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Introduction

1.1 Mass, length and time scales in astrophysics

Astrophysics is the science dealing with stars, galaxies and the entire Universe. The aim of this book is to present astrophysics as a serious science based on quantitative measurements and rigorous theoretical reasoning.

The standard units of mass, length and time that we use (cgs or SI units) are appropriate for our everyday life. For expressing results of astrophysical measurements, however, they are not the most convenient units. Let us begin with a discussion of the basic units we use in astrophysics and the scales of various astrophysical objects we encounter.

Unit of mass

The mass of the Sun is denoted by the symbol $M_\odot$ and is often used as the unit of mass in astrophysics. Its value is

$$M_\odot = 1.99 \times 10^{30}\ \mathrm{kg}. \tag{1.1}$$

Although intrinsic brightnesses and sizes of stars vary over several orders of magnitude, the masses of most stars lie within a relatively narrow range from $0.1 M_\odot$ to $20 M_\odot$. The reason behind this will be discussed in §3.6.1. Hence the solar mass happens to be a very convenient unit in stellar astrophysics. Sometimes, however, we have to deal with objects much more massive than stars. The mass of a typical galaxy can be $10^{11} M_\odot$. Globular clusters, which are dense clusters of stars having nearly spherical shapes, typically have masses around $10^5 M_\odot$.

Unit of length

The average distance of the Earth from the Sun is called the *Astronomical Unit* (abbrev. AU). Its value is

$$\mathrm{AU} = 1.50 \times 10^{11}\ \mathrm{m}. \tag{1.2}$$

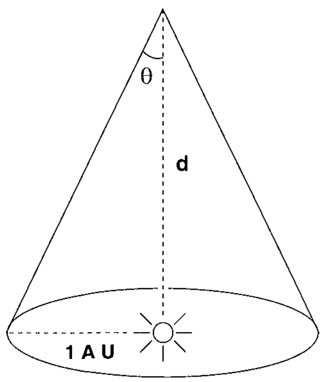

Fig. 1.1 Definition of parsec.

It is a very useful unit for measuring distances within the solar system. But it is too small a unit to express the distances to stars and galaxies.

As the Earth goes around the Sun, the nearby stars seem to change their positions very slightly with respect to the faraway stars. This phenomenon is known as *parallax*. Let us consider a star on the polar axis of the Earth's orbit at a distance d away, as shown in Figure 1.1. The angle θ is half of the angle by which this star appears to shift with the annual motion of the Earth and is defined to be the parallax. It is obviously given by

$$\theta = \frac{1\ \text{AU}}{d}. \tag{1.3}$$

The *parsec* (abbrev. pc) is the distance where the star has to be so that its parallax turns out to be $1''$. Keeping in mind that $1''$ is equal to $\pi/(180 \times 60 \times 60)$ radians, it is easily found from (1.3) that

$$\text{pc} = 3.09 \times 10^{16}\ \text{m}. \tag{1.4}$$

It may be noted that 1 pc is equal to 3.26 light years – a unit very popular with popular science writers, but rarely used in serious technical literature. For even larger distances, the standard units are kiloparsec (10^3 pc, abbrev. kpc), megaparsec (10^6 pc, abbrev. Mpc) and gigaparsec (10^9 pc, abbrev. Gpc).

The star nearest to us, Proxima Centauri, is at about a distance of 1.31 pc. Our Galaxy and many other galaxies like ours are shaped like disks with thickness of order 100 pc and radius of order 10 kpc. The geometric mean between these two distances, which is 1 kpc, may be taken as a measure of the galactic size. The Andromeda Galaxy, one of the nearby bright galaxies, is at a distance of about 0.74 Mpc. The distances to very faraway galaxies are of order Gpc. It should be kept in mind that light from very distant galaxies started when the Universe was much younger and the concept of distance to such galaxies is not a very straightforward concept, as we shall see in §14.4.1. It is useful to keep

Table 1.1 Approximate conversion factors to be memorized.

$M_\odot$	$\approx$	2×10^{30} kg
pc	$\approx$	3×10^{16} m
yr	$\approx$	3×10^{7} s

the following rule of thumb in mind: pc is a measure of interstellar distances, kpc is a measure of galactic sizes, Mpc is a measure of intergalactic distances and Gpc is a measure of the visible Universe.

