MAGNETOHYDRODYNAMIC TURBULENCE

This book presents an introduction to, and state-of-the-art account of, magnetohydrodynamic (MHD) turbulence, an active field both in general turbulence theory and in various areas of astrophysics. The book starts by introducing the MHD equations and certain useful approximations. The transition to turbulence is then discussed, including the problem of finite-time singularities of the ideal equations and the excitation of instabilities. The second part of the book deals with incompressible MHD turbulence, the macroscopic aspects connected with the various self-organization processes, the phenomenology of the turbulence spectra, two-point closure theory, and intermittency. The third part considers two extensions: two-dimensional turbulence and compressible (in particular, supersonic) turbulence. Because of the similarities in the theoretical approach, these chapters start with a brief account of the corresponding methods developed in hydrodynamic turbulence. The final part of the book is devoted to three astrophysical topics: turbulence in the solar wind, in accretion disks, and in the interstellar medium. This book is suitable for graduate students and researchers working in turbulence theory, plasma physics, and astrophysics.

DIETER BISKAMP received his Ph.D. from the University of Munich. Following a postdoctoral period at the Max Planck Institute for Astrophysics, he went on to work at the Space Research Institute in Frascati. In 1972, he became a Senior Research Scientist at the Max Planck Institute for Plasma Physics. From 1981 to 1995 he was head of the General Theory Group and subsequently head of the Nonlinear Plasma Dynamics Group, a position he held until 2001. In 1979 he was visiting Professor at the University of Texas and in 1995 COE visiting Professor at the National Institute for Fusion Science in Nagoya. He currently works as a consultant at the Center for Interdisciplinary Plasma Science at the Max Planck Institute for Extraterrestrial Physics. He is the author of two previous books, *Nonlinear Magnetohydrodynamics* and *Magnetic Reconnection in Plasmas*. 
MAGNETOHYDRODYNAMIC TURBULENCE

DIETER BISKAMP
Center for Interdisciplinary Plasma Science, Garching
To Robert H. Kraichnan
## Contents

**Preface**  
page xi

1 Introduction  

2 Magnetohydrodynamics  
   2.1 MHD equations  
      2.1.1 Dynamic equations  
      2.1.2 The rotating reference frame  
   2.2 Incompressibility and the Boussinesq approximation  
   2.3 Conservation laws  
      2.3.1 Fluid invariants  
      2.3.2 Magnetic invariants  
   2.4 Equilibrium configurations  
   2.5 Linear waves  
      2.5.1 Waves in a homogeneous magnetized system  
      2.5.2 Waves in a stratified system  
   2.6 Elsässer fields and Alfvén time normalization  

3 Transition to turbulence  
   3.1 Singularities of the ideal equations  
      3.1.1 FTS in the Euler equations  
      3.1.2 Formation of current sheets in ideal MHD  
   3.2 Instabilities  
      3.2.1 Kelvin–Helmholtz instability  
      3.2.2 Rayleigh–Taylor instability  
      3.2.3 Kelvin–Helmholtz instability in a stratified medium  
      3.2.4 The tearing instability  

4 Macroscopic turbulence theory  
   4.1 One-point closure  
      4.1.1 Reynolds equations for MHD  
      4.1.2 Turbulent transport coefficients  

© Cambridge University Press  www.cambridge.org
4.1.3 Large-eddy simulations of MHD turbulence 70
4.1.4 Mean-field electrodynamics 72
4.2 Self-organization processes 75
  4.2.1 Selective decay 75
  4.2.2 The Alfvén effect and dynamic alignment 77
  4.2.3 Energy-decay laws 79
5 Spectral properties and phenomenology 86
  5.1 Homogeneous isotropic turbulence 87
  5.2 Ideal systems and turbulent cascades 89
    5.2.1 Absolute equilibrium states 90
    5.2.2 Cascade directions 92
  5.3 Spectra in dissipative MHD turbulence 93
    5.3.1 Magnetic Reynolds numbers 93
    5.3.2 Phenomenology of the inertial-range spectrum 95
  5.3.3 Anisotropy of MHD turbulence 100
  5.3.4 Dissipation scales 102
  5.3.5 Energy spectra in highly aligned turbulence 104
  5.3.6 Results of numerical simulations 107
6 Two-point-closure theory 113
  6.1 Quasi-normal-type approximations 114
    6.1.1 The problem of closure 114
    6.1.2 The quasi-normal approximation 116
    6.1.3 The eddy-damped quasi-normal Markovian approximation (EDQNM) 117
  6.2 The EDQNM theory of MHD turbulence 119
    6.2.1 Helical turbulence 120
    6.2.2 Correlated turbulence 125
  6.3 Shortcomings of closure approximations 129
7 Intermittency 131
  7.1 Self-similarity versus intermittency 133
  7.2 Structure functions 137
    7.2.1 Scaling exponents 138
    7.2.2 Extended self-similarity (ESS) 140
    7.2.3 The refined similarity hypothesis 142
  7.3 Exact turbulence relations 144
    7.3.1 Kolmogorov’s four-fifths law 144
    7.3.2 Yaglom’s four-thirds law 146
    7.3.3 The four-thirds law in MHD turbulence 148
## Contents

