{k^2}{\varepsilon\mu}\right)^{\frac{1}{2}}. \tag{1.3.14}$$

The parameter p is complex because the term with σ^2 in the parentheses is generally smaller than the term containing k^2,

$$E_y = \exp\left[-\frac{\sigma t}{2\varepsilon} \pm \mathrm{i}\left(\frac{k^2}{\varepsilon\mu} - \frac{\sigma^2}{4\varepsilon^2}\right)^{\frac{1}{2}} t\right] \exp\,(\pm\,\mathrm{i}kx). \tag{1.3.15}$$

E_y is an oscillating function of t and x. Before we discuss the physical content of Eq. (1.3.15) we consider the meaning of some of the quantities involved. It is convenient to introduce new terms pertinent to the description of optical phenomena in the infrared. Consider the inverse product $\varepsilon^{-1}\mu^{-1}$, which has the dimension of the square of a velocity, m^2 s^{-2}. This is the propagation velocity, v, of electromagnetic waves in a medium with dielectric constant ε and permeability μ. For free space this velocity is the velocity of light, c. We have

$$v = (\varepsilon\mu)^{-\frac{1}{2}}; \qquad c = (\varepsilon_0\mu_0)^{-\frac{1}{2}}. \tag{1.3.16}$$

Consequently

$$\frac{c}{v} = \left(\frac{\varepsilon\mu}{\varepsilon_0\mu_0}\right)^{\frac{1}{2}} = (\varepsilon_{\mathrm{rel}}\mu_{\mathrm{rel}})^{\frac{1}{2}} = n. \tag{1.3.17}$$

The ratio of the propagation velocity of free space to that of a medium is the refractive index, n, of the medium. In this case both n and k are real quantities. Since μ_{rel} is nearly unity for most materials of importance in the infrared, the refractive index can often be approximated by $n \sim (\varepsilon_{\mathrm{rel}})^{\frac{1}{2}}$.

The constant k has the dimension of inverse length; it is the number of radians per meter, the angular wavenumber. Therefore,

$$k\lambda = 2\pi, \tag{1.3.18}$$

where λ is the wavelength in meters. The angular frequency, ω, measured in radians per second, is then

$$\omega = kv. \tag{1.3.19}$$

The frequency, f, in hertz (cycles per second), and the wavenumber, ν, in m^{-1}, are

$$f = \frac{\omega}{2\pi}; \qquad \nu = \frac{k}{2\pi}. \tag{1.3.20}$$

Even for a wavelength of 1000 μm the frequency is approximately 3×10^{11} Hz, a very high frequency compared with radio waves. The FM broadcast band is about 100 MHz or 10^8 Hz, for comparison. The term $k^2/\varepsilon\mu$ in Eq. (1.3.15) is simply ω^2

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and the solution for E_y becomes:

$$E_y = \exp\left(-\frac{\sigma t}{2\varepsilon}\right) \exp\left\{\pm i\omega \left[1 - \left(\frac{\sigma}{2\varepsilon\omega}\right)^2\right]^{\frac{1}{2}} t\right\} \exp(\pm ikx). \tag{1.3.21}$$

As required for a second order differential equation, Eq. (1.3.21) represents two solutions, indicated by the $\pm$ signs. One solution describes a wave traveling in the direction of x (outgoing wave, opposite signs, $+\,-$ or $-\,+$), and the other, a wave traveling in the opposite direction (incoming wave, equal signs, $+\,+$ or $-\,-$). If the amplitudes of these waves are equal, only a standing wave exists. For a nonconductive medium, where σ is zero, the solution for the outgoing wave simplifies to

$$E_y(\sigma = 0) = e^{\pm i(kx - \omega t)}, \tag{1.3.22}$$

which is a plane, unattenuated wave traveling in the x-direction. This case is shown in Fig. 1.3.1 by the periodic curve marked '0'.

