Elastic Wave Propagation and Generation in Seismology

Seismology has complementary observational and theoretical components, and a thorough understanding of the observations requires a sound theoretical background. Seismological theory, however, can be a difficult mathematical subject and introductory books do not generally give students the tools they need to solve seismological problems by themselves. This book addresses these shortcomings by bridging the gap between introductory textbooks and advanced monographs. It provides the necessary mathematical machinery and demonstrates how to apply it.

The author's approach is to consider seismological phenomena as problems in applied mathematics. To this end, each problem is carefully formulated and its solution is derived in a step-by-step approach. Although some exposure to vector calculus and partial differential equations is expected, most of the mathematics needed is derived within the book. This includes Cartesian tensors, solution of 3-D scalar and vector wave equations, Green's functions, and continuum mechanics concepts. The book covers strain, stress, propagation of body and surface waves in simple models (half-spaces and the layer over a half-space), ray theory for *P* and *S* waves (including amplitude equations), near and far fields generated by moment tensor sources in infinite media, and attenuation and the mathematics of causality.

Numerous programs for the computation of reflection and transmission coefficients, for the generation of *P*- and *S*-wave radiation patterns, and for near- and far-field synthetic seismograms in infinite media are provided by the author on a dedicated website. The book also includes problems for students to work through, with solutions available on the associated website. This book will therefore find a receptive audience among advanced undergraduate and graduate students interested in developing a solid mathematical background to tackle more advanced topics in seismology. It will also form a useful reference volume for researchers wishing to brush up on the fundamentals.

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In memory of my father
Jose A. Pujol

Contents

Preface

The study of the theory of elastic wave propagation and generation can be a daunting task because of its inherent mathematical complexity. The books on the subject currently available are either advanced or introductory. The advanced ones require a mathematical background and/or maturity generally beyond that of the average seismology student. The introductory ones, on the other hand, address advanced subjects but usually skip the more difficult mathematical derivations, with frequent references to the advanced books. What is needed is a text that goes through the complete derivations, so that readers have the opportunity to acquire the tools and training that will allow them to pose and solve problems at an intermediate level of difficulty and to approach the more advanced problems discussed in the literature. Of course, there is nothing new in this idea; there are hundreds of physics, mathematics, and engineering books that do just that, but unfortunately this does not apply to seismology. Consequently, the student in a seismology program without a strong quantitative or theoretical component, or the observational seismologist interested in a clear understanding of the analysis or processing techniques used, do not have an accessible treatment of the theory. A result of this situation is an ever widening gap between those who understand seismological theory and those who do not. At a time when more and more analysis and processing computer packages are available, it is important that their users have the knowledge required to use those packages as something more than black boxes.

This book was designed to fill the existing gap in the seismological literature. The guiding philosophy is to start with first principles and to move progressively to more advanced topics without recourse to "it can be proved" or references to Aki and Richards (1980). To fully benefit from the book the reader is expected to have had exposure to vector calculus and partial differential equations at an introductory level. Some knowledge of Fourier transforms is convenient, but except for a section in Chapter 6 and Chapter 11 they are little used. However, it is also expected that readers without this background will also profit from the book because of its explanatory material and examples.

The presentation of the material has been strongly influenced by the books of Ben-Menahem and Singh (1981) and Aki and Richards (1980), but has also benefited from those of Achenbach (1973), Burridge (1976), Eringen and Suhubi (1975), Hudson (1980), and Sokolnikoff (1956), among others. In fact, the selection of the sources for the different chapters and sections of the book was based on a "pick and choose" approach, with the overall goal of giving a presentation that is as simple as possible while at the same time retaining the inherent level of complexity of the individual topics. Again, this idea is not new, and is summarized in the following statement due to Einstein, "everything should be made as simple as possible, but not simpler" (quoted in Ben-Menahem and Singh's book). Because of its emphasis on fundamentals, the book does not deal with observations or with data analysis. However, the selection of topics was guided in part by the premise that they should be applicable to the analysis of observations. Brief chapter descriptions follow.

The first chapter is a self-contained introduction to Cartesian tensors. Tensors are essential for a thorough understanding of stress and strain. Of course, it is possible to introduce these two subjects without getting into the details of tensor analysis, but this approach does not have the conceptual clarity that tensors provide. In addition, the material developed in this chapter has direct application to the seismic moment tensor, discussed in Chapters 9 and 10. For completeness, a summary of results pertaining to vectors, assumed to be known, is included. This chapter also includes an introduction to dyadics, which in some cases constitute a convenient alternative to tensors and are found in some of the relevant literature.

Chapters 2 and 3 describe the strain and rotation tensors and the stress tensor, respectively. The presentation of the material is based on a continuum mechanics approach, which provides a conceptually clearer picture than other approaches and has wide applicability. For example, although the distinction between Lagrangian and Eulerian descriptions of motion is rarely made in seismology, the reader should be aware of them because they may be important in theoretical studies, as the book by Dahlen and Tromp (1998) demonstrates. Because of its importance in earthquake faulting studies, the Mohr circles for stress are discussed in detail. Chapter 4 introduces Hooke's law, which relates stress and strain, and certain energy relations that actually permit proving Hooke's law. The chapter also discusses several classic elastic parameters, and derives the elastic wave equation and introduces the P and S waves.

