1

Introduction

Control systems are tightly intertwined in our daily lives so much so that we take them for granted. They may be as low tech and unglamorous as our flush toilet. Or they may be as high tech as electronic fuel injection in our cars. In fact, there is more than a handful of computer control systems in a typical car that we now drive. In everything from the engine to transmission, shock absorber, brakes, pollutant emission, temperature, and so forth, there is an embedded microprocessor controller keeping an eye out for us. The more gadgetry, the more tiny controllers pulling the trick behind our backs.[1] At the lower end of consumer electronic devices, we can bet on finding at least one embedded microcontroller.

In the processing industry, controllers play a crucial role in keeping our plants running – virtually everything from simply filling up a storage tank to complex separation processes and chemical reactors.

As an illustration, let's take a look at a bioreactor (Fig. 1.1). To find out if the bioreactor is operating properly, we monitor variables such as temperature, pH, dissolved oxygen, liquid level, feed flow rate, and the rotation speed of the impeller. In some operations, we may also measure the biomass and the concentration of a specific chemical component in the liquid or the composition of the gas effluent. In addition, we may need to monitor the foam head and make sure it does not become too high. We most likely need to monitor the steam flow and pressure during the sterilization cycles. We should note that the schematic diagram is far from complete. By the time we have added enough details to implement all the controls, we may not recognize the bioreactor. These features are not pointed out to scare anyone; on the other hand, this is what makes control such a stimulating and challenging field.

For each quantity that we want to maintain at some value, we need to ensure that the bioreactor is operating at the desired conditions. Let's use the pH as an example. In control calculations, we commonly use a *block diagram* to represent the problem (Fig. 1.2). We will learn how to use mathematics to describe each of the blocks. For now, the focus is on some common terminology.

To consider pH as a *controlled variable*, we use a pH electrode to measure its value and, with a transmitter, send the signal to a controller, which can be a little black box or a computer. The controller takes in the pH value and compares it with the desired pH, what

[1] In the 1999 Mercedes-Benz S-class sedan, there are approximately 40 "electronic control units" that control up to 170 different variables.

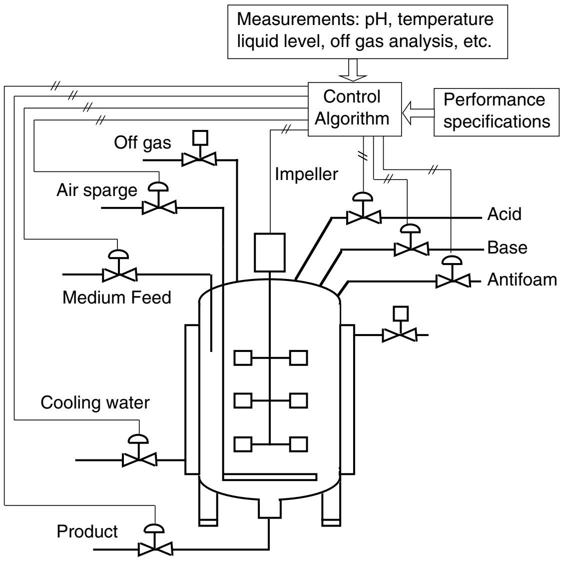

Figure 1.1. Schematic diagram of instrumentation associated with a fermentor. The steam sterilization system and all sensors and transmitters are omitted for clarity. The thick solid lines represent process streams. The thin solid lines represent information flow.

is called the *set point* or the *reference*. If the values are not the same, there is an *error*, and the controller makes proper adjustments by manipulating the acid or the base pump – the *actuator*.[2] The adjustment is based on calculations made with a *control algorithm*, also called the control law. The error is calculated at the summing point, where we take the desired pH minus the measured pH. Because of how we calculate the error, this is a *negative-feedback* mechanism.

