

Cambridge University Press

0521809495 - Dynamics of the Atmosphere: A Course in Theoretical Meteorology

Wilford Zdunkowski and Andreas Bott

Frontmatter

[More information](#)

DYNAMICS OF THE ATMOSPHERE: A COURSE IN THEORETICAL METEOROLOGY

Dynamics of the Atmosphere is a textbook with numerous exercises and solutions, written for senior undergraduate and graduate students of meteorology and related sciences. It may also be used as a reference source by professional meteorologists and researchers in atmospheric science. In order to encourage the reader to follow the mathematical developments in detail, the derivations are complete and leave out only the most elementary steps.

The book consists of two parts, the first presenting the mathematical tools needed for a thorough understanding of the second part. Mathematical topics include a summary of the methods of vector and tensor analysis in generalized coordinates; an accessible presentation of the method of covariant differentiation; and a brief introduction to nonlinear dynamics. These mathematical tools are used later in the book to tackle such problems as the fields of motion over different types of terrain, and problems of predictability.

The second part of the book begins with the derivation of the equation describing the atmospheric motion on the rotating earth, followed by several chapters that consider the kinematics of the atmosphere and introduce vorticity and circulation theorems. Weather patterns can be considered as superpositions of waves of many wavelengths, and the authors therefore present a discussion of wave motion in the atmosphere, including the barotropic model and some Rossby physics. A chapter on inertial and dynamic stability is presented and the component form of the equation of motion is derived in the general covariant, contravariant, and physical coordinate forms. The subsequent three chapters are devoted to turbulent systems in the atmosphere and their implications for weather-prediction equations. At the end of the book newer methods of weather prediction, such as the spectral technique and the stochastic dynamic method, are introduced in order to demonstrate their potential for extending the forecasting range as computers become increasingly powerful.

WILFORD ZDUNKOWSKI received B.S. and M.S. degrees from the University of Utah and was awarded a Ph.D. in meteorology from the University of Munich in 1962. He then returned to the Department of Meteorology at the University of Utah, where he was later made Professor of Meteorology. In 1977, he took up a professorship, at the Universität Mainz, where for twenty years he taught courses related to the topics presented in this book. Professor Zdunkowski has been the recipient of numerous awards from various research agencies in the USA and in Germany, and has travelled extensively to report his findings to colleagues around the world.

Cambridge University Press

0521809495 - Dynamics of the Atmosphere: A Course in Theoretical Meteorology

Wilford Zdunkowski and Andreas Bott

Frontmatter

[More information](#)

ANDREAS BOTT received a Diploma in Meteorology from the Universität Mainz in 1982, and subsequently worked as a research associate under Professor Paul Crutzen at the Max-Planck-Institut für Chemie in Mainz, where he was awarded a Ph.D. in Meteorology in 1986. He held a variety of positions at the Institute for Atmospheric Physics in the Universität Mainz between 1986 and 1999, and during this time he also spent periods as a guest scientist at institutions in the USA, Norway, and Japan. Since 2000, Dr Bott has been a University Professor for Theoretical Meteorology at the Rheinische Friedrich-Wilhelms-Universität, in Bonn. Professor Bott teaches courses in theoretical meteorology, atmospheric thermodynamics, atmospheric dynamics, cloud microphysics, atmospheric chemistry, and numerical modeling.

Cambridge University Press
0521809495 - Dynamics of the Atmosphere: A Course in Theoretical Meteorology
Wilford Zdunkowski and Andreas Bott
Frontmatter
[More information](#)

DYNAMICS OF THE ATMOSPHERE:
A COURSE IN THEORETICAL
METEOROLOGY

WILFORD ZDUNKOWSKI
and
ANDREAS BOTT



Cambridge University Press
0521809495 - Dynamics of the Atmosphere: A Course in Theoretical Meteorology
Wilford Zdunkowski and Andreas Bott
Frontmatter
[More information](#)

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa
<http://www.cambridge.org>

© Cambridge University Press 2003

This book is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without
the written permission of Cambridge University Press.

