1 Introduction

After a short historical summary, the central concepts of magnetic order and hysteresis are presented. Magnet applications are summarized, and magnetism is situated in relation to physics, materials science and industrial technology.

1.1 A brief history of magnetism

The history of magnetism is coeval with the history of science. The magnet's ability to attract ferrous objects by remote control, acting at a distance, has captivated countless curious spirits over two millenia (not least the young Albert Einstein). To demonstrate a force field that can be manipulated at will, you need only two chunks of permanent magnet or one chunk of permanent magnet and a piece of temporary magnet such as iron. Feeble permanent magnets are quite widespread in nature in the form of lodestones – rocks rich in magnetite, the iron oxide Fe_3O_4 – which were magnetized by huge electric currents in lightning strikes. Priests and people in Sumer, ancient Greece, China and pre-Colomban America were familiar with the natural magic of these magnets.

A lodestone carved in the shape of a Chinese spoon was the centrepiece of an early magnetic device, the 'South pointer'. Used for geomancy in China at the beginning of our era (Fig. 1.1), the spoon turns on the base to align its handle with the Earth's magnetic field. Evidence of the South pointer's application can be seen in the grid-like street plans of certain Chinese towns, where the axes of quarters built at different times are misaligned because of the secular variation of the direction of the horizontal component of the Earth's magnetic field.

Shen Kua, 沈括 1031–1095.

A propitious discovery, attributed to Zheng Gongliang in 1064, was that iron could acquire a thermoremanent magnetization when quenched from red heat. Steel needles thus magnetized in the Earth's field were the first artificial permanent magnets. They aligned themselves with the field when floated or suitably suspended. A short step led to the invention of the navigational compass, which was described by Shen Kua around 1088. Reinvented in Europe a century later, the compass enabled the great voyages of discovery, including the European discovery of America by Christopher Columbus in 1492 and the earlier Chinese discovery of Africa by the eunuch admiral Cheng Ho in 1433.

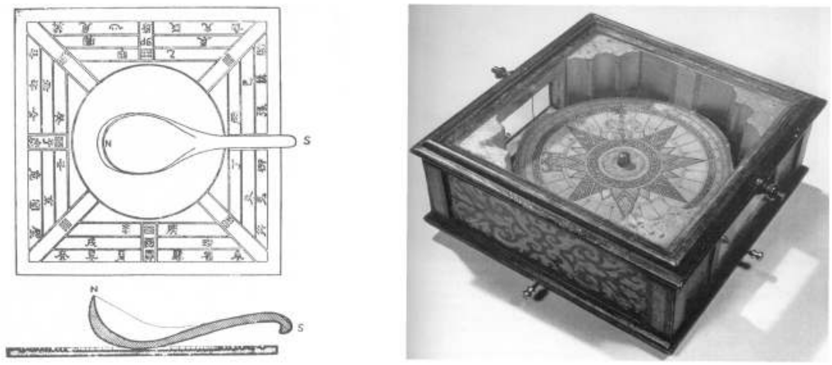
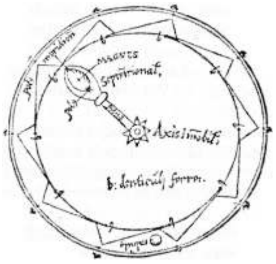


Figure 1.1

Some early magnetic devices: the 'South pointer' used for orientation in China around the beginning of the present era, and a Portuguese mariner's compass from the fifteenth century.



A perpetuum mobile, proposed by Petrus Peregrinus in 1269.