This simple pH control example is what we call a *single-input single-output* (SISO) system; the single input is the set point and the output is the pH value.[3] This simple feedback

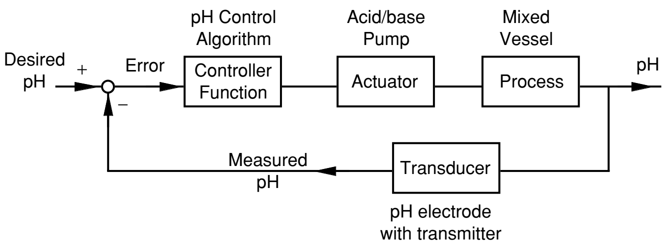

Figure 1.2. Block-diagram representation of a single-input single-output negative-feedback system. Labels within the boxes are general. Labels outside the boxes apply to the simplified pH control discussion.

[2] In real life, bioreactors actually use on–off control for pH.

[3] We will learn how to identify input and output variables, how to distinguish among manipulated variables, disturbances, measured variables, and so forth. Do not worry about remembering all the terms here; they will be introduced properly in subsequent chapters.

mechanism is also what we call a *closed loop*. This single-loop system ignores the fact that the dynamics of the bioreactor involves complex interactions among different variables. If we want to take a more comprehensive view, we need to design a *multiple-input multiple-output* (MIMO), or *multivariable*, system. When we invoke the term *system*, we are referring to the *process*[4] (the bioreactor here), the *controller*, and all other instrumentation, such as *sensors*, *transmitters*, and *actuators* (like valves and pumps) that enable us to control the pH.

When we change a specific operating condition, meaning the set point, we would like, for example, the pH of the bioreactor to follow our command. This is what we call *servocontrol*. The pH value of the bioreactor is subjected to external *disturbances* (also called *load changes*), and the task of suppressing or rejecting the effects of disturbances is called *regulatory control*. Implementation of a controller may lead to instability, and the issue of system *stability* is a major concern. The control system also has to be *robust* such that it is not overly sensitive to changes in process parameters.

What are some of the issues when we design a control system? In the first place, we need to identify the role of various variables. We need to determine what we need to control, what we need to manipulate, what the sources of disturbances are, and so forth. We then need to state our design objective and specifications. It may make a difference whether we focus on the servo or on the regulator problem, and we certainly want to make clear, quantitatively, the desired response of the system. To achieve these goals, we have to select the proper control strategy and controller. To implement the strategy, we also need to select the proper sensors, transmitters, and actuators. After all is done, we have to know how to tune the controller. Sounds like we are working with a musical instrument, but that's the jargon.

The design procedures depend heavily on the dynamic model of the process to be controlled. In more advanced model-based control systems, the action taken by the controller actually depends on the model. Under circumstances for which we do not have a precise model, we perform our analysis with approximate models. This is the basis of a field called *system identification and parameter estimation*. Physical insight that we may acquire in the act of model building is invaluable in problem solving.

Although we laud the virtue of dynamic modeling, we will not duplicate the introduction of basic conservation equations. It is important to recognize that all of the processes that we want to control, e.g., bioreactor, distillation column, flow rate in a pipe, drug delivery system, etc., are what we have learned in other engineering classes. The so-called model equations are conservation equations in heat, mass, and momentum. We need force balance in mechanical devices, and, in electrical engineering, we consider circuit analysis. The difference between what we now use in control and what we are more accustomed to is that control problems are *transient* in nature. Accordingly, we include the time-derivative (also called accumulation) term in our balance (model) equations.

What are some of the mathematical tools that we use? In *classical* control, our analysis is based on linear ordinary differential equations with constant coefficients – what is called *linear time invariant* (LTI). Our models are also called *lumped-parameter* models, meaning that variations in space or location are not considered. Time is the only independent variable. Otherwise, we would need partial differential equations in what is called *distributed-parameter* models. To handle our linear differential equations, we rely heavily

[4] In most of the control world, a process is referred to as a *plant*. Here "process" is used because, in the process industry, a plant carries the connotation of the entire manufacturing or processing facility.

