

MAGNETOHYDRODYNAMIC TURBULENCE

This book presents an introduction to, and state-of-the-art account of, magnetohydrodynamic (MHD) turbulence, an active field both in general turbulence theory and in various areas of astrophysics. The book starts by introducing the MHD equations and certain useful approximations. The transition to turbulence is then discussed, including the problem of finite-time singularities of the ideal equations and the excitation of instabilities. The second part of the book deals with incompressible MHD turbulence, the macroscopic aspects connected with the various self-organization processes, the phenomenology of the turbulence spectra, two-point closure theory, and intermittency. The third part considers two extensions: two-dimensional turbulence and compressible (in particular, supersonic) turbulence. Because of the similarities in the theoretical approach, these chapters start with a brief account of the corresponding methods developed in hydrodynamic turbulence. The final part of the book is devoted to three astrophysical topics: turbulence in the solar wind, in accretion disks, and in the interstellar medium. This book is suitable for graduate students and researchers working in turbulence theory, plasma physics, and astrophysics.

DIETER BISKAMP received his Ph.D. from the University of Munich. Following a postdoctoral period at the Max Planck Institute for Astrophysics, he went on to work at the Space Research Institute in Frascati. In 1972, he became a Senior Research Scientist at the Max Planck Institute for Plasma Physics. From 1981 to 1995 he was head of the General Theory Group and subsequently head of the Nonlinear Plasma Dynamics Group, a position he held until 2001. In 1979 he was visiting Professor at the University of Texas and in 1995 COE visiting Professor at the National Institute for Fusion Science in Nagoya. He currently works as a consultant at the Center for Interdisciplinary Plasma Science at the Max Planck Institute for Extraterrestrial Physics. He is the author of two previous books, *Nonlinear Magnetohydrodynamics* and *Magnetic Reconnection in Plasmas*.

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DIETER BISKAMP

Center for Interdisciplinary Plasma Science, Garching



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To Robert H. Kraichnan

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Preface

Turbulence in electrically conducting fluids is necessarily accompanied by magnetic-field fluctuations, which will, in general, strongly influence the dynamics. It is true that, in our terrestrial world, conducting fluids in turbulent motion are rare. In astrophysics, however, material is mostly ionized and strong turbulence is a widespread phenomenon, for instance in stellar convection zones and stellar winds and in the interstellar medium. Turbulent magnetic fields are therefore expected to play an important role. Despite the fact that, on a microscopic level, astrophysical plasmas exhibit rather diverse properties, a unified macroscopic treatment in the framework of magnetohydrodynamics (MHD) to describe the most important magnetic effects is appropriate. Hence there is much interest in MHD turbulence in the astrophysical community. Considerable interest comes also from the side of pure theory, where MHD turbulence introduces new concepts into turbulence theory, as the large number of articles on this topic in the literature shows. However, to date no monograph on MHD turbulence seems to have been written. I therefore believe that a treatise both introducing the field and reviewing the current state of the art could be welcome.

The book consists of four major parts: an introductory part, Chapters 2 and 3, discusses the MHD model and the transition to turbulence; the second part, Chapters 4–7, focusses on the theory of incompressible turbulence; the third part, Chapters 8 and 9, deals with two important extensions, two-dimensional turbulence, which arises in the presence of a strong magnetic field, and, in a sense the opposite case, compressible, in particular supersonic, turbulence; and finally a part concerning applications, Chapters 10–12, treating three areas in which MHD turbulence is observed, or expected to be excited, namely turbulence in the solar wind, in accretion disks, and in the interstellar medium. A book on MHD turbulence is also a book on hydrodynamic turbulence, using and generalizing methods developed for the latter, which the reader will find in the first parts of most chapters. The chapters dealing with the applications contain

a general introduction to each field and may be read independently of the rest. Apart from elementary fluid dynamics no particular expertise is required, though some knowledge of plasma physics can sometimes be helpful. I hope that the book will be suitable for those just entering the field and also interesting for researchers in the field.

It is a pleasure to express my gratitude to the many colleagues with whom I enjoyed very fruitful discussions on the various topics of this book. In particular, I would like to thank Axel Brandenburg, Rainer Grauer, Eckart Marsch, Wolf-Christian Müller, H el ene Politano, Annick Pouquet, Rainer Schwenn, and Rudolf Treumann. I am grateful to Barbara Mori for preparing the figures included in the book. I should also like to acknowledge the hospitality and financial support provided by the Center for Interdisciplinary Plasma Science, where most of this book was written, with special thanks to Gregor Morfill. Finally, I want to thank Steven Holt for his excellent copy-editing of the manuscript.

Dieter Biskamp