First published 2003

Printed in the United Kingdom at the University Press, Cambridge

Typeface Times 11/14 pt *System* L^AT_EX 2_ε [TB]

A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication data

ISBN 0 521 80949 5 hardback
ISBN 0 521 00666 X paperback

Cambridge University Press
0521809495 - Dynamics of the Atmosphere: A Course in Theoretical Meteorology
Wilford Zdunkowski and Andreas Bott
Frontmatter
[More information](#)

This book is dedicated to the memory of
Professor K. H. Hinkelmann (1915–1989)
and Dr J. G. Korb (1928–1991)
who excelled as theoretical meteorologists and as
teachers of meteorology at the University of Mainz, Germany.

Contents

<i>Preface</i>	<i>page</i> xv
Part 1 Mathematical tools	1
M1 Algebra of vectors	3
M1.1 Basic concepts and definitions	3
M1.2 Reference frames	6
M1.3 Vector multiplication	7
M1.4 Reciprocal coordinate systems	15
M1.5 Vector representations	19
M1.6 Products of vectors in general coordinate systems	22
M1.7 Problems	23
M2 Vector functions	25
M2.1 Basic definitions and operations	25
M2.2 Special dyadics	28
M2.3 Principal-axis transformation of symmetric tensors	32
M2.4 Invariants of a dyadic	34
M2.5 Tensor algebra	40
M2.6 Problems	42
M3 Differential relations	43
M3.1 Differentiation of extensive functions	43
M3.2 The Hamilton operator in generalized coordinate systems	48
M3.3 The spatial derivative of the basis vectors	51
M3.4 Differential invariants in generalized coordinate systems	53
M3.5 Additional applications	56
M3.6 Problems	60
M4 Coordinate transformations	62
M4.1 Transformation relations of time-independent coordinate systems	62

viii	<i>Contents</i>	
	M4.2 Transformation relations of time-dependent coordinate systems	67
	M4.3 Problems	73
M5	The method of covariant differentiation	75
	M5.1 Spatial differentiation of vectors and dyadics	75
	M5.2 Time differentiation of vectors and dyadics	79
	M5.3 The local dyadic of v_p	82
	M5.4 Problems	83
M6	Integral operations	84
	M6.1 Curves, surfaces, and volumes in the general q^i system	84
	M6.2 Line integrals, surface integrals, and volume integrals	87
	M6.3 Integral theorems	90
	M6.4 Fluid lines, surfaces, and volumes	94
	M6.5 Time differentiation of fluid integrals	96
	M6.6 The general form of the budget equation	101
	M6.7 Gauss' theorem and the Dirac delta function	104
	M6.8 Solution of Poisson's differential equation	106
	M6.9 Appendix: Remarks on Euclidian and Riemannian spaces	107
	M6.10 Problems	110
M7	Introduction to the concepts of nonlinear dynamics	111
	M7.1 One-dimensional flow	111
	M7.2 Two-dimensional flow	116
Part 2	Dynamics of the atmosphere	131
1	The laws of atmospheric motion	133
	1.1 The equation of absolute motion	133
	1.2 The energy budget in the absolute reference system	136
	1.3 The geographical coordinate system	137
	1.4 The equation of relative motion	146
	1.5 The energy budget of the general relative system	147
	1.6 The decomposition of the equation of motion	150
	1.7 Problems	154
2	Scale analysis	157
	2.1 An outline of the method	157
	2.2 Practical formulation of the dimensionless flow numbers	159
	2.3 Scale analysis of large-scale frictionless motion	161
	2.4 The geostrophic wind and the Euler wind	167
	2.5 The equation of motion on a tangential plane	169
	2.6 Problems	169