For a weakly conducting material – dry soil or rocks, for example – two effects may be noted. First, due to the factor $\exp(-\sigma t/2\varepsilon)$ in Eq. (1.3.21), the amplitudes of the waves diminish exponentially with time. Materials with good optical transmission properties must, therefore, be electrical insulators, but not all insulators are transparent. For many substances the frequency dependence of the refractive index is due to quantum mechanical resonances. Equation (1.3.17) is valid for low frequencies where v and n can be determined from the static values of ε and μ, but not necessarily at infrared or visible wavelengths. The second effect to be noted in Eq. (1.3.21) concerns a frequency shift by the factor $[1 - (\sigma/2\varepsilon\omega)^2]^{\frac{1}{2}}$. As long as σ is small compared with $2\varepsilon\omega$, as in the case marked 0.05 in Fig. 1.3.1, the frequency shift is negligible, but it becomes noticeable for the case $\sigma/2\varepsilon\omega = 0.2$. If σ is equal to or larger than $2\varepsilon\omega$ – that is, if the conduction current is comparable to or larger than the displacement current, as in metals – then the square root in Eq. (1.3.21) becomes zero or imaginary; in either case periodic solutions disappear and only an exponential decay exists, shown by curve 1 of Fig. 1.3.1.

Now we return to the choice of p in Eq. (1.3.12). With the assumption $p = \pm i\omega$ the solution for T becomes

$$T = e^{\pm i\omega t}, \tag{1.3.23}$$

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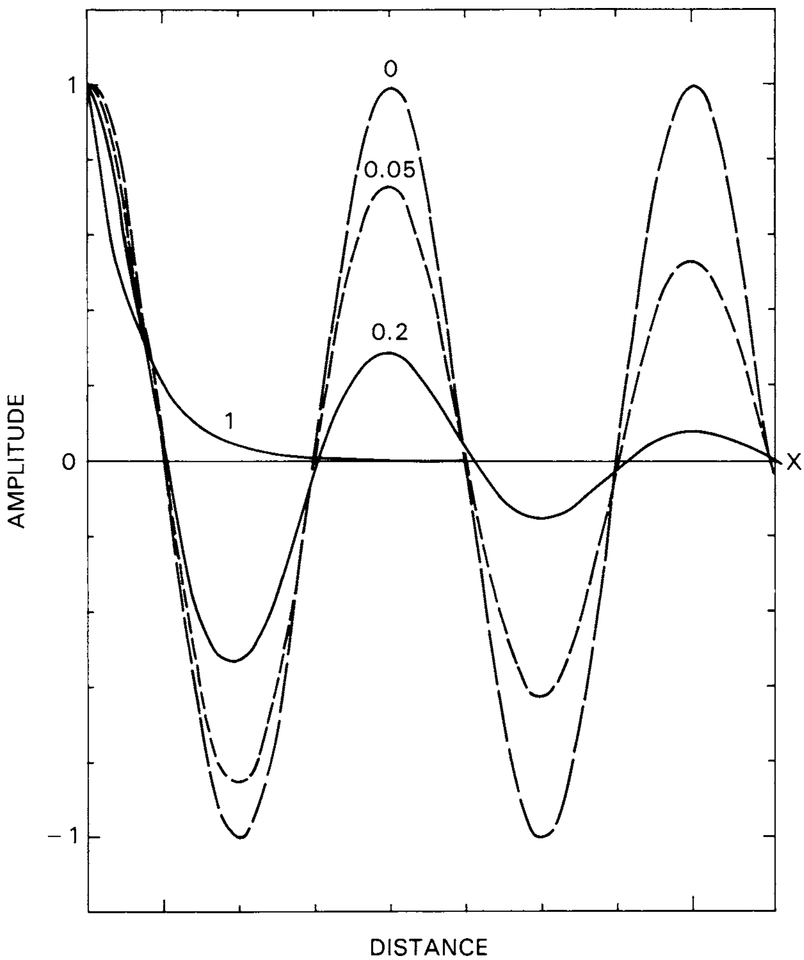

Fig. 1.3.1 Amplitudes of electromagnetic waves propagating in a medium. The parameter refers to the ratio of conduction to displacement current. If this ratio is zero the material is transparent. If this ratio is one or larger, such as in metals, only an exponential decay exists.

but in this case k is complex. We have

$$k = (\varepsilon\mu\omega^2 + \mathrm{i}\sigma\mu\omega)^{\frac{1}{2}} = \frac{\omega}{c}(n_\mathrm{r} + \mathrm{i}n_\mathrm{i}), \tag{1.3.24}$$

where n_r is the real and n_i the imaginary part of the refractive index, n. Squaring Eq. (1.3.24) and setting the real and imaginary parts of both sides equal leads to equations for the real part of k,

$$\frac{\omega n_\mathrm{r}}{c} = \omega\left(\frac{\varepsilon\mu}{2}\left\{\left[1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2\right]^{\frac{1}{2}} + 1\right\}\right)^{\frac{1}{2}}, \tag{1.3.25}$$