PLASTICITY AND GEOMECHANICS

Plasticity theory is widely used to describe the behaviour of soil and rock in many engineering situations. *Plasticity and Geomechanics* presents a concise introduction to the general subject of plasticity with a particular emphasis on applications in geomechanics. Metal plasticity is described and elementary theories are discussed before attention is focused specifically on geomaterials. The greater part of the book is devoted to the classical aspects of plasticity, particularly the use of upper and lower bound theorems and slip line theory. Critical state theory is introduced and Cam Clay is described in detail.

Derived from the authors' own lecture notes, this book is written with students firmly in mind; the main body of the work is concerned with applications, while the more theoretical aspects such as proofs of theorems are placed in appendices. Excessive use of mathematical methods is avoided in the main body of the text and, where possible, physical interpretations are given for important concepts. In this way the authors present a clear introduction to the complex ideas and concepts of plasticity as well as demonstrating how this developing subject is of critical importance to geomechanics and geotechnical engineering.

Although entirely self-contained, this book constitutes a companion volume to the acclaimed *Elasticity and Geomechanics* by the same authors, and will appeal to students and researchers in the fields of civil, mechanical, material and geological engineering. It may be used as a text for senior-level undergraduate and graduate courses in soil mechanics, foundation engineering and geomechanics.

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Preface

Plasticity and Geomechanics follows on from our earlier book *Elasticity and Geomechanics*. Like the earlier book, this one is very much a textbook rather than a treatise or reference book. It has grown from lecture notes and is written with students firmly in mind. Hopefully it will provide an easy, accessible introduction to a subject which, while being widely used in engineering practice, is often difficult for students to assimilate. The plasticity of metals is itself a subject of some complexity. When, instead of metals, the material we are concerned with is either soil or rock, the level of complexity is increased significantly. We have attempted here to untangle the ideas and concepts, and to lay out as clear a picture as possible of a subject area that is still in a state of development and discovery.

The book is organised as follows. Chapters 1 and 2 review some of the basic elements of stress and strain as well as the fundamentals of elasticity. Chapter 2 also presents a general discussion of inelastic response in soil, emphasising the defining characteristics of yield under isotropic compression and dilatancy as a result of shearing. Chapters 3 and 4 set out the fundamental ideas of yield surface and flow rules. The geometry of principal stress space is developed in detail. Yield loci for metals, for Coulomb materials and for some modifications of Coulomb materials are all presented. The Cam Clay and Modified Cam Clay surfaces are summarised. Chapter 4 develops the basic ideas of normality and the associated flow rule as well as non-associated flow. The concepts of perfect plasticity and work hardening are introduced and a complete stress–strain relationship for a general material with non-associated flow is derived. Whenever possible, important concepts such as normality are demonstrated by simple examples. A more complex but practically important example involving cavity expansion is also considered. Chapter 5 introduces the collapse load theorems and limit analysis. This is the longest chapter. In it we attempt to provide a clear introduction to what might be termed the art of finding useful stress (lower bound) and deformation (upper bound) fields for practical problems. Chapter 6 presents an introduction to slip line fields. In the interest of simplicity, the topic is developed initially for purely cohesive materials. Frictional materials are introduced as an embellishment of the purely cohesive case and complicated mathematics is avoided wherever possible. Finally, in Chapter 7, work hardening and critical state soil mechanics are described. As in the preceding chapters we try to avoid excessive detail, but endeavour to demonstrate important concepts by appealing to examples. The fundamentals of critical state theory are developed using Cam Clay together with a simple example problem. A micromechanical theory for normal or virgin compression of an idealised soil is also presented in this chapter. Throughout the book our choice of material is guided by a belief in the importance of simplicity and a desire to make fundamental ideas accessible to students.

Each chapter is followed by a short reading list detailing original sources for the material presented, complemented by references to additional reading of a more general nature. Also, following each chapter is a selection of problems that may be used to help develop the reader's understanding and skill.

The book concludes with a collection of appendices. These expand or elaborate on topics that do not fit easily with the flow of writing in the main text. Most aspects of a more mathematical nature are placed here. In particular, proofs associated with the important theorems of limit analysis as well as a complete development of Mohr's circle, virtual work and uniqueness of solutions are given. The appendices provide rigour for those readers who wish it without interrupting the more physical development in the chapters.

