TOWARDS A PHILOSOPHY OF REAL MATHEMATICS

In this ambitious study, David Corfield attacks the widely held view that it is the nature of mathematical knowledge which has shaped the way in which mathematics is treated philosophically, and claims that contingent factors have brought us to the present thematically limited discipline. Illustrating his discussion with a wealth of examples, he sets out a variety of new ways to think philosophically about mathematics, ranging from an exploration of whether computers producing mathematical proofs or conjectures are doing real mathematics, to the use of analogy, the prospects for a Bayesian confirmation theory, the notion of a mathematical research programme, and the ways in which new concepts are justified. His highly original book challenges both philosophers and mathematicians to develop the broadest and richest philosophical resources for work in their disciplines, and points clearly to the ways in which this can be done.

DAVID CORFIELD holds a Junior Lectureship in Philosophy of Science at the University of Oxford. He is co-editor (with Jon Williamson) of *Foundations of Bayesianism* (2001), and he has published articles in journals including *Studies in History and Philosophy of Science* and *Philosophia Mathematica*.

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From the east to western Ind,
No jewel is like Rosalind.

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Preface

I should probably not have felt the desire to move into the philosophy of mathematics had it not been for my encounter with two philosophical works. The first of these was Imre Lakatos's *Proofs and Refutations* (1976), a copy of which was thrust into my hands by a good friend Darian Leader, who happens to be the godson of Lakatos. The second was an article entitled 'The Uses and Abuses of the History of Topos Theory' by Colin McLarty (1990), a philosopher then unknown to me. What these works share is the simple idea that what mathematicians think and do should be important for philosophy, and both express a certain annoyance that anyone could think otherwise.

Finding a post today as a philosopher of mathematics is no easy task. Finding a post as a philosopher of mathematics promoting change is even harder. When a discipline is in decline, conservatism usually sets in. I am, therefore, grateful beyond words to my PhD supervisor, Donald Gillies, both for his support over the last decade and for going to the enormous trouble of applying for the funding of two research projects, succeeding in both, and offering one to me. The remit of the project led me in directions I would not myself have chosen to go, especially the work reported in chapters 2 and 3, and I rather think chapters 5 and 6 as well, but this is often no bad thing. I am thus indebted to the Leverhulme Trust for their generous financial support. Thanks also to Jon Williamson, the other fortunate recipient, for discussions over tapas.

Colin McLarty has provided immense intellectual and moral support over the years, and also arranged a National Endowment of the Humanities Summer Seminar where sixteen of us were allowed the luxury of talking philosophy of mathematics for six weeks in the pleasant surroundings of Case Western Reserve University. My thanks to the NEH and to the other participants for making it such an enjoyable experience.

I should also like to acknowledge the helpful advice of Ronnie Brown, Jeremy Butterfield, James Cussens, Matthew Donald, Jeremy Gray, Colin

Howson, Mary Leng, Penelope Maddy, Stephen Muggleton, Madeline Muntersbjorn, Jamie Tappenden, Robert Thomas and Ed Wallace. This book could only have benefited from greater exposure to the intellectual ambience of the History and Philosophy of Science Department in Cambridge, where the writing was finished. Unfortunately time was not on my side. I only hope a little of the spirit of the department has trickled through into its pages.

Hilary Gaskin at Cambridge University Press has smoothed the path to publication. Four of the chapters are based on material published elsewhere. Chapter 5 is based on my chapter in Corfield and Williamson 2001, *Foundations of Bayesianism*, Kluwer. Chapters 7 and 9 are based on papers of the same title in *Studies in the History and Philosophy of Science*, 28(1): 99–121 and 32(3): 507–33. Chapter 8 is likewise based on my article in *Philosophia Mathematica* 6: 272–301. I am grateful to Kluwer, Elsevier and Robert Thomas for permission to publish them.

I should like to thank J. Scott Carter and Masahico Saito for kindly providing me with the figure displayed on the cover. It shows one of the ingenious ways they have devised of representing knotted surfaces in four-dimensional space. In chapter 10 we shall see how this type of representation permits diagrammatic calculations to be performed in higher-dimensional algebra.

Love and thanks to Oliver, Kezia and Diggory for adding three more dimensions to my life beyond the computer screen, and to my parents for all their support. This book I dedicate to Ros for fourteen years of sheer bliss.