When we come to the middle ages, virtues and superstitions had accreted to the lodestone like iron filings. Some were associated with its name.[1] People dreamt of perpetual motion and magnetic levitation. The first European text on magnetism by Petrus Peregrinus describes a *perpetuum mobile*. Perpetual motion was not to be, except perhaps in the never-ending dance of electrons in atomic orbitals with quantized angular momentum, but purely passive magnetic levitation was eventually achieved at the end of the twentieth century. Much egregious fantasy was debunked by William Gilbert in his 1600 monograph *De Magnete*, which was arguably the first modern scientific text. Examination of the direction of the dipole field at the surface of a lodestone sphere or 'terella', and relating it to the observation of dip which by then had been measured at many points on the Earth's surface, led Gilbert to identify the source of the magnetic force which aligned the compass needle as the Earth itself, rather than the stars as previously assumed. He inferred that the Earth itself was a great magnet.[2]

The curious Greek notion that the magnet possessed a soul – it was animated because it moved – was to persist in Europe well into the seventeenth century, when it was finally laid to rest by Descartes. But other superstitions regarding the benign or malign influences of magnetic North and South poles remain alive and well, as a few minutes spent browsing the Internet will reveal.

William Gilbert, 1544–1603.

Magnetic research in the seventeenth and eighteenth centuries was mostly the domain of the military, particularly the British Navy. An important civilian advance, promoted by the Swiss polymath Daniel Bernoulli, was the invention in 1743 of the horseshoe magnet. This was to become magnetism's most enduring archetype. The horseshoe is an ingenious solution to the problem of making a reasonably compact magnet which will not destroy itself in its own demagnetizing field. It has remained the icon of magnetism up to the present

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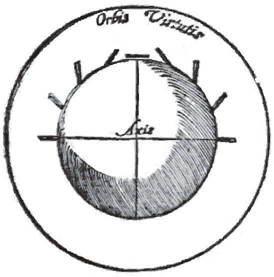
[1] In English, the word 'magnet' is derived through Latin from the Greek for Magnesian stone (*Ŏ μαγνης λῑθος*), after sources of lodestones in Asia Minor. In Sanscrit 'चुम्बक' and Romance languages – French 'l'aimant', Spanish 'imán', Portuguese 'imã' – the connotation is the attraction of opposite poles, like that of man and woman.

[2] '*Magnus magnes ipse est globus terrestris*'.



A lodestone 'terella' used by Gilbert to demonstrate how the magnetic field of the Earth resembles that of a magnet.

day. Usually red, and marked with 'North' and 'South' poles, horseshoe magnets still feature in primary school science books all over the world, despite the fact that these horseshoes have been quite obsolete for the past 50 years.

The obvious resemblances between magnetism and electricity, where like or unlike charges repel or attract, led to a search for a deeper connection between the two cousins. Luigi Galvani's 'animal electricity', stemming from his celebrated experiments on frogs and corpses, had a physical basis – nerves work by electricity. It inspired Anton Messmer to postulate a doctrine of 'animal magnetism' which was enthusiastically embraced in Parisian salons for some years before Louis XVI was moved to appoint a Royal Commission to investigate. Chaired by Benjamin Franklin, the Commission thoroughly discredited the phenomenon, on the basis of a series of blind tests. Their report, published in 1784, was a landmark of scientific rationality.

It was in Denmark in 1820 that Hans-Christian Oersted eventually discovered the true connection between electricity and magnetism by accident. He demonstrated that a current-carrying wire produced a *circumferential* field capable of deflecting a compass needle. Within weeks, André-Marie Ampère and Dominique-François Arago in Paris wound wire into a coil and showed that the current-carrying coil was equivalent to a magnet. The electromagnetic revolution was launched.

Réné Descartes, 1596–1650.

The remarkable sequence of events that ensued changed the world for ever. Michael Faraday's intuition that the electric and magnetic forces could be conceived in terms of all-pervading fields was critical. He discovered electromagnetic induction (1821) and demonstrated the principle of the electric motor with a steel magnet, a current-carrying wire and a dish of mercury. The discovery of a connection between magnetism and light followed with the magneto-optic Faraday effect (1845).

An eighteenth century horseshoe magnet.

