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Notation Index

Symbols assignable to Greek or Latin letters:

\forall	for all, 7, 504
c_0	sequences $\rightarrow 0$, 202
\mathbb{C}	complex numbers, 153
$C(\cdot)$	set of continuous functions, 51
$C_b(\cdot)$	set of bounded continuous functions, 53, 158
d_{sup}	supremum distance, 51, 52
\exists	(there) exists, 7, 504
δ_x	point mass at x , 292
E	expectation $\int \cdot dP$, 251, 306
\in	member of, 3, 505
\notin	not a member of, 4, 505
\emptyset	empty set, 4, 506
F_σ	countable union of closed sets, 60
G_δ	countable intersection of open sets, 60
\liminf	$\liminf x_n = \sup_m \inf_{n \geq m} x_n$, 129, 131
$\limsup_{n \rightarrow \infty}$	$\limsup x_n = \inf_m \sup_{n \geq m} x_n$, 129
$\limsup_{y \rightarrow x}$	44
ℓ^1	space of summable sequences, 72
$\mathcal{L}(\cdot)$	law of random variable, 282
λ	Lebesgue measure, 98
ℓ^p	p -summable sequences, 162
\mathcal{L}^0	set of all measurable functions, 119, 288
L^0	equivalence classes of them, 288
\mathcal{L}^p	p -integrable functions, 153, 157
L^p	equivalence classes of them, 158
\mathcal{L}^∞	essentially bounded functions, 155
L^∞	equivalence classes of them, 158
$N(m, C)$	normal law on \mathbb{R}^k , 307, 309
$N(m, \sigma^2)$	normal law on \mathbb{R} , 299
\mathbb{N}	set of nonnegative integers, 7, 507
\mathbb{Q}	set of rational numbers, 7, 58, 516
\mathbb{R}	set of real numbers, 7–8, 58, 516
\mathbb{R}^2	plane, 8
\mathbb{R}^k	Euclidean space, 38–39
\sup	supremum, 8, 34–35, 517
\mathbb{Z}	set of all integers $\dots -1, 0, 1, \dots$, 7, 516

Symbols not assignable to letters, alphabetized by defining words:

\sim	asymptotic, 477
$[\dots]$	closed interval, 24
\downarrow	decreases to, 53
$:=$	equals by definition, 3
\mapsto	function specifier, 5–6
\subset	included in, 3
\uparrow	increases to, 116
\cap	intersection, 5–7
(\dots)	open interval, 25
\perp	perpendicular, 163
\otimes	product for σ -algebras, 118
\setminus	relative complement, 5
\upharpoonright	restricted to, 13
Δ	symmetric difference, 5
\cup	union, 5–7