	<i>Contents</i>	ix
3	The material and the local description of flow	171
3.1	The description of Lagrange	171
3.2	Lagrange’s version of the continuity equation	173
3.3	An example of the use of Lagrangian coordinates	175
3.4	The local description of Euler	182
3.5	Transformation from the Eulerian to the Lagrangian system	186
3.6	Problems	187
4	Atmospheric flow fields	189
4.1	The velocity dyadic	189
4.2	The deformation of the continuum	193
4.3	Individual changes with time of geometric fluid configurations	199
4.4	Problems	205
5	The Navier–Stokes stress tensor	206
5.1	The general stress tensor	206
5.2	Equilibrium conditions in the stress field	208
5.3	Symmetry of the stress tensor	209
5.4	The frictional stress tensor and the deformation dyadic	210
5.5	Problems	212
6	The Helmholtz theorem	214
6.1	The three-dimensional Helmholtz theorem	214
6.2	The two-dimensional Helmholtz theorem	216
6.3	Problems	217
7	Kinematics of two-dimensional flow	218
7.1	Atmospheric flow fields	218
7.2	Two-dimensional streamlines and normals	222
7.3	Streamlines in a drifting coordinate system	225
7.4	Problems	228
8	Natural coordinates	230
8.1	Introduction	230
8.2	Differential definitions of the coordinate lines	232
8.3	Metric relationships	235
8.4	Blaton’s equation	236
8.5	Individual and local time derivatives of the velocity	238
8.6	Differential invariants	239
8.7	The equation of motion for frictionless horizontal flow	242
8.8	The gradient wind relation	243
8.9	Problems	244

x	<i>Contents</i>	
9	Boundary surfaces and boundary conditions	246
9.1	Introduction	246
9.2	Differential operations at discontinuity surfaces	247
9.3	Particle invariance at boundary surfaces, displacement velocities	251
9.4	The kinematic boundary-surface condition	253
9.5	The dynamic boundary-surface condition	258
9.6	The zeroth-order discontinuity surface	259
9.7	An example of a first-order discontinuity surface	265
9.8	Problems	267
10	Circulation and vorticity theorems	268
10.1	Ertel's form of the continuity equation	268
10.2	The baroclinic Weber transformation	271
10.3	The baroclinic Ertel–Rossby invariant	275
10.4	Circulation and vorticity theorems for frictionless baroclinic flow	276
10.5	Circulation and vorticity theorems for frictionless barotropic flow	293
10.6	Problems	301
11	Turbulent systems	302
11.1	Simple averages and fluctuations	302
11.2	Weighted averages and fluctuations	304
11.3	Averaging the individual time derivative and the budget operator	306
11.4	Integral means	307
11.5	Budget equations of the turbulent system	310
11.6	The energy budget of the turbulent system	313
11.7	Diagnostic and prognostic equations of turbulent systems	315
11.8	Production of entropy in the microturbulent system	319
11.9	Problems	324
12	An excursion into spectral turbulence theory	326
12.1	Fourier Representation of the continuity equation and the equation of motion	326
12.2	The budget equation for the amplitude of the kinetic energy	331
12.3	Isotropic conditions, the transition to the continuous wavenumber space	333
12.4	The Heisenberg spectrum	336
12.5	Relations for the Heisenberg exchange coefficient	340
12.6	A prognostic equation for the exchange coefficient	341

	<i>Contents</i>	xi
	12.7 Concluding remarks on closure procedures	346
	12.8 Problems	348
13	The atmospheric boundary layer	349
	13.1 Introduction	349
	13.2 Prandtl-layer theory	350
	13.3 The Monin–Obukhov similarity theory of the neutral Prandtl layer	358
	13.4 The Monin–Obukhov similarity theory of the diabatic Prandtl layer	362
	13.5 Application of the Prandtl-layer theory in numerical prognostic models	369
	13.6 The fluxes, the dissipation of energy, and the exchange coefficients	371
	13.7 The interface condition at the earth’s surface	372
	13.8 The Ekman layer – the classical approach	375
	13.9 The composite Ekman layer	381
	13.10 Ekman pumping	388
	13.11 Appendix A: Dimensional analysis	391
	13.12 Appendix B: The mixing length	394
	13.13 Problems	396
14	Wave motion in the atmosphere	398
	14.1 The representation of waves	398
	14.2 The group velocity	401
	14.3 Perturbation theory	403
	14.4 Pure sound waves	407
	14.5 Sound waves and gravity waves	410
	14.6 Lamb waves	418
	14.7 Lee waves	418
	14.8 Propagation of energy	418
	14.9 External gravity waves	422
	14.10 Internal gravity waves	426
	14.11 Nonlinear waves in the atmosphere	431
	14.12 Problems	434
15	The barotropic model	435
	15.1 The basic assumptions of the barotropic model	435
	15.2 The unfiltered barotropic prediction model	437
	15.3 The filtered barotropic model	450
	15.4 Barotropic instability	452
	15.5 The mechanism of barotropic development	463
	15.6 Appendix	468
	15.7 Problems	470