All this experimental work inspired James Clerk Maxwell's formulation[3] of a unified theory of electricity, magnetism and light in 1864, which is summarized in the four famous equations that bear his name:

$$\nabla \cdot \boldsymbol{B} = 0, \tag{1.1a}$$

$$\epsilon_0 \nabla \cdot \boldsymbol{E} = \rho, \tag{1.1b}$$

$$(1/\mu_0)\nabla \times \boldsymbol{B} = \boldsymbol{j} + \epsilon_0 \partial \boldsymbol{E}/\partial t, \tag{1.1c}$$

$$\nabla \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t. \tag{1.1d}$$

These equations relate the electric and magnetic fields, $\boldsymbol{E}$ and $\boldsymbol{B}$ at a point in *free space* to the distributions of electric charge and current densities, ρ and $\boldsymbol{j}$ in surrounding space. A spectacular consequence of Maxwell's equations is the existence of a solution representing coupled oscillatory electric and magnetic

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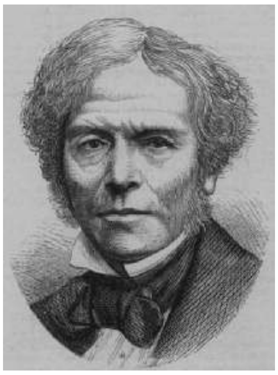
[3] 'From a long view of the history of mankind there can be little doubt that the most significant event of the nineteenth century will be judged as Maxwell's discovery of the laws of electrodynamics' (R. Feynman *The Feynman Lectures in Physics*. Vol. II, Menlo Park: Addison-Wesley (1964)).

André Marie Ampère, 1775–1836.

Hans-Christian Oersted, 1777–1851.



Michael Faraday, 1791–1867.

fields propagating at the speed of light. These electromagnetic waves extend over the entire spectrum, with wavelength Λ and frequency f, related by $c = \Lambda f$. The electric and magnetic constants ϵ_0 and μ_0 depend on definitions and the system of units, but they are related by

$$\sqrt{\epsilon_0\mu_0} = \frac{1}{c}, \tag{1.2}$$

where c is the speed of light in vacuum, 2.998×10^8 m s^{-1}. This is also the ratio of the average values of E and B in the electromagnetic wave. Maxwell's equations are asymmetric in the fields $\boldsymbol{E}$ and $\boldsymbol{B}$ because no magnetic counterpart of electric charge has ever been identified in nature. Gilbert's idea of North and South magnetic poles, somehow analogous to Coulomb's positive and negative electric charges, has no physical reality, although poles remain a conceptual convenience and they simplify certain calculations. Ampère's approach, regarding electric currents as the source of magnetic fields, has a sounder physical basis. Either approach can be used to *describe* ferromagnetic material such as magnetite or iron, whose magnetism is equally well represented by distributions of magnetic poles or electric currents. Nevertheless, the real building blocks of electricity and magnetism are *electric charges* and *magnetic dipoles*; the dipoles are equivalent to electric current loops. Dielectric and magnetic materials are handled by introducing two auxiliary fields $\boldsymbol{D}$ and $\boldsymbol{H}$, as discussed in Chapter 2.

An additional equation, due to Lorentz, gives the force on a particle with charge q moving with velocity $\boldsymbol{v}$, which is subject to electric and magnetic fields:

$$\boldsymbol{f} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}). \tag{1.3}$$

Units of $\boldsymbol{E}$ are volts per metre (or newtons per coulomb), and the units of $\boldsymbol{B}$ are newtons per ampere per metre (or tesla).

A technical landmark in the early nineteenth century was William Sturgeon's invention of the iron-cored electromagnet in 1824. The horseshoe-shaped core was temporarily magnetized by the magnetic field produced by current flowing in the windings. Electromagnets proved more effective than the weak permanent magnets then available for excitation of electric motors and generators. By the time the electron was discovered in 1897,[4] the electrification of the planet was already well advanced. Urban electrical distribution networks dispelled the tyranny of night with electric light and the stench of public streets was eliminated as horses were displaced by electric trams. Telegraph cables spanned the Earth, transmitting messages close to the speed of light for the equivalent of €20 a word.

