

Index

- acceleration of a particle in a continuous medium, 62
 Airy function, 228, 230
 Airy phase, 228, 229
 alternating symbol, *see* permutation symbol
 anisotropic medium, 92
 attenuation, anelastic, 357, *see* Q , scattering
 and linearity, 368
 coefficient, 360, 366
 causality constraints, 360, 376
 complex velocity, 365, 367, 383
 dispersion relations, 372
 intrinsic, 359
 Kramers–Kronig relations, 372
 limiting phase velocity, 369
 mechanisms, 358
 of seismic waves, 367–370
 auxiliary plane, 335
 axis
 B or null, 307, 309, 345
 compression or P , 307, 308, 345
 tension or T , 307, 308, 345
 Azimi's relation, 382
- balance
 of angular momentum, 62
 of linear momentum, 62
 Betti's reciprocal theorem, 319
 body force, 64
 equivalent, 316, 323
 body waves, 98
 boundary conditions, 131
 free surface, 134
 homogeneous, 320, 324
 solid–liquid, 135
 solid–solid, 134
 bulk modulus, 95
 for incompressible materials, 95
- calculus of variations, 254
 Cauchy principal value, 151, 417
 example, 404
 causality, 131, 279, 369, 370, 419, 422, *see* noncausal
 caustic, 257, 260
- center of
 compression, 300
 dilatation, 300
 rotation, 299
 compression axis, *see* axis
 concentrated force, 278
 conjugate planes, 335
 relations between their parameters, 338
 conservation of
 angular momentum, 331
 linear momentum, 331
 mass, 57, 62, 82
 constitutive equation, 40, 84
 constructive interference
 Love waves, 198, 199
 Rayleigh waves, 209
 contraction of two tensors, 16
 convolution theorem, 150, 418
 coordinate system
 Cartesian, 2
 unit vectors, 3
 epicentral, 332
 geographical, 331
 non-Cartesian, 2
 orthogonal, 2
 spherical, 291
 centered at the source, 303
 unit vectors, 303
 couple, 297
 double, 298
 equivalence to a pair of dipoles, 305
 representation of slip on a horizontal plane, 327
 moment of, 298
 crack opening, 300
 critical angle, *see* reflection and transmission
 cross product, *see* vector
 cutoff frequency, *see* Love, Rayleigh waves
- dashpot, 359
 deformation, 41
 elastic, 84, 90
 hyperelastic, 90

deformation (*cont.*)
 plastic, 84
 small, 45
 diagonalization of a second-order tensor, 23–25
 eigenvalues and eigenvectors, 25, 39
 dilatation, 52
 dip
 angle, 332, 345
 direction, 332, 345
 dipole, 297, 411
 compressional, 305
 tensional, 305
 Dirac's delta, 279, 280, 281, 295, 407, 409, 418
 Fourier transform, 416
 Hilbert transform, 187
 dislocation, 317
 dispersion, 188, 213, *see* Airy, Love waves, Rayleigh waves
 anomalous, 216
 associated with attenuation, 369
 broad-band waves, 220–225
 dispersive string, 214, 225
 group velocity, 216, 224
 inverse, 216
 local frequency, 223
 local wavenumber, 223
 method of stationary phase, 220
 narrow-band waves, 215–220
 normal, 216
 phase velocity, 215, 224
 relations, 372
 displacement
 gradient, 51
 static, 348
 vector, 44
 distributions, theory of, 407–418
 dot product, 2
 double couple, *see* couple, moment tensor, radiation pattern
 double force, 297
 duration of a function, 373
 dyads, 32–34
 dyadic, 32, 34
 antisymmetric, 37
 conjugate, 37
 nonion form, 35
 symmetric, 37
 unit, 36
 earthquake model, 316
 eigenvalues
 antisymmetric second-order tensor, 39
 symmetric second-order tensor, 25, 39
 eigenvectors, *see* eigenvalues
 eikonal equation, *see* ray theory
 elastic solid, 84
 elastic tensor c_{ijkl} , 11, 90
 isotropic, 92
 symmetry properties, 90, 92
 elastic wave equation for homogeneous media, 97, 98, 282