Unit of time

Astrophysicists have to deal with very different time scales. On the one hand, the age of the Universe is of the order of a few billion years. On the other hand, there are pulsars which emit pulses periodically after intervals of fractions of a second. There is no special unit of time. Astrophysicists use years for large time scales and seconds for small time scales, the conversion factor being

$$\mathrm{yr} = 3.16 \times 10^7 \ \mathrm{s}. \tag{1.5}$$

The stars typically live for millions to billions of years. Occasionally, one uses the unit gigayear (10^9 yr, abbrev. Gyr). The age of the Sun is believed to be about 4.5 Gyr.

The importance of order of magnitude estimates

We can often have good guesses of the values of various quantities around us even without making accurate measurements. By looking at a table, I may make a rough estimate that its side is about 1 m long. By lifting a sack of potatoes, I may make a rough estimate that it weighs about 5 kg. Careful measurements usually show that such guesses are not very much off the mark. We never have the suspicion that a measurement of the length of a table would yield values like either 10^{-2} cm or 100 km. For astrophysical quantities, we usually do not have any such direct feeling. If somebody tells us that the mass of the Sun is either 10^{20} kg or 10^{40} kg, there would be nothing in our everyday experiences on the basis of which we could say that these values are unreasonable. Hence, in astrophysics, it is often very useful first to make order of magnitude estimates of various quantities before embarking on a more detailed calculation. Throughout this book, we shall be making various order of magnitude estimates. For such purposes, it is useful to remember the conversion factors given in Table 1.1. The accurate values of these conversion factors are given in (1.1), (1.4) and (1.5).

Although the emphasis in this book will be on understanding things and not memorizing things, we would urge the readers to commit the conversion factors of Table 1.1 to memory. They are used too often in making various order of magnitude estimates!

1.2 The emergence of modern astrophysics

From the dawn of civilization, human beings have wondered about the starry sky. Astronomy is one of the most ancient sciences. Perhaps mathematics and medicine are the only other sciences which can claim as ancient a tradition as astronomy. But modern astrophysics, which arose out of a union between astronomy and physics, is a fairly recent science; it can be said to have been born in the middle of the nineteenth century.

Let us say a few words about ancient astronomy. Early humans noticed that most stars did not seem to change their positions with respect to each other. The seven stars of the Great Bear occupy the same relative positions night after night. But a handful of starlike objects – the planets – kept on changing their positions with respect to the background stars. It was noticed that there was a certain regularity in the movements of the planets. Building a model of the planetary motions was the outstanding problem of ancient astronomy, which reached its culmination in the geocentric theory of Hipparchus (second century BC) and Ptolemy (second century AD). Ptolemy's *Almagest*, which luckily survived the ravages of time, has come down to us as one of the greatest classics of science and provides the definitive account of the geocentric model. The scientific Renaissance of Europe began with Copernicus (1543) showing that a heliocentric model provided a simpler explanation of the planetary motions than the geocentric model. The new physics developed by Galileo and Newton finally provided a dynamical theory which could be used to calculate the orbits of planets around the Sun.

Only very rarely a branch of science reaches a phase when the practitioners of that science feel that all the problems which that branch of science had set out to solve had been adequately solved. With the development of Newtonian mechanics, planetary astronomy reached a kind of finality. Even the complicated techniques of calculating perturbations to planetary orbits due to the larger planets got perfected by the nineteenth century. Astronomers then turned their attention beyond the solar system. Telescopes also became sufficiently large by the middle of the nineteenth century to reveal some of the secrets of the stellar world to us. It may be mentioned that, with the heralding of the Space Age in the middle of the twentieth century, research in planetary science has blossomed again. However, modern planetary science has become a scientific discipline quite distinct from astrophysics and we shall not discuss about planets in this book.

If stars are distributed in a three-dimensional space and the Earth is going round the Sun, then nearby stars should appear to change their positions with the movement of the Earth, i.e. they should display parallax. Now we know that even the nearest stars have too little parallax to be detected by the naked eye. Certainly no parallax observations were available at the time of Copernicus. While proposing that the Earth moves around the Sun, Copernicus (1543) himself was bothered by the question why stars showed no parallax and correctly guessed that the stars may just be too far away. Ever since the invention of the telescope, astronomers have been on the lookout for parallax. Finally, in the fateful year 1838, three astronomers working in three different countries almost simultaneously reported the first parallax measurements (Bessel in Germany, Struve in Russia and Henderson in South Africa). This forever demolished the Aristotelian belief that stars are studded on the two-dimensional inner surface of a crystal sphere. Suddenly the sky ceased to be a two-dimensional globe and opened into an apparently limitless three-dimensional space! The stars are not static objects in space. The component of velocity perpendicular to the line of sight would lead to the change of position of a star in the sky. Such motions in the globe of the sky are called *proper motions*. Even Barnard's star, which has the largest proper motion of about $10''$ per yr, would take 360 yr to move through 1° in the sky. Most stars have much smaller proper motions and it is no wonder the appearance of the sky has not changed that much in the last 2000 yr. Some of the first measurements of proper motions were also made in the middle of the nineteenth century and it became clear that stars are luminous objects wandering around in the vast, dark three-dimensional space.