xii	<i>Contents</i>	
16	Rossby waves	471
16.1	One- and two-dimensional Rossby waves	471
16.2	Three-dimensional Rossby waves	476
16.3	Normal-mode considerations	479
16.4	Energy transport by Rossby waves	482
16.5	The influence of friction on the stationary Rossby wave	483
16.6	Barotropic equatorial waves	484
16.7	The principle of geostrophic adjustment	487
16.8	Appendix	493
16.9	Problems	494
17	Inertial and dynamic stability	495
17.1	Inertial motion in a horizontally homogeneous pressure field	495
17.2	Inertial motion in a homogeneous geostrophic wind field	497
17.3	Inertial motion in a geostrophic shear wind field	498
17.4	Derivation of the stability criteria in the geostrophic wind field	501
17.5	Sectorial stability and instability	504
17.6	Sectorial stability for normal atmospheric conditions	509
17.7	Sectorial stability and instability with permanent adaptation	510
17.8	Problems	512
18	The equation of motion in general coordinate systems	513
18.1	Introduction	513
18.2	The covariant equation of motion in general coordinate systems	514
18.3	The contravariant equation of motion in general coordinate systems	518
18.4	The equation of motion in orthogonal coordinate systems	520
18.5	Lagrange's equation of motion	523
18.6	Hamilton's equation of motion	527
18.7	Appendix	530
18.8	Problems	531
19	The geographical coordinate system	532
19.1	The equation of motion	532
19.2	Application of Lagrange's equation of motion	536
19.3	The first metric simplification	538
19.4	The coordinate simplification	539
19.5	The continuity equation	540
19.6	Problems	541

	<i>Contents</i>	xiii
20	The stereographic coordinate system	542
20.1	The stereographic projection	542
20.2	Metric forms in stereographic coordinates	546
20.3	The absolute kinetic energy in stereographic coordinates	549
20.4	The equation of motion in the stereographic Cartesian coordinates	550
20.5	The equation of motion in stereographic cylindrical coordinates	554
20.6	The continuity equation	556
20.7	The equation of motion on the tangential plane	558
20.8	The equation of motion in Lagrangian enumeration coordinates	559
20.9	Problems	564
21	Orography-following coordinate systems	565
21.1	The metric of the η system	565
21.2	The equation of motion in the η system	568
21.3	The continuity equation in the η system	571
21.4	Problems	571
22	The stereographic system with a generalized vertical coordinate	572
22.1	The ξ transformation and resulting equations	573
22.2	The pressure system	577
22.3	The solution scheme using the pressure system	579
22.4	The solution to a simplified prediction problem	582
22.5	The solution scheme with a normalized pressure coordinate	584
22.6	The solution scheme with potential temperature as vertical coordinate	587
22.7	Problems	589
23	A quasi-geostrophic baroclinic model	591
23.1	Introduction	591
23.2	The first law of thermodynamics in various forms	592
23.4	The vorticity and the divergence equation	593
23.5	The first and second filter conditions	595
23.6	The geostrophic approximation of the heat equation	597
23.7	The geostrophic approximation of the vorticity equation	603
23.8	The ω equation	605
23.9	The Philipps approximation of the ageostrophic component of the horizontal wind	609
23.10	Applications of the Philipps wind	614
23.11	Problems	617