Green's function, 295
 harmonic plane wave solutions, 121–123
 Lamé's solution, 282
 P , SH , SV solutions, 123, 124
 vector solutions in the frequency domain, 119–121
 with a concentrated force in the x_j direction,
 285–288
 far field of a force in the x_3 direction, 291–295
 near and far fields, 289–291
 type of motion, 288
 with a concentrated force in arbitrary direction, 296
 elastic wave equation for vertically heterogeneous media, 199
 for SH motion, 200
 energy, *see* power density
 conservation, 86
 flux, 126
 P , SH , SV waves, 127
 free, 87
 inhomogeneous waves, 153, 156, 187
 intrinsic, 86
 kinetic, 86, 89
 partial, 375
 strain, 89
 density, 89, 92, 96
 energy equation, *see* reflection–transmission problems
 entropy, 86
 equation of
 continuity, 82
 motion, 70, 85, 237
 Euler angles, 336
 Eulerian description of motion, 42
 explosion model, 302
 far field
 of a concentrated force
 in the x_j direction, 289–291
 in the x_3 direction, 291–295
 of a moment tensor source, 302, 311, 312
 fault
 dip-slip, 333
 left-lateral, 334
 normal, 333
 oblique-slip, 334
 reverse, 333
 right-lateral, 334
 strike-slip, 334
 fault parameters, *see* dip, slip, strike
 fault plane–auxiliary plane ambiguity, 335
 fault plane solution, 345
 Fermat's principle, 254, 257
 finite strain tensors, 43–45
 focal
 mechanism, 345
 sphere, 343
 focus, 343
 footwall, of a fault, 332
 Fourier transform, 110
 inverse, 110
 involving derivatives, 314, 373, 415
 of distributions, 413–418

- Fourier transform (*cont.*)
 of a space-scaled function, 232
 of a time-shifted function, 151
 sign convention, 110
 space domain, 110
 time domain, 110
 frequency, 108, 109
 complex, 366
 local, 223
 frequency equation, *see* period equation
 fundamental mode, *see* Love, Rayleigh waves
 Futterman's relations, 377–381
- Gauss' theorem, 29
 in the presence of a surface of discontinuity, 321–322
 generalized Hooke's law, 90
 geometric spreading factor, 112, 355
 gradient
 of a scalar function, 19
 of a vector, 22
 Green's function
 for the elastic wave equation, 295
 space–time reciprocity, 320
 tensor nature, 295, 296
 time translation property, 329
 for the scalar wave equation, 279, 422–424
 group velocity, *see* dispersion, Love waves
- hanging wall, of a fault, 332
 Hansen vectors
 for the elastic wave equation in the frequency domain, 119–123
 for the vector Helmholtz equation, 117
 for the vector wave equation, 115
 harmonic, 116
 properties, 115
 harmonic oscillator, 360
 damped, 361
 harmonic waves, *see* waves
 Heaviside's unit-step function
 derivative, 411
 Fourier transform, 417
 Helmholtz decomposition theorem, 113, 281–282
 Helmholtz equation
 scalar, 107
 vector, 116
 Hilbert transform, 151, 187, 275, 371, 419–421
 Hooke's law, 90
 hydrostatic pressure, 64, 73, 94
 hyperelastic solid, 84
- impedance, 163
 acoustic, 163
 impulse response, 386
 index
 dummy, 9
 free, 9
 indicial notation, 3
 indigenous source, 331
 inhomogeneous waves, *see* waves
- intermediate field, 312
 isotropic
 medium, 92
 solid, 92
 tensor, 28
- Jacobian, 42
 material derivative, 82
- Kalinin and Azimi's relation, 382
 Kramers–Kronig dispersion relations, 372, 380
 Kronecker delta, 4, 20, 21
- Lagrangian description of motion, 42
 Lamé's parameters, 92
 Laplacian
 of a scalar function, 19
 of a vector, 20
 Levi-Civita symbol, *see* permutation symbol
 Love waves
 as the result of constructive interference, 198–199
 homogeneous half-space, 191
 layer over a half-space, 191
 cut-off frequency, 195
 displacement, 191, 193
 fundamental mode, 195
 group velocity, 232
 modes, 195
 nodal planes, 197
 period equation, 194
 phase velocity, 190, 194, 196, 197
 vertically heterogeneous medium, 199–202
 low-velocity layer, 177
- material derivative, 60
 minimum-delay function, 373
 minimum-phase-shift function, 372
 mode conversion, 129
 modulus of compression, 95
 moment
 mechanical, 298
 of a couple, 298
 scalar seismic, 327
 moment tensor
 decomposition into spatial and temporal factors, 301
 density, 329
 for a center of compression, 301
 for a center of rotation, 301
 for slip on a horizontal plane, 329, 330
 for slip on a fault of arbitrary orientation, 331–338
 for the double couple $M_{13} + M_{31}$, 301
 for the opening of a crack, 302, 355
 for the single couple M_{31} , 301
 source, 301
 seismic, 329–331
 multiple reflections, 270, 387
- Navier's equation, 98
 near field
 of a concentrated force in the x_j direction, 289–291
 of a moment tensor source, 311

nodal planes
 of Love waves, 197
 of radiation patterns, 308
 noncausal arrivals, 151, 275, 369

P waves in homogeneous isotropic media, 98, *see*
 radiation patterns
 direction of particle motion, 119, 124
 displacement, 118, 124
 one-dimensional problem, 99
 potential, 284
 velocity, 98