Another momentous event took place in the middle of the nineteenth century. Bunsen and Kirchhoff (1861) provided the first correct explanation of the dark lines observed by Fraunhofer (1817) in the solar spectrum and realized that the presence of various chemical elements in the Sun can be inferred from those dark lines. As soon as astronomers started looking carefully at the stellar spectra, it became clear that the Sun and the stars are made up of the same chemical elements which are found on the Earth. This discovery provided a death blow to the other Aristotelian doctrine that heavenly bodies are made up of the element ether which is different from terrestrial elements and obeyed different laws of physics. Newton had shown that planets obeyed the same laws of physics as falling objects at the Earth's surface. It now became clear that stars are made up of the same stuff as the Earth and the laws of physics discovered in the terrestrial laboratories should hold for them.

With the realization that the laws of physics can be applied to understand the behaviour of stars, the modern science of astrophysics was born. Nowadays the words 'astronomy' and 'astrophysics' are used almost interchangeably. Although modern astrophysicists study problems completely different from the problems studied by ancient astronomers, two very useful concepts introduced by ancient astronomers are still universally used. One is the concept of celestial coordinates, and the other is the magnitude scale for describing the brightness of a celestial object. We now turn to these two topics.

1.3 Celestial coordinates

The sky appears as a spherical surface above our heads. We call it the *celestial sphere*. Just as the position of a place on the Earth's surface can be specified with the latitude and longitude, the position of an astronomical object on the celestial sphere can be specified with two similar coordinates. These coordinates are defined in such a way that faraway stars which appear immovable with respect to each other have fixed coordinates. Objects like planets which move with respect to them will have their coordinates changing with time.

The coordinate corresponding to latitude is called the *declination*. The points where the Earth's rotation axis would pierce the celestial sphere are called *celestial poles*. The north celestial pole is at present close to the pole star. The great circle on the celestial sphere vertically above the Earth's equator is called the *celestial equator*. The declination is essentially the latitude on the celestial sphere defined with respect to the celestial poles and equator. Something lying on the celestial equator has declination zero, whereas the north pole has declination $+\pi/2$.

The coordinate corresponding to longitude is called the *right ascension* (R.A. in brief). Just as the zero of longitude is fixed by taking the longitude of Greenwich as zero, we need to fix the zero of R.A. for defining it. This is done with the help of a great circle called the *ecliptic*. Since the Earth goes around the Sun in a year, the Sun's position with respect to the distant stars, as seen by us, keeps changing and traces out a great circle in the sky. The ecliptic is this great circle. Twelve famous constellations (known as the *signs of the zodiac*) appear on the ecliptic. It was noted from almost prehistoric times that the Sun happens to be in different constellations in different times of the year. We cannot, of course, directly see a constellation when the Sun lies in it. But, by looking at the stars just after sunset and just before sunrise, ancient astronomers could infer the position of the Sun in the celestial sphere. The celestial equator and the ecliptic are inclined at an angle of about $23\frac{1}{2}^\circ$ and intersect at two points, as shown in Figure 1.2. One of these points, lying in the constellation Aries, is taken as the zero of R.A. When the Sun is at this point, we have the vernal equinox. It is a standard convention to express the R.A. in hours rather than in degrees. The celestial sphere rotates around the polar axis by 15° in one hour. Hence one hour of R.A. corresponds to 15°.

The declination and R.A. are basically defined with respect to the rotation axis of the Earth, which fixes the celestial poles and equator. One problematic aspect of introducing coordinates in this way is that the Earth's rotation axis

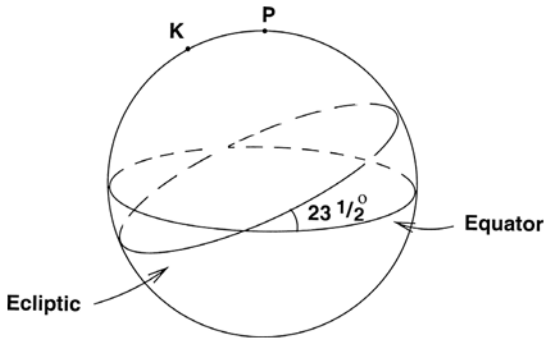

Fig. 1.2 The celestial sphere with the equator and the ecliptic indicated on it. The celestial pole is denoted by P, whereas K is the pole of the ecliptic.