The bulk of the book is devoted to perfectly plastic materials. This may seem odd in light of the current interest in critical state theories for soils, but in our view it is essential knowledge. A firm understanding of basic principles is the foundation for expertise in any subject, and plasticity is no exception. We share in a growing concern that the demands on engineering curricula in current times are such that many students have had little opportunity to gain an adequate background in what might be termed the more 'classical' aspects of plasticity theory. This occurs because of two recent developments. The first is critical state soil mechanics. Critical state theory has become the new paradigm for the analysis of geotechnical problems. This is quite proper but, as with any rapidly developing paradigm, there is a tendency for a gold-rush attitude to infiltrate and subvert the normal course of study. The second development is the advent of computer methods in engineering. The widespread availability of powerful, inexpensive computers together with commercial software has revolutionised all aspects of engineering design over the last 20 years. This all too often creates a culture of uninspired thought, sometimes lacking in judgement. Numerical solutions now proliferate where once thoughtful, critical analysis was the 'only game in town'. Of course, there is no doubt that both critical state soil mechanics and numerical solutions are positive developments, but, to use them safely and efficiently, these advances must be underpinned by a well-developed understanding of both basic plasticity and elements of continuum mechanics. Our purpose here is to provide an introduction to the basic concepts in as painless a way as possible.

There are of course many excellent books on the theory of plasticity. For beginning students, Calladine's monograph *Engineering Plasticity* (full citation given at the end of Chapter 4) is a superb introduction to aspects of metal plasticity. Nadai's treatise, *Theory of Flow and Fracture in Solids* (cited in Chapter 6), is not only a reference work of great depth and scope but is also notable for taking pains to develop a variety of ideas in the context of modern soil mechanics, together with strong links to continuum mechanics. In the realm of geotechnical literature, nearly all modern textbooks contain varying amounts of material related to both the theory and the application of plasticity. A number of books more or less devoted to critical state theory have appeared since the seminal work *Critical State Soil Mechanics* by Schofield and Wroth (cited in Chapter 3). Our book in no way competes with any of these. Indeed, the exact opposite is true. We delve into critical state theory but only in the most elementary way and only after we have dealt with the classical topics of limit analysis and slip line theory. We merely wish to expose the reader to the potential of critical state analysis in the hope of encouraging further study. Among the more specialised geotechnical literature, two books deserve special mention. Chen's *Limit Analysis and Soil Plasticity* (cited in Chapter 5) contains a wealth of solutions in limit analysis covering many topics of practical interest to geotechnical engineering, and Sokolovski's *Statics of Soil Media* (cited in Chapter 6) presents the most thorough development of slip line analysis. Both books are dedicated to specific aspects of plasticity and could be regarded as required reading for research students. Neither, however, would be especially suitable as an introductory text. Our aim in this book is to fill the gap between elementary soil mechanics and more specialised books such as those by Chen or Sokolovski, as well as the books devoted to critical state theory.

Finally, there are several individuals and organisations to whom we express thanks for their assistance in preparing this book. We are indebted to the Institut A für Mechanik, University of Stuttgart, Germany and Ecole National des Ponts et Chaussées in Paris, where parts of the work were researched and written. One of us (APSS) thanks the University of Canterbury for the award of a Visiting Erskine Fellowship and the Canadian Council for the Arts for the award of a Killam Research Fellowship. The libraries of Cambridge University were a

great help in obtaining original references, both early and modern. Professor Malcolm Bolton of Cambridge University first showed us how simply the Cam Clay model can be developed in the context of simple shearing. His development is reiterated in Chapter 7. Dr Glenn McDowell of the University of Nottingham and Professor Jim Hill of the University of Wollongong reviewed parts of the manuscript and made many constructive comments. Much of the writing was done in a Hertfordshire cottage belonging to Mr and Mrs K.A. Maclean. Their hospitality is acknowledged with gratitude. The friendship and encouragement of Mr Norman Travis is also acknowledged with special thanks. Finally, we thank Anne and Sally for their patience and understanding throughout the trials and tribulations of lost files, crashing hard disks, jammed printers, headaches, backaches and all the other joys of writing, and Sally is specially thanked for compiling the index for this volume.

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