Table 1.1. Examples used in different chapters

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on *Laplace transform*, and we invariably rearrange the resulting algebraic equation into the so-called *transfer functions*. These algebraic relations are presented graphically as block diagrams (as in Fig. 1.2). However, we rarely go as far as solving for the time-domain solutions. Much of our analysis is based on our understanding of the roots of the characteristic polynomial of the differential equation – what we call the *poles*.

At this point, a little secret should be disclosed. Just from the terminology, it may be inferred that control analysis involves quite a bit of mathematics, especially when we go over stability and frequency-response methods. That is one reason why these topics are not immediately introduced. Nonetheless, we have to accept the prospect of working with mathematics. It would be a lie to say that one can be good in process control without sound mathematical skills.

Starting in Chap. 6, a select set of examples is repeated in some subsections and chapters. To reinforce the thinking that different techniques can be used to solve the same problem, these examples retain the same numeric labeling. These examples, which do not follow conventional numbering, are listed in Table 1.1 to help you find them.

It may be useful to point out a few topics that go beyond a first course in control. With certain processes, we cannot take data continuously, but rather in certain selected slow intervals (e.g., titration in freshmen chemistry). These are called *sampled-data* systems. With computers, the analysis evolves into a new area of its own – *discrete-time* or *digital* control systems. Here, differential equations and Laplace transform do not work anymore. The mathematical techniques to handle discrete-time systems are difference equations and *z transforms*. Furthermore, there are *multivariable* and *state-space* controls, which we will encounter in a brief introduction. Beyond the introductory level are optimal control, nonlinear control, adaptive control, stochastic control, and fuzzy-logic control. Do not lose the perspective that control is an immense field. Classical control appears insignificant, but we have to start somewhere, and onward we crawl.

2

Mathematical Preliminaries

Classical process control builds on linear ordinary differential equations (ODEs) and the technique of the Laplace transform. This is a topic that we no doubt have come across in an introductory course on differential equations – like two years ago? Yes, we easily have forgotten the details. Therefore an attempt is made here to refresh the material necessary to solve control problems; other details and steps will be skipped. We can always refer back to our old textbook if we want to answer long-forgotten but not urgent questions.

What Are We Up to?

- The properties of Laplace transform and the transforms of some common functions. We need them to construct a table for doing an **inverse transform**.
- Because we are doing an inverse transform by means of a look-up table, we need to break down any given transfer functions into smaller parts that match what the table has – what are called **partial fractions**. The time-domain function is the sum of the inverse transform of the individual terms, making use of the fact that Laplace transform is a linear operator.
- The time-response characteristics of a model can be inferred from the poles, i.e., the roots of the characteristic polynomial. This observation is independent of the input function and singularly the most important point that we must master before moving onto control analysis.
- After a Laplace transform, a differential equation of deviation variables can be thought of as an input–output model with transfer functions. The causal relationship of changes can be represented by block diagrams.
- In addition to transfer functions, we make extensive use of steady-state gain and time constants in our analysis.
- Laplace transform is applicable to only *linear* systems. Hence we have to **linearize** nonlinear equations before we can go on. The procedure of linearization is based on a first-order Taylor series expansion.

2.1. A Simple Differential Equation Model

First an impetus is provided for solving differential equations in an approach unique to control analysis. The mass balance of a well-mixed tank can be written (see Review Problems) as

$$\tau \frac{\mathrm{d}C}{\mathrm{d}t} = C_{\text{in}} - C, \quad \text{with } C(0) = C_0,$$

where C is the concentration of a component, C_{in} is the inlet concentration, C_0 is the initial concentration, and τ is the space time. In classical control problems, we invariably rearrange the equation as

$$\tau \frac{\mathrm{d}C}{\mathrm{d}t} + C = C_{\text{in}} \tag{2.1}$$

and further redefine variables $C' = C - C_0$ and $C'_{\text{in}} = C_{\text{in}} - C_0$.[1] We designate C' and C'_{in} as **deviation variables** – they denote how a quantity deviates from the original value at $t = 0$.[2] Because C_0 is a constant, we can rewrite Eq. (2.1) as

$$\tau \frac{\mathrm{d}C'}{\mathrm{d}t} + C' = C'_{\text{in}}, \quad \text{with } C'(0) = 0. \tag{2.2}$$

