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978-0-521-80717-3 - Industrial Mathematics: Case Studies in the Diffusion of Heat and Matter

Glenn R. Fulford and Philip Broadbridge

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Industrial Mathematics

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“Dedicated to my niece, Elise Champion”

G.F.

“Dedicated to Matthew and Daniel, the two fine sons of Phil and Alice”

P.B.

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Preface

At a pragmatic level, there are often no alternatives to mathematical models to test new industrial designs. Physical prototypes may be too expensive or too time consuming. In both the design phase and the operations phase, the direct measurement of important operational variables may be impossible or uneconomic. The regions of interest may be inaccessible because of mechanical barriers, high temperatures or hazardous chemical environments.

Mathematical modelling is an efficient and relatively inexpensive device for testing the effect of changing operating conditions in an industrial process. It is far easier and less costly to change a small number of parameters in a mathematical model than to shut down an industrial plant and modify its large-scale equipment. In most circumstances this should not be done as a trial-and-error experiment.

In this era of rapidly changing technology, the efficacy of mathematical modelling in industry should be appreciated more than ever before. Therefore, it is frustrating to note a recent trend towards reducing the amount of core mathematics subjects in engineering degree programmes. It is our expectation that this book could at least be used for an optional course for the more mathematically-oriented students of engineering. Such a course is even more important in the education of those with an interest in the logical design of new technology when the majority of graduates have insufficient mathematical training for this purpose. This course would meet one of the needs of a modern industrial mathematics course, namely that of relevance to real industrial problems. This would alleviate a common criticism by recent engineering students that

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little attempt is made to relate mathematics service courses to their professional practice.

On the other hand, real-world mathematical modelling courses are good preparation for mathematics students who hope to work in the industrial environment. In too many mathematics degree programmes, applied mathematics courses are presented solely as mathematical methods courses, i.e. solution techniques for various types of equations and optimisation problems. In typical methods courses, artificial applications are often added almost as an afterthought following long sessions of formal theoretical development. Like most mathematicians, we can understand the power and beauty of mathematical rigour in the development of useful mathematical methods. However, without some practice in real-world mathematical modelling, mathematics graduates do not have the best preparation for work in industry.

In the industrial context, mathematicians are not presented with a set of equations ready to be analysed. Instead they are confronted with a set of practical problems that have not yet been expressed in mathematical terms. This translation to mathematical terms is a difficult step if one has had no experience at this activity. From our dealings with many scientists, engineers and mathematicians, we have found that the most proficient mathematical modellers are the experienced modellers, who have learnt to listen to specialists from other non-mathematical backgrounds. However, we still believe that good mathematical modelling skills are based on fundamental principles that can be taught in a course based on case studies.

Good mathematical models must respect accepted scientific laws such as physical conservation laws. Insight can be gained by idealising a model so that only the most important factors, processes and parameters are retained. This is also an aim of experimental control. Secondary effects may be added later as perturbations on the leading terms. Mathematical predictions of physical processes (and perhaps of economic and behavioural activity) should ultimately be expressible in terms of dimensionless variables and the key factors must be expressible in terms of dimensionless parameters.

This text represents a course that has undergone many modifications since 1986 after the authors have presented it to third year mathematics students at three Australian universities; La Trobe University, The Aus-

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tralian Defence Force Academy (University College of The University of New South Wales) and The University of Wollongong.

We will comment on some features of the course. First, industrial case studies are presented at the outset rather than as add-on examples. These case studies lead to mathematical models that motivate development and reinforcement of mathematical methods. It is our experience that the students understand the mathematical methods better after they have applied them to case studies. The level of mathematics used here is not advanced; some of it will already have been encountered at second year level. Assumed mathematical techniques include exact solution methods for constant-coefficient ordinary differential equations, systems of linear algebraic equations, graphical solution of nonlinear transcendental equations.

Other mathematical techniques, that many students may not have seen at second year level, are introduced in a rudimentary way. These include:

- free boundary value problems for partial differential equations (Chapter 2);
- Stretching transformations (Chapter 3);
- perturbation expansions (Chapter 4);
- bifurcation analysis (Chapter 5);
- Fourier series and nonlinear transformations of nonlinear partial differential equations (Chapter 6).

These chapters, containing the case studies, are self-contained and may be studied in any order (after Chapter 1) to suit the backgrounds of the students. However, the techniques introduced in Chapter 3 are an extension of those introduced in Chapter 2, so Chapter 3 should ideally be studied after Chapter 2.

Given the necessity of placing no more than reasonable demands on students, it has been possible to concentrate on only one illustrative area of activity in industrial modelling. We have chosen this area to be continuum modelling involving diffusion and heat conduction. Of course it would be equally possible to choose another area such as queueing theory, operations research, number theory or coding theory. In order to make our course accessible to mathematics students with very little training in physics, we have had to make brief oversimplified accounts of

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the underlying physical processes. Also, we have not done justice to the techniques of numerical analysis that are useful tools in mathematical modelling. Almost all of our students have recently or concurrently taken courses in numerical methods and we have referenced these in our lecture classes.

As evidence that this kind of applications-oriented course is uncommon, many of our students have stated that this course is unlike any other. However, after taking this course, many have felt a greater degree of confidence in their mathematical skills and in applying them to industrial problems.

We wish to thank our colleagues who have given useful suggestions for the development and improvement of this course. Among them are Yvonne Stokes, David Clements, Kerry Landman, Stephen Bedding, Timothy Marchant, Edgar Smith, Rodney Weber and Havinder Sidhu. We would also like to thank Bradley Loh and Lance Miller for their assistance with proof reading the manuscript. We would also like to thank our editor, Roger Astley.

We have presented this course to several hundred students. Many of these have inspired improvements through their feedback. Their professional development has been a source of great pleasure to us.

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