Paley–Wiener theorem, 375
 applications, 376–377

Parseval’s formula, 373

period, 108
 equation, *see* Love, Rayleigh waves
 permutation symbol, 18, 20, 21, 38, 39
 and determinants, 20

phase velocity, 157, *see* dispersion, Love, Rayleigh
 waves
 for an attenuating medium, 380, 383

plane, *see* auxiliary, conjugate plane
 of incidence, 124

plane waves, *see* waves

point source, 278
 approximation, 326

Poisson solid, 96

Poisson’s ratio, 94, 95, 99
 negative, 96

polarization
 angle, 124, 304, 339
 of particle motion, 124
 circular, 125
 elliptical, 125
 linear, 125

potency, 327

power density, 126
P, *SH*, *SV* waves, 127
 average, 127

pressure, 59, *see* hydrostatic

process
 adiabatic, 87
 isothermal, 88
 reversible, 86

Q, 358
 frequency dependence, 360, 367, 368, 376
 frequency-dependent model, 380
 nearly constant model, 381
 spatial, 366
 temporal, 363

quadratic form, 57
 positive definite, 57
 eigenvalues, 57

quarter-wavelength rule, 176

radiation pattern, 291
 for a moment tensor source, 303–305, 313
 extremal values of the *P* wave pattern, 305
 for a point force in the x_3 direction, 291–295
 for the double couple $M_{13} + M_{31}$, 308–311

and the *B*, *P*, *T* axes, 308, 309
 for the single couple M_{31} , 308

ray amplitudes, 258
 elastic wave equation, 261
P waves, anisotropic and isotropic media, 263
 polarization trihedral, 267
 ray-centered coordinate system, 267
S wave polarization vectors, 267
S waves, isotropic media, 263–268
 scalar wave equation, 258–261
 transport equation, 258

ray theory
 anisotropic media, 237–239
 Christoffel matrix, 239
 conditions for validity, 236, 237, 239
 eikonal equation, 236, 240
 elastic wave equation, 237–242
 effect of discontinuities in elastic parameters,
 268–269
 for *SH* waves in a layer over a half-space at normal
 incidence, 270–274
 high-frequency approximation, 236, 239
P and *S* waves, isotropic media, 240–242
 direction of motion, 241, 242
 velocities, 240
 scalar wave equation, 235–237
 transport equation, 258
 wave front, 242
 zeroth-order solution, 238

rays, 242
 differential geometry, 248
 binormal, 249
 curvature, 248
 Darboux vector, 251
 Frenet trihedral, 249
 moving trihedral, 249
 osculating circle, 249
 principal normal, 248
 radius of curvature, 249
 tangent vector, 248
 torsion, 250

Fermat’s principle, 254, 257
 in a medium with a depth-dependent velocity, 246
 in a medium with constant velocity, 244–246
 in a medium with spherical symmetry, 247–248,
 257
 noncausal arrivals, 275
 ray parameter, 246, 247, 248
 takeoff angle, 246, 343

Rayleigh waves
 homogeneous half-space, 202
 ground motion, 205–206
 period equation, 203
 layer over a half-space, 206
 cut-off frequency, 208
 fundamental mode, 209
 M1 and M2 branches, 208
 particle motion, 208
 phase velocity, 208, 209
 symmetric and antisymmetric modes, 208
 vertically heterogeneous medium, 209–212