is not fixed, but precesses around an axis perpendicular to the plane of the Earth's orbit around the Sun. This means that the point P in Figure 1.2 traces out an approximate circle in the celestial sphere slowly in about 25,800 years, around the pole K of the ecliptic. This phenomenon is called *precession* and was discovered by Hipparchus (second century BC) by comparing his observations with the observations made by earlier astronomers about 150 years previously. The precession is caused by the gravitational torque due to the Sun acting on the Earth and can be explained from the dynamics of rigid bodies (see, for example, Goldstein, 1980, §5–8). Due to precession, the positions of the celestial poles and the celestial equator keep changing slowly with respect to fixed stars. Hence, if the declination and the R.A. of an astronomical object at a time are defined with respect to the poles and the equator at that time, then certainly the values of these coordinates will keep changing with time. The current convention is to use the coordinates defined with respect to the positions of the poles and the equator in the year 2000.

Many ground-based optical telescopes have been traditionally designed to have *equatorial mounting*, which means that the main axis of the telescope is parallel to the rotation axis of the Earth. The telescope is designed such that it can have two kinds of motion. Firstly, it can be rotated towards or away from the axis of mounting (which is the Earth's rotation axis). Secondly, the telescope can be moved to generate a conical surface with this axis as the central axis. Suppose we want to turn the telescope towards an object of which the declination and R.A. are known. The first kind of motion enables us to set the telescope at the correct declination. The second kind of motion allows us to turn it to various values of R.A. at that declination.

The main advantage of using the declination and R.A. is that an equatorially mounted telescope can easily be turned to an object of which we know the declination and R.A. However, there is another coordinate system, called *galactic coordinates*, widely used in galactic studies. In this system, the plane of our

Galaxy is taken as the equator and the direction of the galactic centre as seen by us (in the constellation Sagittarius) is used to define the zero of longitude.

1.4 Magnitude scale

Suppose we have two series of lamps – the first series with lamps having intensities I_0, $2I_0$, $3I_0$, $4I_0$ …, whereas the lamps in the second series have intensities $I_0, 2I_0, 4I_0, 8I_0$ …. When we look at the two series of lamps, it is the second series which will appear to have lamps of steadily increasing intensity. In other words, the human eye is more sensitive to a geometric progression of intensity rather than an arithmetic progression. The magnitude scale for describing apparent brightnesses of celestial objects is based on this fact.

On the basis of naked eye observations, the Greek astronomer Hipparchus (second century BC) classified all the stars into six classes according to their apparent brightnesses. We can now of course easily measure the apparent brightness quantitatively. It appears that stars in any two successive classes, on the average, differ in apparent brightness by the same common factor. A quantitative basis of the magnitude scale was given by Pogson (1856) by noting that the faintest stars visible to the naked eye are about 100 times fainter compared to the brightest stars. Since the brightest and faintest stars differ by five magnitude classes, stars in two successive classes should differ in apparent brightness by a factor $(100)^{1/5}$. Suppose two stars have apparent brightnesses l_1 and l_2, whereas their magnitude classes are m_1 and m_2. It is clear that

$$\frac{l_2}{l_1} = (100)^{\frac{1}{5}(m_1 - m_2)}. \tag{1.6}$$

Note that the magnitude scale is defined in such a fashion that a fainter object has a higher value of magnitude. On taking the logarithm of (1.6), we find

$$m_1 - m_2 = 2.5 \log_{10} \frac{l_2}{l_1}. \tag{1.7}$$

This can be taken as the definition of *apparent magnitude* denoted by m, which is a measure of the apparent brightness of an object in the sky.

Since a star emits electromagnetic radiation in different wavelengths, one important question is: what is the wavelength range over which we consider the electromagnetic radiation emitted by a star to measure its apparent brightness quantitatively? If we use apparent brightnesses based on the radiation in all wavelengths, then the magnitude defined from it is called the *bolometric magnitude*. Since any device for measuring intensity of light does not respond to all wavelengths in the same way, finding the bolometric magnitude from measurements with a particular device is not straightforward. A much more convenient system, called the *Ultraviolet–Blue–Visual system* or the *UBV* *system*, was introduced by Johnson and Morgan (1953) and is now universally used by astronomers. In this system, the light from a star is made to pass through filters which allow only light in narrow wavelength bands around the three wavelengths: 3650 Å, 4400 Å and 5500 Å. From the measurements of the intensity of light that has passed through these filters, we get magnitudes in ultraviolet, blue and visual, usually denoted by U, B and V. Typical examples of V magnitudes are: the Sun, $V = -26.74$; Sirius, the brightest star, $V = -1.45$; faintest stars measured, $V \approx 27$.