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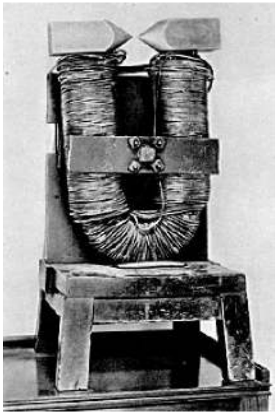
[4] The decisive step for the discovery of the electron was taken in England by Joseph John Thompson, who measured the ratio of its charge to mass. The name, derived from ἤλεκτρον the Greek word for amber, had been coined earlier (1891 in Dublin) by George Johnston Stoney.



A nineteenth century electromagnet.

James Clerk Maxwell, 1831–1879.

Despite the dazzling technical and intellectual triumphs of the electromagnetic revolution, the problem of explaining how a solid could possibly be ferromagnetic was unsolved. The magnetization of iron, $M = 1.76 \times 10^6$ amperes per metre, implies a perpetually circulating Ampèrian surface current density of the same magnitude. Currents of hundreds of thousands of amperes coursing around the surface of a magnetized iron bar appeared to be a wildly implausible proposition. Just as preposterous was Pierre Weiss's molecular field theory, dating from 1907, which successfully explained the phase transition at the Curie point where iron reversibly loses its ferromagnetism. The theory postulated an internal magnetic field parallel to, but some three orders of magnitude greater than, the magnetization. Although Maxwell's equation (1.1a) proclaims that the magnetic field $\boldsymbol{B}$ should be continuous, no field remotely approaching that magnitude has ever been detected outside a magnetized iron specimen. Ferromagnetism therefore challenged the foundations of classical physics, and a satisfactory explanation only emerged after quantum mechanics and relativity, the twin pillars on which modern physics rests, were erected in the early years of the twentieth century.

Strangely, the Ampèrian currents turned out to be associated with quantized angular momentum, and especially with the intrinsic spin of the electron, discovered by George Uhlenbeck and Samuel Goudsmit in 1925. The spin is quantized in such a way that it can have just two possible orientations in a magnetic field, 'up' and 'down'. Spin is the source of the electron's intrinsic magnetic moment, which is known as the Bohr magneton: $\mu_B = 9.274 \times 10^{-24}$ A m^2. The magnetic properties of solids arise essentially from the magnetic moments of their atomic electrons. The interactions responsible for ferromagnetism represented by the Weiss molecular field were shown by Werner Heisenberg in 1929 to be *electrostatic* in nature, originating from the quantum mechanics of the Pauli principle. Heisenberg formulated a Hamiltonian to represent the interaction of two neighbouring atoms whose total electronic spins, in units of Planck's constant $\hbar = 1.055 \times 10^{-34}$ J s, are $\boldsymbol{S}_i$ and $\boldsymbol{S}_j$, namely

$$\mathcal{H} = -2\mathcal{J}\boldsymbol{S}_i \cdot \boldsymbol{S}_j, \tag{1.4}$$

where $\mathcal{J}$ is the exchange constant; $\mathcal{J}/k_B$ is typically in the range 1–100 K. Here k_B is Boltzmann's constant, 1.3807×10^{-23} J K^{-1}. Atomic magnetic moments are associated with the electronic spins. The quantum revolution underpinning modern atomic and solid state physics and chemistry was essentially complete at the time of the sixth Solvay Congress in 1930 (Fig. 1.2). Filling in the details has proved to be astonishingly rich and endlessly useful.[5] For instance, when the exchange interaction $\mathcal{J}$ is negative (antiferromagnetic) rather than

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[5] Already in 1930 there was the conviction that all the basic problems of the physics of solids had been solved in principle; Paul Dirac said 'The underlying physical phenomena necessary for a mathematical explanation of a large part of physics and all of chemistry are now understood in principle, the only difficulty being that the exact application of these laws leads to equations much too complicated to be soluble' (P. Dirac, *Proc. Roy. Soc.* **A123**, 714 (1929)).