Note that the equation now has a zero initial condition. For reference, the solution to Eq. (2.2) is[3]

$$C'(t) = \frac{1}{\tau} \int_0^t C'_{\text{in}}(z) e^{-(t-z)/\tau} \mathrm{d}z. \tag{2.3}$$

If C'_{in} is zero, we have the trivial solution $C' = 0$. It is obvious from Eq. (2.2) immediately. For a more interesting situation in which C' is nonzero or for C to deviate from the initial C_0, C'_{in} must be nonzero, or in other words, C_{in} is different from C_0. In the terminology of differential equations, the right-hand side (RHS) C'_{in} is called the **forcing function**. In control, it is called the *input*. Not only is C'_{in} nonzero, it is, under most circumstances, a function of time as well, $C'_{\text{in}} = C'_{\text{in}}(t)$.

In addition, the time dependence of the solution, meaning the exponential function, arises from the left-hand side (LHS) of Eq. (2.2), the linear differential operator. In fact, we may recall that the LHS of Eq. (2.2) gives rise to the so-called characteristic equation (or characteristic polynomial).

Do not worry if you have forgotten the significance of the characteristic equation. We will come back to this issue again and again. This example is used just as a prologue. Typically in a class on differential equations, we learn to transform a *linear* ordinary equation into

[1] At steady state, $0 = C^s_{\text{in}} - C^s$, and if $C^s_{\text{in}} = C_0$, we can also define $C'_{\text{in}} = C_{\text{in}} - C^s_{\text{in}}$. We will come back to this when we learn to linearize equations. We will see that we should choose $C_0 = C^s$.

[2] Deviation variables are analogous to *perturbation variables* used in chemical kinetics or in fluid mechanics (linear hydrodynamic stability). We can consider a deviation variable as a measure of how far it is from steady state.

[3] When you come across the term convolution integral later in Eq. (4.10) and wonder how it may come about, take a look at the form of Eq. (2.3) again and think about it. If you wonder where Eq. (2.3) comes from, review your old ODE text on integrating factors. We skip this detail as we will not be using the time-domain solution in Eq. (2.3).

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an *algebraic* equation in the *Laplace domain*, solve for the transformed dependent variable, and finally get back the *time-domain* solution with an inverse transformation.

In classical control theory, we make extensive use of a Laplace transform to analyze the dynamics of a system. The key point (and at this moment the trick) is that we will try to predict the time response *without* doing the inverse transformation. Later, we will see that the answer lies in the roots of the characteristic equation. This is the basis of classical control analyses. Hence, in going through Laplace transform again, it is not so much that we need a remedial course. Our old differential equation textbook would do fine. The key task here is to pitch this mathematical technique in light that may help us to apply it to control problems.

2.2. Laplace Transform

Let us first state a few important points about the application of Laplace transform in solving differential equations (Fig. 2.1). After we have formulated a model in terms of a *linear* or a *linearized* differential equation, $dy/dt = f(y)$, we can solve for $y(t)$. Alternatively, we can transform the equation into an algebraic problem as represented by the function $G(s)$ in the Laplace domain and solve for $Y(s)$. The time-domain solution $y(t)$ can be obtained with an inverse transform, but we rarely do so in control analysis.

What we argue (of course it is true) is that the Laplace-domain function $Y(s)$ must contain the same information as $y(t)$. Likewise, the function $G(s)$ must contain the same dynamic information as the original differential equation. We will see that the function $G(s)$ can be "clean looking" if the differential equation has zero initial conditions. That is one of the reasons why we always pitch a control problem in terms of deviation variables.[4] We can now introduce the definition.