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.4</td>
<td>Phenomenological models of intermittency</td>
<td>150</td>
</tr>
<tr>
<td>7.4.1</td>
<td>The log-normal model</td>
<td>150</td>
</tr>
<tr>
<td>7.4.2</td>
<td>The log-Poisson model</td>
<td>153</td>
</tr>
<tr>
<td>7.5</td>
<td>Intermittency in MHD turbulence</td>
<td>156</td>
</tr>
<tr>
<td>7.5.1</td>
<td>Log-Poisson models for MHD turbulence</td>
<td>156</td>
</tr>
<tr>
<td>7.5.2</td>
<td>The effect of the mean magnetic field</td>
<td>159</td>
</tr>
<tr>
<td>8</td>
<td>Two-dimensional turbulence</td>
<td>161</td>
</tr>
<tr>
<td>8.1</td>
<td>Two-dimensional hydrodynamic turbulence</td>
<td>164</td>
</tr>
<tr>
<td>8.1.1</td>
<td>Properties of the ideal system</td>
<td>165</td>
</tr>
<tr>
<td>8.1.2</td>
<td>The decay of enstrophy</td>
<td>166</td>
</tr>
<tr>
<td>8.1.3</td>
<td>The phenomenology of the dual cascade</td>
<td>168</td>
</tr>
<tr>
<td>8.1.4</td>
<td>The enstrophy cascade</td>
<td>170</td>
</tr>
<tr>
<td>8.1.5</td>
<td>The inverse energy cascade</td>
<td>171</td>
</tr>
<tr>
<td>8.2</td>
<td>Two-dimensional MHD turbulence</td>
<td>173</td>
</tr>
<tr>
<td>8.2.1</td>
<td>Properties of the ideal MHD system</td>
<td>174</td>
</tr>
<tr>
<td>8.2.2</td>
<td>Decay of 2D MHD turbulence</td>
<td>175</td>
</tr>
<tr>
<td>8.2.3</td>
<td>Spectra in 2D MHD turbulence</td>
<td>178</td>
</tr>
<tr>
<td>8.2.4</td>
<td>Intermittency in 2D MHD turbulence</td>
<td>181</td>
</tr>
<tr>
<td>9</td>
<td>Compressible turbulence and turbulent convection</td>
<td>183</td>
</tr>
<tr>
<td>9.1</td>
<td>MHD shock waves</td>
<td>185</td>
</tr>
<tr>
<td>9.2</td>
<td>Compressible homogeneous turbulence</td>
<td>191</td>
</tr>
<tr>
<td>9.2.1</td>
<td>Supersonic hydrodynamic turbulence</td>
<td>191</td>
</tr>
<tr>
<td>9.2.2</td>
<td>Supersonic MHD turbulence</td>
<td>199</td>
</tr>
<tr>
<td>9.2.3</td>
<td>Numerical methods in compressible hydrodynamics</td>
<td>201</td>
</tr>
<tr>
<td>9.3</td>
<td>Turbulent convection</td>
<td>203</td>
</tr>
<tr>
<td>9.3.1</td>
<td>Turbulence in a Boussinesq fluid</td>
<td>203</td>
</tr>
<tr>
<td>9.3.2</td>
<td>Passive scalar turbulence</td>
<td>207</td>
</tr>
<tr>
<td>9.3.3</td>
<td>Compressible turbulent convection</td>
<td>212</td>
</tr>
<tr>
<td>9.3.4</td>
<td>Magnetoconvection</td>
<td>214</td>
</tr>
<tr>
<td>10</td>
<td>Turbulence in the solar wind</td>
<td>217</td>
</tr>
<tr>
<td>10.1</td>
<td>Mean properties of the solar wind</td>
<td>217</td>
</tr>
<tr>
<td>10.1.1</td>
<td>The hydrodynamic model of the solar wind</td>
<td>218</td>
</tr>
<tr>
<td>10.1.2</td>
<td>Effects of the solar magnetic field</td>
<td>220</td>
</tr>
<tr>
<td>10.1.3</td>
<td>Fast and slow winds</td>
<td>222</td>
</tr>
<tr>
<td>10.2</td>
<td>MHD fluctuations in the solar wind</td>
<td>224</td>
</tr>
<tr>
<td>10.2.1</td>
<td>Wave types and turbulence spectra</td>
<td>225</td>
</tr>
<tr>
<td>10.2.2</td>
<td>Intermittency in solar-wind turbulence</td>
<td>229</td>
</tr>
</tbody>
</table>
Contents