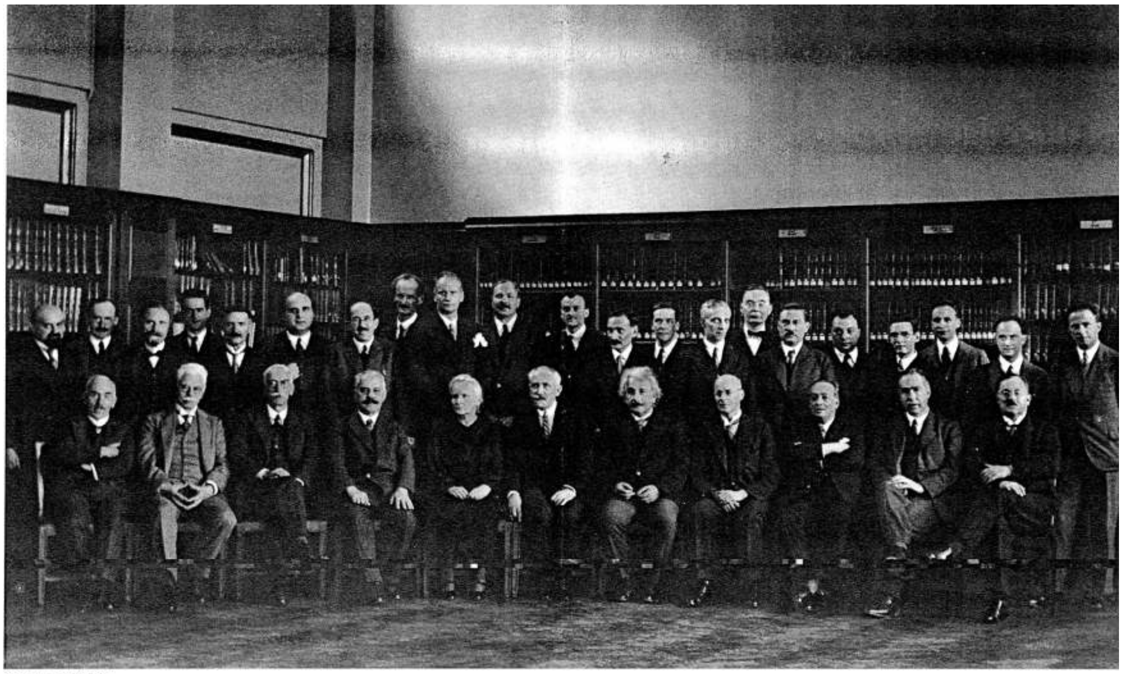

Photo Benjamin Couprie.

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Th. DE DONDER P. ZEEMAN P. WEISS A. SOMMERFELD Mme CURIE P. LANGEVIN A. EINSTEIN O. RICHARDSON B. CABRERA N. BOHR W.J. DE HAAS

Figure 1.2

Participants at the 1930 Solvay Congress, which was devoted to magnetism.

positive (ferromagnetic) there is a tendency for the spins at sites *i* and *j* to align antiparallel rather than parallel. Louis Néel pointed out in 1936 and 1948 that this leads to antiferromagnetism or ferrimagnetism, depending on the topology of the crystal lattice. Magnetite, the archetypal natural magnetic material, is a ferrimagnet.

Louis Néel, 1904–2000.

One lesson from a study of the history of magnetism is that fundamental understanding of the science may not be a prerequisite for technological progress. Yet fundamental understanding helps. The progression from the poorly differentiated set of hard and soft magnetic steels that existed at the start of the twentieth century to the wealth of different materials available today, with all sorts of useful properties described in this book, owes more to metallurgy and systematic crystal chemistry than it does to quantum physics. Only since the rare-earth elements began to be alloyed with cobalt and iron in new permanent magnets from the late 1960s onwards has quantum mechanics contributed significantly to magnetic materials development. Much progress in science is made empirically, with no recourse to basic theory. One area, however, where quantum mechanics has been of central importance for magnetism is in its interaction with electromagnetic radiation in the radiofrequency, microwave and optical ranges. The discovery of magnetic resonance methods in the 1940s