- reciprocal theorem, 318
- reflection at a free surface
 - incident P waves, 137
 - energy equation, 142–144
 - special cases, 142
 - surface displacement, 141
 - incident SH waves, 136–137
 - incident SV waves, 144
 - critical angle, 146
 - displacement in the time domain, 148–153
 - energy equation, 153
 - inhomogeneous waves, 147
 - special cases, 144
- reflection and transmission, layer over a half-space, 170
 - incident P waves, 179, 183
 - ray theory example, 274–276
 - surface displacement, 184–185
 - incident SH waves, 172
 - low-velocity layer, 177
 - quarter-wavelength rule, 176
 - ray theory for normal incidence, 270–274
 - special cases, 175–177
 - surface displacement, 174
 - incident SV waves, 179, 183
 - surface displacement, 184–186
- reflection and transmission, liquid–solid boundary
 - incident P waves, 169–170
- reflection and transmission, solid–liquid boundary
 - incident P waves, 168–169
 - incident SH waves, 187
 - incident SV waves, 169
- reflection and transmission, solid–solid boundary
 - incident P waves, 157
 - critical angles, 163
 - energy equation, 164
 - inhomogeneous waves, 163–164
 - normal incidence, 161–163
 - incident SH waves, 153, 187
 - critical angle, 155
 - displacements for angles larger than the critical, 187
 - energy equation, 156
 - inhomogeneous waves, 155–156, 187
 - incident SV waves, 164
 - critical angles, 165–167
 - energy equation, 167–168
 - inhomogeneous waves, 165–166
- representation theorem, 318
- retarded potential (or solution) 281
- rigidity, 94
 - for inviscid fluids, 95
- rise time, 347
- rotation
 - infinitesimal, 30–32, 39
 - of Cartesian coordinates, 7
 - pure, 56
 - tensor, 50
 - dyadic form, 52, 57
- S waves in homogeneous isotropic media, 98, *see* radiation patterns
 - direction of particle motion, 119
 - displacement, 118, 123
 - one-dimensional problem, 99
 - velocity, 98
- scalar, 13
 - product, 2
- scattering attenuation, 359, 386–388
- separation of variables, 101, 104
- SH waves in homogeneous isotropic media, *see* radiation patterns
 - direction of particle motion, 124
 - displacement, 123, 124
 - potential, 284
- shear
 - pure, 55
 - simple, 54
- shear modulus, 94
- sgn function, 146, 220
 - derivative, 416
 - Fourier transform, 417
- slip
 - angle, 333, 345
 - vector, 333, 336, 345
- slowness, 251
- Snell's law, 138, 157
- source time function, 290
- spalling, 142
- spectral ratio method, 384–385
 - window bias, 385–386
- standard linear solid, 359
- stationary phase, method of, 220
- Stoneley waves, 212–213
- strain, *see* energy
 - ellipsoid, 58
 - normal, 45
 - principal, 45
 - quadric, 49
 - reference state, 90
 - shearing, 45
- strain tensor
 - finite, 43–45
 - infinitesimal, 45
 - compatibility equations, 46
 - dyadic form, 51
 - geometrical meaning, 46
 - principal directions, 45
 - proof that it is a tensor, 49
- stress
 - Mohr's circles, 79–81
 - normal, 70
 - reference state, 90
 - shearing, 70
- stress tensor, 68
 - deviatoric part, 73
 - dyadic form, 69
 - isotropic part, 73
 - Piola–Kirchhoff, 66
 - principal directions, 72, 99
 - principal planes, 72
 - symmetry, 71

stress vector, 64–67
 continuity, 134
 decomposition into normal and shearing vectors, 73–74
 stationary values and directions, 75–79
 in the principal axes coordinate system, 72

strike, 332, 345
 direction, 332

summation convention, 8

superposition principle, 110

surface force, 64

surface waves, 188
 displacements for, 190, 191

SV waves in homogeneous isotropic media, *see* radiation patterns
 direction of particle motion, 124
 displacement, 123, 124
 potential, 284

synthetic seismograms
 for concentrated forces in unbounded media, 290–291
 for the double couple $M_{13} + M_{31}$, 349–352

t^* , 384

take-off angle, 246, 343

tension axis, *see* axis

tensor, Cartesian, 2
 antisymmetric, 16, 21, 39
 associated vector, 28–29
 differentiation, 17
 isotropic, 28
 of order n , 13
 of order one, 13
 of order zero, 13
 operations, 14–16
 pseudo, 19
 second order, 13, *see* diagonalization
 symmetric, 16, 21, 38, 39
 trace, 15, 39
 transformation law, 13

thermodynamics
 first law, 86, 87
 second law, 86

Thomson–Haskell method, 171

total field
 of a concentrated force in the x_j direction, 291
 of a moment tensor source, 311–312

traction, 65

transfer function, crustal, 177

transmission, *see* reflection and transmission

uncertainty principle, 218

vector
 associated with an antisymmetric tensor, 28–29
 curl of, 20
 divergence of, 19
 gradient of, 22
 irrotational, 115
 Laplacian of, 20
 normal to a fault, 336
 product, 2, 19, 20, 39
 pseudo, 19
 solenoidal, 115
 transformation law, 8

velocity, *see* P , S waves
 apparent, 157
 complex, 365, 367, 383
 of a particle in a continuum medium, 61

vertical seismic profiling, 386

viscoelasticity, 359

wave equation
 D'Alembert solution, 105
 elastic, *see* elastic wave equation
 for a medium with spherical symmetry, 111
 scalar, one-dimensional, 100, 128
 with complex velocity, 365
 scalar, three-dimensional, 103
 with a source term, 279
 string in a viscous medium, 364
 string subject to a restoring force, 214
 vector, 112

wave front, 107, 112, 242, 245
 triplication, 260–261

wavelength, 108

wavenumber, 109
 complex, 365
 local, 223

waves, *see* P , S , SH , SV waves
 equivoluminal, 119
 inhomogeneous, 147, 156, *see* reflection and transmission
 longitudinal, 119
 plane, 107
 harmonic, 108
 progressive, 109
 spherical, 112
 standing, 109
 transverse, 119
 traveling, 109

work, 86, 319
 rate of, 87, 88, 125

Young's modulus, 94

Zoeppritz equations, 131