10.2.3 Dissipation of turbulence 230
10.2.4 Compressive fluctuations 231

11 Turbulence in accretion disks 233
11.1 Basic properties of accretion disks 234
11.2 The standard disk model 238
11.2.1 Keplerian disks 238
11.2.2 The $\alpha$-disk model 242
11.3 Hydrodynamic stability of accretion disks 244
11.3.1 Shear-flow stability of a Keplerian disk 245
11.3.2 Effects of convective turbulence 247
11.4 Magnetorotational instability 248
11.4.1 Linear instability 249
11.4.2 Nonlinear saturation and magnetoviscosity 252

12 Interstellar turbulence 256
12.1 The main properties of the interstellar medium 257
12.2 Observational results on molecular clouds 259
12.2.1 Supersonic turbulence 260
12.2.2 Gravity in molecular clouds 261
12.2.3 The density spectrum and mass distribution 262
12.2.4 Magnetic fields 264
12.3 Stability of molecular clouds 267
12.3.1 The virial theorem 268
12.3.2 Ambipolar diffusion 272
12.3.3 Generation of turbulence in molecular clouds 273

References 277
Index 293
Preface

Turbulence in electrically conducting fluids is necessarily accompanied by magnetic-field fluctuations, which will, in general, strongly influence the dynamics. It is true that, in our terrestrial world, conducting fluids in turbulent motion are rare. In astrophysics, however, material is mostly ionized and strong turbulence is a widespread phenomenon, for instance in stellar convection zones and stellar winds and in the interstellar medium. Turbulent magnetic fields are therefore expected to play an important role. Despite the fact that, on a microscopic level, astrophysical plasmas exhibit rather diverse properties, a unified macroscopic treatment in the framework of magnetohydrodynamics (MHD) to describe the most important magnetic effects is appropriate. Hence there is much interest in MHD turbulence in the astrophysical community. Considerable interest comes also from the side of pure theory, where MHD turbulence introduces new concepts into turbulence theory, as the large number of articles on this topic in the literature shows. However, to date no monograph on MHD turbulence seems to have been written. I therefore believe that a treatise both introducing the field and reviewing the current state of the art could be welcome.

The book consists of four major parts: an introductory part, Chapters 2 and 3, discusses the MHD model and the transition to turbulence; the second part, Chapters 4–7, focusses on the theory of incompressible turbulence; the third part, Chapters 8 and 9, deals with two important extensions, two-dimensional turbulence, which arises in the presence of a strong magnetic field, and, in a sense the opposite case, compressible, in particular supersonic, turbulence; and finally a part concerning applications, Chapters 10–12, treating three areas in which MHD turbulence is observed, or expected to be excited, namely turbulence in the solar wind, in accretion disks, and in the interstellar medium. A book on MHD turbulence is also a book on hydrodynamic turbulence, using and generalizing methods developed for the latter, which the reader will find in the first parts of most chapters. The chapters dealing with the applications contain
a general introduction to each field and may be read independently of the rest. Apart from elementary fluid dynamics no particular expertise is required, though some knowledge of plasma physics can sometimes be helpful. I hope that the book will be suitable for those just entering the field and also interesting for researchers in the field.

It is a pleasure to express my gratitude to the many colleagues with whom I enjoyed very fruitful discussions on the various topics of this book. In particular, I would like to thank Axel Brandenburg, Rainer Grauer, Eckart Marsch, Wolf-Christian Müller, Hélène Politano, Annick Pouquet, Rainer Schwenn, and Rudolf Treumann. I am grateful to Barbara Mori for preparing the figures included in the book. I should also like to acknowledge the hospitality and financial support provided by the Center for Interdisciplinary Plasma Science, where most of this book was written, with special thanks to Gregor Morfill. Finally, I want to thank Steven Holt for his excellent copy-editing of the manuscript.

Dieter Biskamp