Table 1.1. The seven ages of magnetism

Period	Dates	Icon	Drivers	Materials
Ancient period	−2000–1500	Compass	State, geomancers	Iron, lodestone
Early modern age	1500–1820	Horseshoe magnet	Navy	Iron, lodestone
Electromagnetic age	1820–1900	Electromagnet	Industry/infrastructure	Electrical steel
Age of understanding	1900–1935	Pauli matrices	Academic	(Alnico)
High-frequency age	1935–1960	Magnetic resonance	Military	Ferrites
Age of applications	1960–1995	Electric screwdriver	Consumer market	Sm-Co, Nd-Fe-B
Age of spin electronics	1995–	Read head	Consumer market	Multilayers

Samuel Goudsmit, 1902–1978.

Georg Uhlenbeck, 1900–1988.

and 1950s and the introduction of powerful spectroscopic and diffraction techniques led to new insights into the magnetic and electronic structure of solids. Technology for generating and manipulating microwaves had been developed in Great Britain for the Second World War.

Recent decades have witnessed an immense expansion of magnetic applications. The science developed over a century, mostly in Europe, was ripe for exploitation throughout the industrialized world. Advances in permanent magnetism, magnetic recording and high-frequency materials underpin much of the progress that has been made with computers, telecommunications equipment and consumer goods that benefit most people on Earth. Permanent magnets have come back to replace electromagnets in a billion tiny motors manufactured every year. Magnetic recording sustains the information revolution and the Internet. There have been seminal advances in earth science, medical imaging and the theory of phase transitions that can be laid at the door of magnetism. This long and promising history of magnetism can be envisaged as seven ages, which are summarized in Table 1.1. The third millenium sees us at the threshold of the seventh age, that of spin electronics. Conventional electronics has ignored the spin on the electron. We are just now beginning to learn how to manipulate spin currents and to make good use of them.

1.2 Magnetism and hysteresis

The most striking manifestation of magnetism in solids is the spontaneous magnetization of ferromagnetic materials such as iron or magnetite. Spontaneous magnetism is usually associated with hysteresis,[6] a phenomenon studied by James Ewing, and named by him in 1881.[7]

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[6] ‘Hysteresis’ was coined from the greek ὕστερειν, to lag behind.

[7] Ewing, a Scot, was appointed as a foreign Professor of Engineering at the University of Tokyo by the Meiji government in 1878. He is regarded as the founder of magnetic research in Japan.

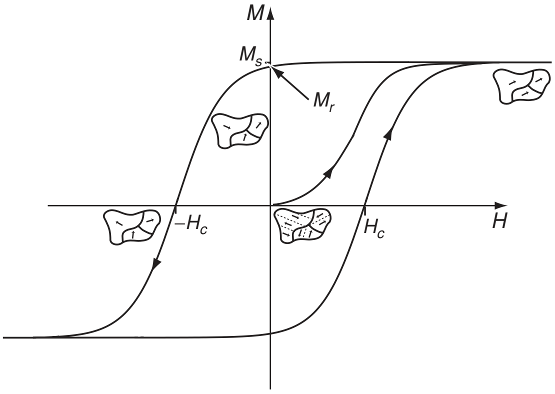


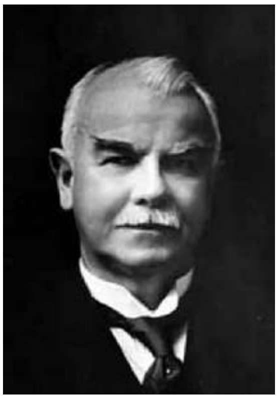
Figure 1.3

The hysteresis loop of a ferromagnet. Initially in an unmagnetized, virgin state. Magnetization appears as an imposed magnetic field $\boldsymbol{H}$, modifies and eventually eliminates the microstructure of ferromagnetic domains magnetized in different directions, to reveal the spontaneous magnetization M_s. The remanence M_r which remains when the applied field is restored to zero, and the coercivity H_c, which is the reverse field needed to reduce the magnetization to zero, are marked on the loop.

