THE LOGIC OF CONCEPT EXPANSION

Scientists and mathematicians frequently describe the development of their field as a process that includes the expansion of concepts. Logicians traditionally deny the possibility of conceptual expansion and the coherence of this description. Meir Buzaglo's innovative study proposes a way of expanding logic to include the stretching of concepts, while modifying the principles which apparently block this possibility. He offers stimulating discussions of the idea of conceptual expansion as a normative process, and of the relation of conceptual expansion to truth, meaning, reference, ontology and paradox. He analyzes the views of Kant, Wittgenstein, Gödel, and others, paying especially close attention to Frege. His book will be of interest to a wide range of readers, from philosophers (of logic, mathematics, language, and science) to logicians, mathematicians, linguists, and cognitive scientists.

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To the memory
of my brother Raphael

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Preface

Two issues have interested me for a long time. One is Kant's perception of metaphysics as an illusion-prone area, while the other involves the intriguing way mathematicians expand their concepts. Although mathematicians may talk about the sine of a complex number, they do not try to define the sine function to apply to the moon. The connection between the two areas becomes clearer when one recalls that Kant has argued that the antinomies of reason derive from illegitimate expansions of concepts beyond their range of application (e.g., applying the categories of causality to the whole world of phenomena). It is important to note that this connection has reappeared and even intensified in contemporary thought. One cannot imagine modern mathematics and physics without the procedure of expansions of concepts, and the analyses of Russell's paradoxes by Russell, Gödel, and others are echoes of Kant's view that one cannot view certain totalities as genuine objects. A parallel approach, with certain changes, may even be attributed to Wittgenstein, whose aim in philosophy was to "bring words back home," as well as Brouwer's diagnosis that classical logic was derived from a careless expansion of logical laws that are valid for finite collections to unbounded ranges.

These developments raise an interesting question about Kant's analysis of the source of illusions, as they make it clear that modern scientists and mathematicians do not respect the boundaries within which the concepts they use were originally defined. It even seems as if they take the opposite tack, expanding whenever possible. To put it differently, while philosophical disillusionment with metaphysics seemed to demand restraint, scientists kept pushing at the borders of meaning, trying to go beyond the accepted sense of terms.

All this implies that the fact that philosophers projected their words beyond their original use does not suffice to render their efforts as mere illusion. We should therefore either improve the Kantian picture

on antinomies or reject it. For example, it is clear that the expansion of causality on the world as a whole is of a different kind than expanding the sine function to the moon; expansions in philosophy seem more natural. Could we therefore view them as expansions in science? Having formulated this question, I had to face another difficulty, for I did not have a frame of reference that would allow me to connect a phenomenon from the sciences to pure metaphysics. By this I do not mean that I could not generalize this process and suggest a logic wherein one could analyze and classify expansions, but what I lacked was a philosophical standpoint that would help me to judge matters better.

At this point I discovered Frege's opposition to the whole idea of expanding concepts. Frege's rejection was grounded in the main principles of his system. These principles served as a middle term and have allowed me to connect a phenomenon from mathematics to general questions on meaning and reference, and to start a new train of thought. Thus, in contrast to Frege's position, I am convinced that the phenomenon of expansions is essential to concepts, and in the concluding chapters I show that the notion of a dichotomy between meaningful and meaningless sentences is inconclusive (although this is not directly connected with the issue of vagueness). This allows me to return to my original question on the way natural non-arbitrary expansions lead us to antinomies, and, what is even more important, to uncover a prevalent logical form that may be applicable to other areas which I delineate in the introduction to this book.

Having finished the book I realize that there are many issues I have left open. What I would most like to do is to analyze Putnam's position on conceptual change, which is a major axis of his thought, as well as the role of expansions in Quine's views, and it would be interesting to read some of Wittgenstein's writings in light of my present suggestions. The reason I do not go into these issues here is that I prefer to develop the instrument proposed here as far as possible before confronting it with more mature philosophies. From this standpoint, the present book is a preparation for a richer dialogue, as well as an introduction to a better understanding of the above-mentioned philosophers.

Acknowledgments

I would like to thank a number of teachers and friends. For my interest in Frege I am indebted to my teacher and mentor Gilead Bar-Elli. Without Frege I would not have had a worthy opponent against whom to construct my present view, and without Gilead I would not have been intrigued by Frege. (This does not mean, of course, that Gilead accepts my criticism of Frege.)

Saharon Shelah helped me prove two theorems about the logic of expansions that are presented here. Even though it was not very difficult for him (I can attest that it took him no more than five minutes), it saved me a great deal of hard work.

I began the work leading to this book at Harvard, in the company of Hilary Putnam, and I am very lucky to have benefited from his encouragement and guidance. At the Hebrew University I was fortunate in being able to study in two of its excellent departments. In the Department of Mathematics I studied logic, and I began my investigations of the logic of changes in extension under the guidance of Menachem Magidor. In the Department of Philosophy I also received much advice and encouragement. Mark Steiner encouraged me to work on the present book. Carl Posy helped me with his generous attentiveness, and thanks to him I was able to improve the presentation of my research. Yemima Ben-Menachem read drafts of my work and made helpful comments. The students in my seminars at the Hebrew University also helped me a great deal, most especially Hilly Razinsky and Gall Elster. This research was supported by the Israel Science Foundation, founded by the Israel Academy of Sciences and Humanities.

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