xiv	<i>Contents</i>	
24	A two-level prognostic model, baroclinic instability	619
24.1	Introduction	619
24.2	The mathematical development of the two-level model	619
24.3	The Phillips quasi-geostrophic two-level circulation model	623
24.4	Baroclinic instability	624
24.5	Problems	633
25	An excursion concerning numerical procedures	634
25.1	Numerical stability of the one-dimensional advection equation	634
25.2	Application of forward-in-time and central-in-space difference quotients	640
25.3	A practical method for the elimination of the weak instability	642
25.4	The implicit method	642
25.5	The aliasing error and nonlinear instability	645
25.6	Problems	648
26	Modeling of atmospheric flow by spectral techniques	649
26.1	Introduction	649
26.2	The basic equations	650
26.3	Horizontal discretization	655
26.4	Problems	667
27	Predictability	669
27.1	Derivation and discussion of the Lorenz equations	669
27.2	The effect of uncertainties in the initial conditions	681
27.3	Limitations of deterministic predictability of the atmosphere	683
27.4	Basic equations of the approximate stochastic dynamic method	689
27.5	Problems	692
	Answers to Problems	693
	List of frequently used symbols	702
	References and bibliography	706

Preface

This book has been written for students of meteorology and of related sciences at the senior and graduate level. The goal of the book is to provide the background for graduate studies and individual research. The second part, *Thermodynamics of the Atmosphere*, will appear shortly. To a considerable degree we have based our book on the excellent lecture notes of Professor Karl Hinkelmann on various topics in dynamic meteorology, including Prandtl-layer theory and turbulence. Moreover, we were fortunate to have Dr Korb's outstanding lecture notes on kinematics of the atmosphere and on mathematical tools for the meteorologist at our disposal.

Quite early on during the writing of this book, it became apparent that we had to replace various topics treated in their notes by more modern material in order to give a reasonably up-to-date account of theoretical meteorology. We were guided by the idea that any topic we have selected for presentation should be treated in some depth in order for it to be of real value to the reader. Insofar as space would permit, all but the most trivial steps have been included in every development. This is the reason why our book is somewhat more bulky than some other books on theoretical meteorology. The student may judge for himself whether our approach is profitable.

The reader will soon recognize that various interesting and important topics have been omitted from this textbook. Including these and still keeping the book of the same length would result in the loss of numerous mathematical details. This, however, might discourage some students from following the discussion in depth. We believe that the approach we have chosen is correct and smoothes the path to additional and more advanced studies.

This book consists of two separate parts. In the first part we present the mathematical techniques needed to handle the various topics of dynamic meteorology which are presented in the second part of the book. The modern student of meteorology and of related sciences at the senior and the graduate level has accumulated a sufficient working knowledge of vector calculus applied to the Cartesian coordinate

Cambridge University Press

0521809495 - Dynamics of the Atmosphere: A Course in Theoretical Meteorology

Wilford Zdunkowski and Andreas Bott

Frontmatter

[More information](#)

system. We are safe to assume that the student has also encountered the important integral theorems which play a dominant role in many branches of physics and engineering. The required extension to more general coordinate systems is not difficult. Nevertheless, the reader may have to deal with some unfamiliar topics. He should not be discouraged since often unfamiliarity is mistaken for inherent difficulty. The unavoidable formality presented in the introductory chapters on first reading looks worse than it really is. After overcoming some initial difficulties, the student will soon gain confidence in his ability to handle the new techniques. The authors came to the conclusion, as the result of many years of learning and teaching, that a mastery of the mathematical introduction is surely worth what it costs in effort.

All mathematical operations have been restricted to three dimensions in space. However, many important formulas can be easily extended to higher-order spaces. Some knowledge of tensor analysis is required for our studies. Since three-dimensional tensor analysis in generalized coordinates can be handled very effectively with the help of dyadics, we have introduced the necessary operations. Only as the last step do we write down the tensor components. By proceeding in this manner, we are likely to avoid errors that may occur quite easily with use of the index notation throughout. We admit that dyadics are quite dispensable when one is working with Cartesian tensors, but they are of great help when one is working with generalized coordinate systems.

The second part of the book treats some of the major topics of dynamic meteorology. As is customary in many textbooks, the introductory chapters discuss some basic topics of thermodynamics. We will depart from this much-trodden path. The reason for this departure is that modern thermodynamics cannot be adequately dealt with in this manner. If formulas from thermodynamics are required, they will be carefully stated. Detailed derivations, however, will be omitted since these will be presented in part II of *A Course in Theoretical Meteorology*. When reference to this book on thermodynamics is made we will use the abbreviation TH.