Chapter 5 deals with solutions to the scalar and vector wave equations and to the elastic wave equation in unbounded media. The treatment of the scalar equation is fairly conventional, but that of the vector equations is not. The basic idea in solving them is to find vector solutions, which in the case of the elastic wave equation immediately lead to the concept of P, SV, and SH wave motion. This approach, used by Ben-Menahem and Singh, bypasses the more conventional approach based on potentials. Because displacements, not potentials, are the observables, it makes sense to develop a theory based on them, particularly when no additional complexity is involved.

The P, SV, and SH vector solutions derived in Chapter 5 are used in Chapter 6, which covers body waves in simple models (half-spaces and a layer over a half-space). Because of their importance in applications, the different cases are discussed in detail. Two important problems, generally ignored in introductory books, also receive full attention. One is the change in waveform shapes that takes place for angles of incidence larger than the critical. The second problem is the amplification of ground motion caused by the presence of a surficial low-velocity layer, which is of importance in seismic risk studies.

Chapter 7 treats surface waves in simple models, including a model with continuous vertical variations in elastic properties, and presents a thorough analysis of dispersion. Unlike the customary discussion of dispersion in seismology books, which is limited to showing the existence of phase and group velocities, here I provide an example of a dispersive system that actually shows how the period of a wave changes as a function of time and position.

Chapter 8 deals with ray theory for the scalar wave equation and the elastic wave equation. In addition to a discussion of the kinematic aspects of the theory, including a proof of Fermat's principle, this chapter treats the very important problem of P and S amplitudes. This is done in the so-called ray-centered coordinate system, for which there are not readily available derivations. This coordinate system simplifies the computation of amplitudes and constitutes the basis of current advanced techniques for the computation of synthetic seismograms using ray theory.

Chapter 9 begins the discussion of seismic point sources in unbounded media. The simplest source is a force along one of the coordinate axes, but obtaining the corresponding solution requires considerable effort. Once this problem has been solved it is easy to find solutions to problems involving combinations of forces such as couples and combinations thereof, which in turn lead to the concept of the moment tensor. Chapter 10 specializes to the case of earthquakes caused by slip on a fault, which is shown to be equivalent to a double couple. After this major result it is more or less straightforward to find the moment tensor corresponding to a fault of arbitrary orientation. Although the Earth is clearly bounded and not homogeneous, the theory developed in these two chapters has had a major impact in seismology and still constitutes a major tool in the study of earthquakes, particularly in combination with ray theory.

Chapter 11 is on attenuation, a subject that is so vast and touches on so many aspects of seismology and physics that actually it deserves a book devoted exclusively to it. For this reason I restricted my coverage to just the most essential facts. One of these is the constraint imposed by causality, which plays a critical role in attenuation studies, and because it is not well covered in most seismology books it has been given prominence here. Causality has been studied by mathematicians, physicists and engineers, and a number of basic theorems rely on the theory of complex variables. Consequently, some of the basic results had to be quoted without even attempting to give a justification, but aside from that the treatment is self-contained. The measurement of attenuation using seismic data has received a large amount of attention in the literature, and for this reason it is not discussed here except for a brief description of the widely used spectral ratio method, and a little-known bias effect introduced when the method is applied to windowed data. As scattering may be a strong contributor to attenuation, this chapter closes with an example based on the effect of a finely layered medium on wave amplitudes and shapes.

The book ends with several appendices designed to provide background material. Appendix A introduces the theory of distributions, which is an essential tool in the study of partial differential equations (among other things). Dirac's delta is the best known example of a distribution, and is customarily introduced as something that is not a "normal" function but then is treated as such. This lack of consistency is unnecessary, as the most basic aspects of distribution theory are easy to grasp when presented with enough detail, as is done in this Appendix. Distributions are already part of the geophysical and seismological literature (e.g., Bourbié *et al.*, 1987; Dahlen and Tromp, 1998), and this Appendix will allow the reader to gain a basic understanding of the concepts discussed there. Appendix B discusses the Hilbert transform, which is a basic element in studies involving causality, and is needed to describe waves incident on a boundary at angles larger than the critical angle. A recipe for the numerical computation of the transform is also given. Appendix C derives the Green's function for the 3-D scalar wave equation. This Green's function is essential to solving the problems discussed in Chapter 9, but in spite of its importance it is usually quoted, rather than derived. The last two appendices derive in detail two fundamental equations given in Chapter 9 for the displacements caused by a single force and by an arbitrary moment tensor.

The book includes some brief historical notes. More often than not, scientific books present a finished product, without giving the readers a sense of the struggle that generally preceded the formalization of a theory. Being aware of this struggle should be part of the education of every future scientist because science rarely progresses in the neat and linear way it is usually presented. Seismology, as well as elasticity, upon which seismology is founded, also had their share of controversy, and it is instructive to learn how our present understanding of seismic theory has come about. In this context, an enlightening review of the history of seismology by Ben-Menahem (1995) is highly recommended.

An important component of the book are the problems. Going through them will help the reader solidify the concepts and techniques discussed in the text. All the problems have hints, so that solving them should not be difficult after the background material has been mastered. In addition, full solutions are provided on a dedicated website:

`http://publishing.cambridge.org/resources/0521817307`

Because of the obvious importance of computers in research and because going from the theory to a computer application is not always straightforward, Fortran and Matlab codes used to generate most of the figures in Chapters 6–10 have been provided on the same website.

The book is based on class notes developed for a graduate course I taught at the University of Memphis over more than ten years. The amount of material covered depended on the background of the students, but about 70% was always covered.

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