1.2.1 The ferromagnetic hysteresis loop

The essential practical characteristic of any ferromagnetic material is the irreversible nonlinear response of magnetization $\boldsymbol{M}$ to an imposed magnetic field $\boldsymbol{H}$. This response is epitomized by the hysteresis loop. The material responds to $\boldsymbol{H}$, rather than $\boldsymbol{B}$, for reasons discussed in the next chapter where we distinguish the applied and internal fields. In free space, $\boldsymbol{B}$ and $\boldsymbol{H}$ are simply proportional. Magnetization, the magnetic dipole moment per unit volume of material, and the H-field are both measured in amperes per metre (A m^{-1}). Since this is a rather small unit – the Earth's magnetic field is about 50 A m^{-1} – the multiples kA m^{-1} and MA m^{-1} are often employed. The applied field must be comparable in magnitude to the magnetization in order to trace a hysteresis loop. The values of spontaneous magnetization M_s of the ferromagnetic elements Fe, Co and Ni at 296 K are 1720, 1370 and 485 kA m^{-1}, respectively. That of magnetite, Fe_3O_4, is 480 kA m^{-1}. A large electromagnet may produce a field of 1000 kA m^{-1} (1 MA m^{-1}), using coils carrying currents of order 100 A.

Hard magnetic materials[8] have broad, square $M(H)$ loops. They are suitable for permanent magnets because, once magnetized by applying a field $H \geq M_s$ sufficient to saturate the magnetization, they remain in a magnetized state when the field is removed. **Soft magnetic materials** have very narrow loops. They are temporary magnets, readily losing their magnetization as soon as the field is removed. The applied field serves to unveil the spontaneous ferromagnetic order that already exists on the scale of microscopic domains. These domain structures are illustrated schematically on the hysteresis loop of Fig. 1.3 for the unmagnetized state at the origin, the saturated state where $M = M_s$, the remanent state in zero field where $M = M_r$ and the state at $H = H_c$, the coercive field where M changes sign. M_r and H_c are known as the **remanence** and the **coercivity**. Magnetic domains were proposed by James Ewing and the principles of domain theory were established by Lev Landau and Evgenii Lifschitz in 1935.



James Ewing, 1855–1935.

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[8] The terms *hard* and *soft* for magnets originated from the mechanical properties of the corresponding magnetic steels.

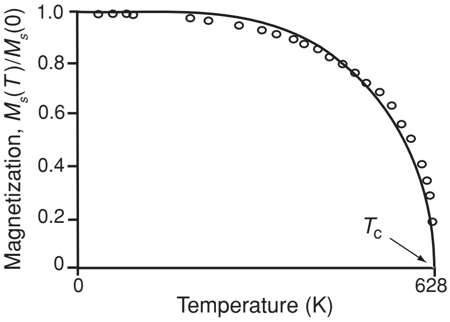


Figure 1.4

Temperature dependence of the spontaneous magnetization of nickel. The Curie point at 628 K is marked.

The hysteresis loop is central to technical magnetism; physicists endeavour to explain it, materials scientists aim to improve it and engineers work to exploit it. The loop combines information on an intrinsic magnetic property, the spontaneous magnetization M_s which exists within a domain of a ferromagnet, and two extrinsic properties, the remanence M_r and coercivity H_c, which depend on a host of extraneous factors including the sample shape, surface roughness, microscopic defects and thermal history, as well as the rate at which the field is swept in order to trace the loop.

1.2.2 The Curie temperature

The spontaneous magnetization due to alignment of the atomic magnetic moments depends on temperature, and it falls precipitously to zero at the Curie temperature T_C. The magnetic ordering is a continuous thermodynamic phase transition with a λ-shaped anomaly in specific heat, associated with disordering of the atomic dipole moments. Above T_C, $M_s(T)$ is zero; below T_C, $M_s(T)$ is reversible. This behaviour is illustrated for nickel in Fig. 1.4.