The **Laplace transform** of a function $f(t)$ is defined as

$$\mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}\,dt, \tag{2.4}$$

where s is the transform variable.[5] To complete our definition, we have the inverse transform,

$$f(t) = \mathcal{L}^{-1}[F(s)] = \frac{1}{2\pi j}\int_{\gamma - j\infty}^{\gamma + j\infty} F(s)e^{st}\,ds, \tag{2.5}$$

where γ is chosen such that the infinite integral can converge.[6] Do not be intimidated by

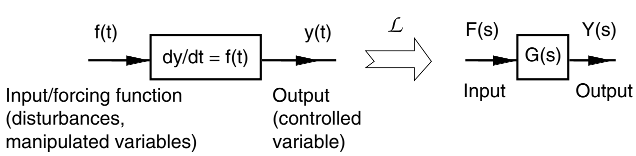

Figure 2.1. Relationship between time domain and Laplace domain.

[4] But! What we measure in an experiment is the "real" variable. We have to be careful when we solve a problem that provides real data.

[5] There are many acceptable notations for a Laplace transform. Here we use a capital letter, and, if confusion may arise, we further add (s) explicitly to the notation.

[6] If you insist on knowing the details, they can be found on the *Web Support*.

Eq. (2.5). In a control class, we never use the inverse transform definition. Our approach is quite simple. We construct a table of the Laplace transform of some common functions, and we use it to do the inverse transform by means of a look-up table.

An important property of the Laplace transform is that it is a **linear operator**, and the contribution of individual terms can simply be added together (superimposed):

$$\mathcal{L}[af_1(t) + bf_2(t)] = a\mathcal{L}[f_1(t)] + b\mathcal{L}[f_2(t)] = aF_1(s) + bF_2(s). \tag{2.6}$$

Note: The linear property is one very important reason why we can do partial fractions and inverse transforms by means of a look-up table. This is also how we analyze more complex, but linearized, systems. Even though a text may not state this property explicitly, we rely heavily on it in classical control.

We now review the Laplace transforms of some common functions – mainly the ones that we come across frequently in control problems. We do not need to know all possibilities. We can consult a handbook or a mathematics textbook if the need arises. (A summary of the important transforms is in Table 2.1.) Generally, it helps a great deal if you can do the following common ones without having to use a look-up table. The same applies to simple algebra, such as partial fractions, and calculus, such as linearizing a function.

(1) A **constant:**

$$f(t) = a, \qquad F(s) = (a/s). \tag{2.7}$$

The derivation is

$$\mathcal{L}[a] = a\int_0^\infty e^{-st}\,\mathrm{d}t = -\frac{a}{s}e^{-st}\Big|_0^\infty = a\left(0 + \frac{1}{s}\right) = \frac{a}{s}.$$

(2) An **exponential function** (Fig. 2.2):

$$f(t) = e^{-at}, \quad \text{with } a > 0, \qquad F(s) = [1/(s+a)], \tag{2.8}$$

$$\mathcal{L}[e^{-at}] = a\int_0^\infty e^{-at}e^{-st}\,\mathrm{d}t = \frac{-1}{(s+a)}e^{-(a+s)t}\Big|_0^\infty = \frac{1}{(s+a)}.$$

(3) A **ramp function** (Fig. 2.2):

$$f(t) = at \quad \text{for } t \geq 0 \text{ and } a = \text{constant}, \qquad F(s) = (a/s^2), \tag{2.9}$$

$$\mathcal{L}[at] = a\int_0^\infty t\,e^{-st}\,\mathrm{d}t = a\left(-t\frac{1}{s}e^{-st}\Big|_0^\infty + \int_0^\infty \frac{1}{s}e^{-st}\,\mathrm{d}t\right)$$
$$= \frac{a}{s}\int_0^\infty e^{-st}\,\mathrm{d}t = \frac{a}{s^2}.$$

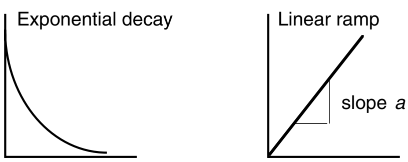

Figure 2.2. Illustration of exponential and ramp functions.

