

Chaos in Dynamical Systems

In the new edition of this classic textbook Ed Ott has added much new material and has significantly increased the number of homework problems. The most important change is the addition of a completely new chapter on control and synchronization of chaos. Other changes include new material on riddled basins of attraction, phase locking of globally coupled oscillators, fractal aspects of fluid advection by Lagrangian chaotic flows, magnetic dynamos and strange nonchaotic attractors.

Over the past few decades scientists, mathematicians and engineers have come to understand that a large variety of systems exhibit complicated evolution with time. This complicated behavior, known as chaos, occurs so frequently that it has become important for workers in many disciplines to have a good grasp of the fundamentals and basic tools of the science of chaotic dynamics.

Topics in the book include: attractors; basins of attraction; one-dimensional maps; fractals; Hausdorff dimensions; symbolic dynamics; stable and unstable manifolds; Lyapunov exponents; metric and topological entropy; chaotic transients; fractal basin boundaries; chaotic scattering; quasiperiodicity; Hamiltonian systems; KAM tori; period doubling cascades; the intermittency transition to chaos; crises; bifurcations to chaos in scattering problems and in fractal basin boundaries; the characterization of dynamics by unstable periodic orbits; control and synchronization of chaos; and quantum chaos in time-dependent bounded systems, as well as in temporarily kicked and scattering problems. Homework problems are included throughout the book.

This new edition will be of interest to advanced undergraduates and graduate students in science, engineering and mathematics taking courses in chaotic dynamics, as well as to researchers in the subject.

EDWARD OTT is currently on the faculty of the University of Maryland where he holds the title of Distinguished University Professor of Physics and of Electrical and Computer Engineering. Before coming to Maryland in 1979, he was a Professor of Electrical Engineering at Cornell University (1968–1979). Prof. Ott's early research was on plasma physics and charged particle beams, including research on space plasmas, fusion plasmas, intense ion beams and electromagnetic wave generation by electron beams. Since the early 1980s, Prof. Ott's main research interests have been in nonlinear dynamics and its applications to problems in science and engineering. Some of this work includes contributions in the areas of bifurcations of chaotic sets, the fractal dimension of strange attractors, the structure of basin boundaries, applications of chaotic dynamics to problems in fluids and plasmas, and the control and synchronization of chaos. Prof. Ott has also been active in the education of students in nonlinear dynamics. He is an author of over 300 research articles in scientific journals.

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Preface to the first edition

Although chaotic dynamics had been known to exist for a long time, its importance for a broad variety of applications began to be widely appreciated only within the last decade or so. Concurrently, there has been enormous interest both within the mathematical community and among engineers and scientists. The field continues to develop rapidly in many directions, and its implications continue to grow. Naturally, such a situation calls for textbooks to serve the need of providing courses to students who will eventually utilize concepts of chaotic dynamics in their future careers. A variety of chaos texts now exists. In my teaching of several courses on chaos, however, I found that the existing texts were not altogether suitable for the type of course I was giving, with respect to both level and coverage of topics. Hence I was motivated to prepare and circulate notes for my class, and these notes led to this book. The book is intended for use in a graduate course for scientists and engineers. Accordingly, any mathematical concepts that such readers may not be familiar with (e.g., measure, Cantor sets, etc.) are introduced and informally explained as needed. While the intended readers are not mathematicians, there is a greater emphasis on basic mathematical concepts than in most other books that address the same audience. The style is pedagogical, and it is hoped that the very interesting, sometimes difficult, concepts that are the backbone for studies of chaos are made clear. The coverage is broad, including such topics as multifractals, quantum chaos, embedding, chaotic scattering, etc. Thus the book can also serve as a reference for workers in the field. There is too much in this book for a single one semester course. Hence it is expected that a teacher would select parts in designing a course; for example, one choice might be to base a one semester

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introductory course on Chapters 1–4, possibly supplemented by a few sections from later chapters. The author has also taught more advanced courses that utilized material now contained in Chapters 7, 9 and 10,* supplemented by readings from current research papers.

I wish to thank my students and colleagues who read and commented on various versions and parts of the manuscript. Special thanks in this regard are owed to George Schmidt, Artur Lopes, Mingzhou Ding and Reinhold Blümel. I also wish to thank Denise Best and, especially, Patsy Keehn for their expert typing of the manuscript. Finally, I thank my wife, Mary, and my children, William and Susan, for their patience and support while this book was being prepared.

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* Chapter 10 of the first edition corresponds to Chapter 11 of this second edition.

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Preface to the second edition

This second edition updates and expands the first edition. The most important change is a new chapter on control and synchronization of chaos (Chapter 10). Further additions have been made throughout the book, including new material on riddled basins of attraction, phase locking of globally coupled oscillators, fractal aspects of fluid advection by Lagrangian chaotic flows, magnetic dynamos and strange nonchaotic attractors. Also, twenty-eight new homework problems for students have been added.

February 2002
College Park

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