Suppose we consider a reddish star. It will have less brightness in B band compared to V band. Hence its B magnitude should have a larger numerical value than its V magnitude. So we can use $(B - V)$ as an indication of a star's colour. The more reddish a star, the larger will be the value of $(B - V)$.

The *absolute magnitude* of a celestial object is defined as the magnitude it would have if it were placed at a distance of 10 pc. The relation between relative magnitude m and absolute magnitude M can easily be found from (1.7). If the object is at a distance d pc, then $(10/d)^2$ is the ratio of its apparent brightness and the brightness it would have if it were at a distance of 10 pc. Hence

$$m - M = 2.5 \log_{10} \frac{d^2}{10^2},$$

from which

$$m - M = 5 \log_{10} \frac{d}{10}. \tag{1.8}$$

The absolute magnitude in the V band, denoted by M_V, is often used as a convenient quantity to indicate the intrinsic brightness of an object.

1.5 Application of physics to astrophysics. Relevance of general relativity

Astrophysics is a supreme example of applied physics. To be a competent astrophysicist, first and foremost one has to be a competent physicist. Virtually all branches of physics are needed in the study of astrophysics. Classical mechanics, electromagnetic theory, optics, thermodynamics, statistical mechanics, fluid dynamics, plasma physics, quantum mechanics, atomic physics, nuclear physics, particle physics, special and general relativity – there is no branch of physics which does not find application in some astrophysical problem or other. We shall use results from all these branches of physics in this book. In the astrophysical setting, however, the laws of physics are often applied to extremes of various physical conditions like density, pressure, temperature, velocity, angular velocity, gravitational field, magnetic field, etc. – well beyond the limits for which the laws have been tested in the laboratory. For example, the vacuousness of the intergalactic space is much more than the best vacuums we can create at the present time, whereas the interiors of neutron stars may have the almost inconceivable density of 10^{17} kg m^{-3}. Only in one case, human beings may have been able to surpass Nature. There are good reasons to suspect that temperatures lower than 2.73 K never existed anywhere in the Universe until scientists succeeded in creating such temperatures about a century ago.

At the first sight, it may seem that the astrophysicists are concerned with the macro-world of very large systems like stars and galaxies, which is far removed from the micro-world of atoms, nuclei and elementary particles. However, it turns out very often that we need the physics of the micro-world to make sense of the macro-world of astrophysics. One example is the famous Chandrasekhar mass limit of white dwarf stars, which will be derived in §5.3. It was found by Chandrasekhar (1931) that the maximum mass which white dwarfs (which are compact dead stars in which no more energy generation takes place) can have is given by

$$M_{\rm Ch} = 2.018 \frac{\sqrt{6}}{8\pi} \left(\frac{hc}{G} \right)^{3/2} \frac{1}{m_{\rm H}^2 \mu_{\rm e}^2}, \tag{1.9}$$

where h is Planck's constant, $m_{\rm H}$ is the mass of hydrogen atom and $\mu_{\rm e}$ is something called the mean molecular weight of electrons (to be introduced in §5.2) having a value close to 2. On putting numerical values of various quantities, $M_{\rm Ch}$ turns out to be about $1.4 M_\odot$. Thus the constants of the atomic world like h and $m_{\rm H}$ determine the mass limit of a vast object like a white dwarf star. It is this interplay between the physics of the micro-world and the physics of the macro-world which makes modern astrophysics such a fascinating scientific discipline. Very often major breakthroughs in micro-physics have a big impact in astrophysics, and occasionally discoveries in astrophysics have provided new insights in micro-physics.

We shall assume the readers of this book to have a working knowledge of mechanics, electromagnetic theory, thermal physics and quantum physics at an advanced undergraduate or beginning graduate level. General relativity happens to be a branch of physics which is often not included in a regular physics curriculum, but which is applied in some areas of astrophysics. Till Chapter 11, we proceed without assuming any background of general relativity. Then, only in the last three chapters of this book, we give an introduction to general relativity and consider its applications to astrophysical problems. Readers unwilling to learn general relativity can still get a reasonably rounded background of modern astrophysics from this book by studying till Chapter 11. We now make a few comments on the circumstances in which general relativity is expected to be important and what a reader misses if he or she is ignorant of general relativity.

Even readers without any technical knowledge of general relativity would have heard of black holes, which are objects with gravitational fields so strong that even light cannot escape. Let us try to find out when this happens.