We will now give a brief description of the various chapters of the dynamics part of the book. Chapter 1 presents the laws of atmospheric motion. The method of scale analysis is introduced in Chapter 2 in order to show which terms in the component form of the equation of motion may be safely neglected in large-scale flow fields. Chapters 3–10 discuss some topics that traditionally belong to the kinematics part of theoretical meteorology. Included are discussions on the material and the local description of flow, the Navier–Stokes stress tensor, the Helmholtz theorem, boundary surfaces, circulation, and vorticity theorems. Since atmospheric flow, particularly in the air layers near the ground, is always turbulent, in Chapters 11 and 12 we present a short introduction to turbulence theory. Some important aspects of boundary-layer theory will be given in Chapter 13. Wave motion in the

Cambridge University Press

0521809495 - Dynamics of the Atmosphere: A Course in Theoretical Meteorology

Wilford Zdunkowski and Andreas Bott

Frontmatter

[More information](#)*Preface*

xvii

atmosphere, some stability theory, and early weather-prediction models are introduced in Chapters 14–17. Lagrange's and Hamilton's treatments of the equation of motion are discussed in Chapter 18.

The following chapters consider flow fields in various coordinate systems. In Chapters 19 and 20 we give a fairly detailed account of the air motion described with the help of the geographic and the stereographic coordinate systems. This description and the following topics are of great importance for numerical weather prediction. In order to study the airflow over irregular terrain, the orography-following coordinate system is introduced in Chapter 21. The air motion in stereographic coordinate systems with a generalized vertical coordinate is discussed in Chapter 22.

Some earlier baroclinic weather-prediction models employed the so-called quasi-geostrophic theory which is discussed in some detail in Chapters 23 and 24. Modern numerical weather prediction, however, is based on the numerical solutions of the primitive equations, i.e. the scale-analyzed original equations describing the flow field. Nevertheless, the quasi-geostrophic theory is still of great value in discussing some major features of atmospheric motion. We will employ this theory to construct weather-prediction models and we show the operational principle.

A brief and very incomplete introduction of numerical methods is given in Chapter 25 to motivate the modeling of atmospheric flow by spectral techniques. Some basic theory of the spectral method is given in Chapter 26. The final chapter of this book, Chapter 27, introduces the problems associated with atmospheric predictability. The famous Lorenz equations and the strange attractor are discussed. The method of stochastic dynamic prediction is introduced briefly.

Problems of various degrees of difficulty are given at the end of most chapters. The almost trivial problems were included to provide the opportunity for the student to become familiar with the new material before he is confronted with more demanding problems. Some answers to these problems are provided at the end of the book. To a large extent these problems were given to the meteorology students of the University of Mainz in their exercise classes. We were very fortunate to be assisted by very able instructors, who conducted these classes independently. We wish to express our sincere gratitude to them. These include Drs G. Korb, R. Schrodin, J. Siebert, and T. Trautmann. It would be impossible to name all contributors to the exercise classes. Our special gratitude goes to Dr W.-G. Panhans for his splendid cooperation with the authors in organizing and conducting these classes. Whenever asked, he also taught some courses to lighten the burden.

It seems to be one of the unfortunate facts of life that no book as technical as this one can be published free of error. However, we take some comfort in the thought that any errors appearing in this book were made by the co-author. To remove these, we would be grateful to anyone pointing out to us misprints and other mistakes they have discovered.

In writing this book we have greatly profited from Professor H. Fortak, whose lecture notes were used by K. Hinkelmann and G. Korb as a guide to organize their manuscripts. We are also indebted to the late Professor G. Hollmann and to Professor F. Wippermann. Parts of their lecture notes were at our disposal.

We also wish to thank our families for their constant support and encouragement.

Finally, we express our gratitude to Cambridge University Press for their effective cooperation in preparing the publication of this book.

W. Zdunkowski
A. Bott