Table 2.1. Summary of a handful of common Laplace transforms

Function	$F(s)$	$f(t)$
The very basic functions	a/s	a or $au(t)$
	a/s^2	at
	$1/(s+a)$	e^{-at}
	$\omega/(s^2+\omega^2)$	$\sin \omega t$
	$s/(s^2+\omega^2)$	$\cos \omega t$
	$\omega/[(s+a)^2+\omega^2]$	$e^{-at} \sin \omega t$
	$(s+a)/[(s+a)^2+\omega^2]$	$e^{-at} \cos \omega t$
	$s^2F(s) - sf(0) - f'(0)$	$\dfrac{d^2 f}{dt^2}$
	$\dfrac{F(s)}{s}$	$\int_0^t f(t)\,dt$
	$e^{-st_0}F(s)$	$f(t-t_0)$
	A	$A\delta(t)$
Transfer functions in time-constant form	$1/(\tau s+1)$	$(1/\tau)e^{-t/\tau}$
	$\dfrac{1}{(\tau s+1)^n}$	$\dfrac{1}{\tau^n(n-1)!}t^{n-1}e^{-t/\tau}$
	$1/[s(\tau s+1)]$	$1-e^{-t/\tau}$
	$1/[(\tau_1 s+1)(\tau_2 s+1)]$	$\left(e^{-t/\tau_1}-e^{-t/\tau_2}\right)/\tau_1-\tau_2$
	$\dfrac{1}{s(\tau_1 s+1)(\tau_2 s+1)}$	$1+\dfrac{\tau_1 e^{-t/\tau_1}-\tau_2 e^{-t/\tau_2}}{\tau_2-\tau_1}$
	$\dfrac{(\tau_3 s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$	$\dfrac{1}{\tau_1}\dfrac{\tau_1-\tau_3}{\tau_1-\tau_2}e^{-t/\tau_1}+\dfrac{1}{\tau_2}\dfrac{\tau_2-\tau_3}{\tau_2-\tau_1}e^{-t/\tau_2}$
	$\dfrac{(\tau_3 s+1)}{s(\tau_1 s+1)(\tau_2 s+1)}$	$1+\dfrac{\tau_3-\tau_1}{\tau_1-\tau_2}e^{-t/\tau_1}+\dfrac{\tau_3-\tau_2}{\tau_2-\tau_1}e^{-t/\tau_2}$
Transfer functions in pole-zero form	$1/(s+a)$	e^{-at}
	$1/[(s+a)^2]$	$t\,e^{-at}$
	$\dfrac{1}{(s+a)^n}$	$\dfrac{1}{(n-1)!}t^{n-1}e^{-at}$
	$1/[s(s+a)]$	$(1/a)\,(1-e^{-at})$
	$1/[(s+a)(s+b)]$	$[1/(b-a)](e^{-at}-e^{-bt})$
	$s/[(s+a)^2]$	$(1-at)\,e^{-at}$
	$s/[(s+a)(s+b)]$	$[1/(b-a)]\,(be^{-bt}-ae^{-at})$
	$\dfrac{1}{s(s+a)(s+b)}$	$\dfrac{1}{ab}\left[1+\dfrac{1}{a-b}(be^{-at}-ae^{-bt})\right]$

Note: We may find many more Laplace transforms in handbooks or texts, but here we stay with the most basic ones. The more complex ones may actually be a distraction to our objective, which is to understand pole positions.

(4) **Sinusoidal functions**

$$f(t) = \sin \omega t, \qquad F(s) = [\omega/(s^2+\omega^2)], \tag{2.10}$$

$$f(t) = \cos \omega t, \qquad F(s) = [s/(s^2+\omega^2)]. \tag{2.11}$$