The Curie temperatures of the three ferromagnetic metals, iron, cobalt and nickel, are 1044 K, 1388 K and 628 K, respectively. No material is known to have a higher Curie temperature than cobalt. Magnetite has a Curie temperature of 856 K.

1.2.3 Coercivity

Pierre Curie, 1859–1906.

The progress in the twentieth century which has spawned such a range of magnetic applications can be summarized in three words – *mastery of coercivity*. No new ferromagnetic material has been discovered with a magnetization greater than that of 'permendur', $Fe_{65}Co_{35}$, for which $M_s = 1950$ kA m^{-1}, but coercivity which barely spanned two orders of magnitude in 1900, from the softest soft iron to the hardest magnet steel, now ranges over eight orders of

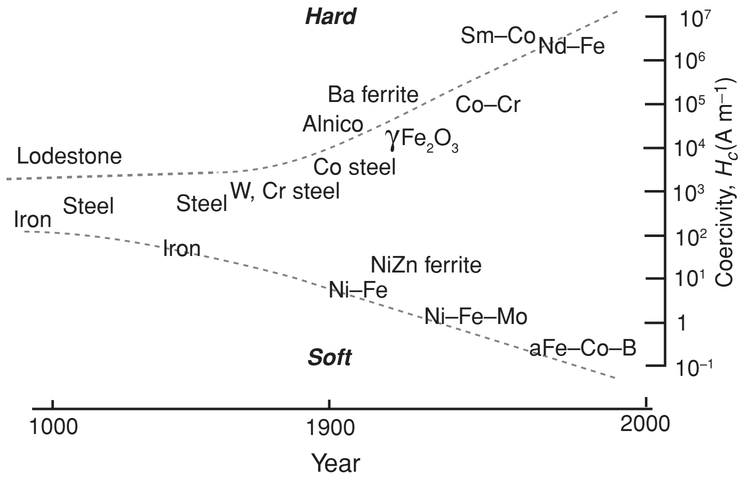
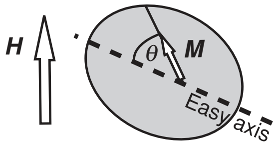


Figure 1.5

Progress in expanding the range of coercivity of magnetic materials during the twentieth century.

magnitude, from less than 0.1 A m^{-1} to more than 10 MA m^{-1}, as shown in Fig. 1.5.

1.2.4 Anisotropy



Magnetization is not necessarily parallel to applied field, unless $\boldsymbol{H}$ is applied in an easy direction.

The natural direction of magnetization in a microscopic ferromagnetic domain is usually constrained to lie along one or more easy axes. Since magnetism is associated with circulating electron currents, time reversal symmetry requires that a state with a certain magnetization distribution $\boldsymbol{M}(r)$ should have the same energy as the state with reversed magnetization along the same axis, $-\boldsymbol{M}(r)$. This tendency is represented by the anisotropy energy E_a, of which the leading term is

$$E_a = K_u \sin^2 \theta, \tag{1.5}$$

where θ is the angle between the direction of $\boldsymbol{M}$ and the easy axis. Here E_a and K_u, the anistropy constant, are measured in J m^{-3}. Typical values range from less than 1 kJ m^{-3} to more than 10 MJ m^{-3}. Anistropy sets on upper bound on the coercivity available in hard magnets. We show in Chapter 7 that

$$H_c < 2K_u/\mu_0 M_s, \tag{1.6}$$

where the magnetic constant μ_0 is $4\pi \times 10^{-7}$ J A^{-2} m^{-1}. Anisotropy also leads to unwanted coercivity in soft magnets. It may be noted from the units that μ_0 always multiplies H^2 or MH in expressions for magnetic energy per unit volume.

Atomic densities in solids are around $n = 10^{29}$ m^{-3}, so if anisotropy energy per atom is expressed in terms of an equivalent temperature using $E_a/n = k_B T$, it is in the range 1 mK–10 K. The energy is usually small in relation to the Curie temperature, but it is nevertheless decisive in determining the hysteresis.